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Performances Comparison Between Ultra-Local Model Control, Integral Sliding Mode Control and PID Control for A Coupled Tanks System

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Abstract: This paper deals with the comparison of robust control approaches for the level water control of coupled tanks system. A new ultra-local model control (ULMC) approach leading to adaptive controller is proposed. The parameter identification of the ultra-local model is based on the algebraic derivation techniques. The main advantages of this control strategy are its simplicity and robustness. A comparison study with the integral sliding mode control (ISMC) approach is carried out. The perfect knowledge of the output variable degree, which is a standard assumption for sliding modes, is assumed here. The comparison of the simulation results for the proposed adaptive controller with the ISMC controller and the classical PID controller has a better performances in the presence of external perturbations and parameter uncertainties.

Keywords: Ultra-local model control, Adaptive PID controller, Integral sliding mode control, Robustness, Coupled tanks system.

1 Introduction

The liquid flow control of a single or multiple tanks has been widely investigated via model-based techniques such as the sliding mode control (SMC) (20), (21) which is a type of variable structure control where the dynamics of a nonlinear system is changed by switching discontinuously on time on a predetermined sliding surface with a high speed. The excellent

robustness properties of sliding mode control with respect to perturbations and uncertainties explain its great popularity (see, e.g., (6), (17), (39)).

We mention that the sliding mode controllers had led to a huge number of exciting applications involving real systems as in (20), (23), (24), (40). The main drawback of sliding mode control is the chattering effect which can excite undesirable high frequency dynamics. Different methods of chattering reduction have been reported. The integral sliding mode control (ISMC) with the boundary layer (34), (38), is one of these methods. The basic idea of ISMC is to include an integral term in the sliding surface such that the system trajectories start on the sliding surface from the initial time instant.

Most of the existing control approaches requires to have a "good" model, *i.e.*, a model combining simplicity and exactness. The identification of physical parameters involved in such models is nevertheless a difficult task. Both integral sliding mode control and ultra-local model control aim to bypass this last step. The ultra-local model control is a recently introduced approach by M. Fliess and C. Join (7), (8), (9), (11), which does not necessitate any mathematical modeling. The unknown dynamics are approximated on a very small time interval by a very simple model which is continuously updated with the aid of online estimation techniques (10), (13), (14), (25), (29). The loop is closed thanks to an adaptive PID (α -PID) controller, which provides the feedforward compensation and is easily tuned. This control approach have already been successfully applied in several case-studies for different fields (1), (11), (12), (18), (19), (22), (27), (35), (36).

The algebraic derivation method developed in (7) is restricted by the estimation of a single parameter, and the second parameter is considered constant and imposed by the practitioner. In practice, this standpoint is a first point that renders a delicate choice for the adaptive PID control strategy. Therefore, it is more judicious to estimate both parameters of ultra-local model. The main contribution of this work is to design a new adaptive PID controller able to estimate the two ultra-local model parameters. A large literature exists on the algebraic estimation of derivatives for noisy signals. However, a new approach based on the algebraic derivation has been implemented in this paper in order to increase the control performances.

The strong industrial ubiquity of classical PID controllers (4), and the great difficulty for tuning them in complex situations is deduced, via an elementary sampling from their connections with α -PID (26), (37). A comparison between the proposed ultra-local model, the integral sliding mode control and the classical PID control is developed. This comparison is kept here in order to clarify the performance improvement and effectiveness of the proposed controller design. In this paper, the control approaches are applied To control the water level of a coupled tanks system which is considered as a nonlinear system of first-order. This implementation is carried out to test the robustness performances with respect to noises, external disturbances and parameter uncertainties.

This paper is organized as follows. In section 2, the dynamic model of the coupled tanks system is obtained. In section 3, the integral sliding mode control of considered system is developed. Section 4 presents the proposed ultra-local model control approach and the corresponding adaptive PID controller. The simulation results are presented in section 5. Finally, concluding remarks are given in section 6.

2 Dynamic model of coupled tanks system

The system, shown in the figure 1, consists of two identical tanks coupled by an orifice. The input is supplied by a variable speed pump which supplies water to the first tank. The orifice

allows the water to flow into the second tank and hence out to a reservoir. The objective of the control problem is to adjust the inlet flow rate $q(t)$ so as to maintain the level in the second tank, $h_2(t)$ close to a desired level water, $h_2^d(t)$. The nonlinear model of the considered system is written as follows:

$$\begin{aligned} S\dot{h}_1(t) &= q(t) - q_1(t) \\ S\dot{h}_2(t) &= q_1(t) - q_2(t) \end{aligned} \quad (1)$$

where:

$$\begin{aligned} q_1(t) &= c_{12}\sqrt{2g(h_1(t) - h_2(t))} \quad \text{for } h_1 \geq h_2 \\ q_2(t) &= c_2\sqrt{2gh_2(t)} \quad \text{for } h_2 > 0 \end{aligned} \quad (2)$$

and:

$h_1(t)$: the level in the first tank;

$h_2(t)$: the level in the second tank;

$q(t)$: the inlet flow rate;

$q_1(t)$: the flow rate from tank 1 to tank 2;

$q_2(t)$: the flow rate out of tank 2;

g : the gravitational constant;

S : the cross-section area of tank 1 and tank 2;

c_{12} : the area of coupling orifice;

c_2 : the area of the outlet orifice;

The fluid flow rate $q(t)$ cannot be negative because the pump can only pump water into the first tank. Therefore, the constraint on the inflow rate is given by:

$$q(t) \geq 0 \quad (3)$$

Now the governing dynamical equations of the coupled tanks system can be written as follows (2):

$$\begin{aligned} \dot{h}_1(t) &= -\frac{c_{12}}{S}\sqrt{2g|h_1(t) - h_2(t)|}\operatorname{sgn}(h_1(t) - h_2(t)) + \frac{q(t)}{S} \\ \dot{h}_2(t) &= \frac{c_{12}}{S}\sqrt{2g|h_1(t) - h_2(t)|}\operatorname{sgn}(h_1(t) - h_2(t)) - \frac{c_2}{S}\sqrt{2gh_2(t)} \end{aligned} \quad (4)$$

At equilibrium, for constant desired water level, the derivatives must be zero, i.e., $\dot{h}_1 = \dot{h}_2 = 0$. Thus, at equilibrium, the following algebraic equations must hold:

$$\begin{aligned} -\frac{c_{12}}{S}\sqrt{2g|h_1(t) - h_2(t)|}\operatorname{sgn}(h_1(t) - h_2(t)) + \frac{q(t)}{S} &= 0 \\ \frac{c_{12}}{S}\sqrt{2g|h_1(t) - h_2(t)|}\operatorname{sgn}(h_1(t) - h_2(t)) - \frac{c_2}{S}\sqrt{2gh_2(t)} &= 0 \end{aligned} \quad (5)$$

From the equation (5), and to satisfy the constraint in equation (3) on the input flow rate, we should have $\operatorname{sgn}(h_1(t) - h_2(t)) \geq 0$, which implies:

$$h_1(t) \geq h_2(t) \quad (6)$$

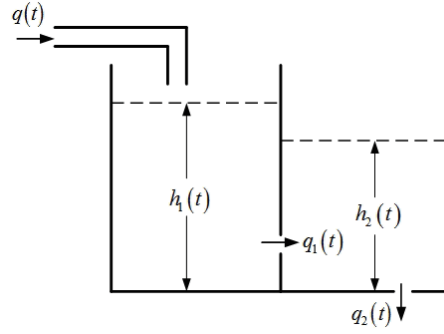


Figure 1 Schematic of the coupled tanks system.

Therefore, in order to satisfy the constraint in equation (3) on the input inflow rate for given values of system parameter c_{12} and c_2 , the water levels in the tanks must satisfy the constraint in equation (6). In addition, for the case when $h_1(t) = h_2(t)$, the system model is decoupled.

Let: $z_1(t) = h_2(t) > 0$, $z_2(t) = h_1(t) - h_2(t) > 0$, $u(t) = q(t)$

and $k_1 = \frac{c_2\sqrt{2g}}{S}$, $k_2 = \frac{c_{12}\sqrt{2g}}{S}$

Due to the nonlinearity of dynamic model of the coupled tanks system, we will define a transformation so that the dynamic model given in equation (4) can be transformed into a form which facilitates the control design.

Let $x(t) = [x_1(t) \ x_2(t)]^T$ defined as follows:

$$\begin{aligned} x_1(t) &= z_1(t) \\ x_2(t) &= -k_1\sqrt{z_1(t)} + k_2\sqrt{z_2(t)} \end{aligned} \quad (7)$$

Hence, the dynamic model of the system can be written in a compact form as:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= f(t) + \phi(t)u(t) \\ y(t) &= x_1(t) \end{aligned} \quad (8)$$

where:

$$\begin{aligned} f(t) &= \frac{k_1k_2}{2} \left(\frac{\sqrt{z_1(t)}}{\sqrt{z_2(t)}} - \frac{\sqrt{z_2(t)}}{\sqrt{z_1(t)}} \right) + \frac{k_1^2}{2} - k_2^2 \\ \phi(t) &= \frac{k_2}{2S} \frac{1}{\sqrt{z_2(t)}} \end{aligned}$$

The dynamic model in equation (8) will be used to design robust control schemes for the coupled tanks system.

3 Integral sliding mode control (ISMC)

3.1 Basic idea

The integral sliding mode control (ISMC) theory is adopted to design the controllers because of its robustness. Consider a nonlinear system which can be represented by the following state space model:

$$\begin{aligned}\dot{x}(t) &= f(x(t), t) + \phi(x(t), t) u(t) \\ y(t) &= x(t)\end{aligned}\tag{9}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $f(x(t), t)$ and $\phi(x(t), t)$ are nonlinear functions and $u(t) \in \mathbb{R}$ is the control input.

The design of ISMC involves two steps such that the first one is to select the switching hyperplane to prescribe the desired dynamic characteristics of the controlled system. The second one is to design the discontinuous control such that the system enters the integral sliding mode $s(t) = 0$ and remains in it forever (33).

In this paper, we use the sliding surface proposed by J.J. Slotine, and defined as follows:

$$s(t) = \left(\frac{d}{dt} + \lambda \right)^n \int_0^t e(t) dt\tag{10}$$

In which $e(t) = x(t) - x^d(t)$, λ is a positive coefficient, and n is the system order. Consider a Lyapunov function:

$$\begin{aligned}V &= \frac{1}{2} s^2 \\ \dot{V} &= s \dot{s}\end{aligned}\tag{11}$$

The simplified first order problem of keeping the scalar $s(x, t)$ at zero can be achieved by choosing the control law $u(t)$. A sufficient condition for the stability of the system is written as:

$$\frac{1}{2} \frac{d}{dt} s^2 \leq -\eta |s|\tag{12}$$

where η is a positive constant. The equation (12) is called reaching condition or sliding condition. $s(x, t)$ verifying (12) is referred to as sliding surface, and the system's behavior once on the surface is called sliding mode.

The process of ISMC can be divided in two phases, that is, the approaching phase and the sliding phase. The integral sliding mode control law $u(t)$ consists of two terms, equivalent term $u_{eq}(t)$, and switching term $u_s(t)$.

In order to satisfy sliding conditions (12) and to despite uncertainties on the dynamic of the system, we add a discontinuous term across the surface $s(x, t) = 0$, so the integral sliding mode control law $u(t)$ has the following form:

$$\begin{aligned}u(t) &= u_{eq}(t) + u_s(t) \\ u_s(t) &= -K \operatorname{sgn}(s(t))\end{aligned}\tag{13}$$

where K is the control gain, and the signum function is defined as:

$$\text{sgn}(s) = \begin{cases} +1 & \text{if } s > 0 \\ 0 & \text{if } s = 0 \\ -1 & \text{if } s < 0 \end{cases} \quad (14)$$

3.2 Integral sliding mode control design

To reduce the chattering problem, the integral sliding mode control design for the coupled tanks system has been developed in the works (2). Let $h_2^d(t)$ be the desired output level of the system, i.e. $y^d(t) = h_2^d(t)$. The sliding surface $s(t)$ developed by Slotine and Li (33) is defined by:

$$s(t) = \left(\frac{d}{dt} + \lambda \right)^n \int_0^t \tilde{x}_1(t) dt \quad (15)$$

where $\tilde{x}_1(t) = x_1(t) - h_2^d(t)$ is the tracking error and n is the order of system. The sliding surface can be written as:

$$\begin{aligned} s(t) &= \dot{x}_1(t) + 2\lambda(x_1(t) - h_2^d(t)) + \lambda^2 \int_0^t (x_1(t) - h_2^d(t)) dt \\ &= -k_1\sqrt{z_1(t)} + k_2\sqrt{z_2(t)} + 2\lambda(x_1(t) - h_2^d(t)) \\ &\quad + \lambda^2 \int_0^t (x_1(t) - h_2^d(t)) dt \end{aligned} \quad (16)$$

Taking the derivative of the equation (16) with respect to time, we get:

$$\begin{aligned} \dot{s}(t) &= \frac{k_1 k_2}{2} \left(\frac{\sqrt{z_1(t)}}{\sqrt{z_2(t)}} - \frac{\sqrt{z_2(t)}}{\sqrt{z_1(t)}} \right) + \frac{k_1^2}{2} - k_2^2 \\ &\quad + \frac{k_2}{2S} \frac{1}{\sqrt{z_2(t)}} u(t) \\ &\quad + 2\lambda(-k_1\sqrt{z_1(t)} + k_2\sqrt{z_2(t)}) - \lambda^2(z_1(t) - h_2^d(t)) \end{aligned} \quad (17)$$

By taking $\dot{s}(t) = 0$, we get:

$$u_{eq}(t) = \frac{1}{\phi(t)} \begin{bmatrix} -f(t) - 2\lambda(-k_1\sqrt{z_1(t)} + k_2\sqrt{z_2(t)}) \\ -\lambda^2(z_1(t) - h_2^d(t)) \end{bmatrix} \quad (18)$$

Since $u(t) = u_{eq}(t) - K \text{sgn}(s(t))$, the integral sliding mode controller is defined by:

$$\begin{aligned} u(t) &= \frac{2S\sqrt{z_2(t)}}{k_2} \left[-\frac{k_1 k_2}{2} \left(\frac{\sqrt{z_1(t)}}{\sqrt{z_2(t)}} - \frac{\sqrt{z_2(t)}}{\sqrt{z_1(t)}} \right) - \frac{k_1^2}{2} + k_2^2 \right. \\ &\quad \left. - 2\lambda(-k_1\sqrt{z_1(t)} + k_2\sqrt{z_2(t)}) - \lambda^2(z_1(t) - h_2^d(t)) \right] \\ &\quad - K \text{sgn}(s(t)) \end{aligned} \quad (19)$$

where λ and K are strictly positive constant.
Consider the Lyapunov function:

$$\begin{aligned} V &= \frac{1}{2}s^2 \\ \dot{V} &= s \left[f + \phi u + 2\lambda (-k_1\sqrt{z_1} + k_2\sqrt{z_2}) - \lambda^2 (z_1 - h_2^d) \right] \end{aligned} \quad (20)$$

By using the equation (19), the following Lyapunov function is obtained:

$$\begin{aligned} \dot{V} &= s \left[(f - \hat{f}) - K \operatorname{sgn}(s) \right] \\ &= Fs - K|s| \quad \text{where} \quad |f - \hat{f}| \leq F \end{aligned} \quad (21)$$

with $\hat{f}$ is estimation of f . If the control gain $K = F + \eta$, then, we get:

$$\dot{V} = -\eta|s| \quad (22)$$

where η is a strictly positive constant. The equation (22) shows that the ISMC technique guarantees the asymptotic stability of the closed loop system.

In order to remove the chattering effect, we consider the boundary layer solution proposed by (34), which seeks to avoid control discontinuities and switching action in the control loop. The discontinuous control law is replaced by a saturation function which approximates the $\operatorname{sgn}(s)$ term in a boundary layer of the sliding manifold $s(t) = 0$. The saturation function is defined as:

$$\operatorname{sat}(s(t)) = \begin{cases} \operatorname{sgn}(s(t)) & \text{for } |s(t)| > \varepsilon \\ \frac{s(t)}{\varepsilon} & \text{for } |s(t)| \leq \varepsilon \end{cases} \quad (23)$$

From the equation (23), for $|s(t)| > \varepsilon$, $\operatorname{sat}(s(t)) = \operatorname{sgn}(s(t))$. However, in a small ε -vicinity of the origin, the so-called boundary layer, $\operatorname{sat}(s(t)) \neq \operatorname{sgn}(s(t))$ is continuous.

4 Ultra-local model control (ULMC)

4.1 Ultra-local model

For the sake of notational simplicity, let us restrict ourselves to single-input single-output (SISO) systems. The unknown global description is replaced by the following ultra-local model:

$$y^{(\nu)}(t) = \hat{F}(t) + \hat{\alpha}(t)u(t) \quad (24)$$

where:

- The control and output variables are respectively $u(t)$ and $y(t)$,
- The derivation order ν of $y(t)$ is generally equal to 1 or 2.

- The time-varying functions $\hat{F}(t)$ and $\hat{\alpha}(t)$ are estimated via the input and the output measurements. These functions subsume not only the unknown structure of the system, which most of the time will be nonlinear, but also of any disturbance.

Remark 1: *In all the existing concrete examples, $\nu = 1$ or 2 . In the context of ultra-local model control, the only concrete case-study where such an extension was until now needed, with $\nu = 2$, has been provided by a magnetic bearing (see (5)). This is explained by a very low friction (see (8)).*

4.2 Adaptive Controllers

Consider the case where $\nu = 2$ in (24):

$$\ddot{y}(t) = \hat{F}(t) + \hat{\alpha}(t) u(t) \quad (25)$$

The corresponding adaptive Proportional-Integral-Derivative controller, or *a*-PID, reads:

$$u(t) = \frac{-\hat{F}(t) + \ddot{y}^d(t) + K_P e(t) + K_I \int e(t) + K_D \dot{e}(t)}{\hat{\alpha}(t)} \quad (26)$$

where:

- $y^d(t)$ is the output reference trajectory, obtained according to the precepts of the flatness-based control (15), (31);
- $e(t) = y^d(t) - y(t)$ is the tracking error;
- $K_P, K_I, K_D \in \mathbb{R}$ are the usual tuning gains (3), (28).

Combining the equations (25) and (26) yields:

$$\ddot{e}(t) + K_D \dot{e}(t) + K_P e(t) + K_I \int e(t) = 0 \quad (27)$$

where $\hat{F}(t)$ and $\hat{\alpha}(t)$ do not appear anymore. the gain tuning becomes therefore quite straightforward. This is a major benefit when compared to "classic" PIDs.

Set $\nu = 1$ in the equation (24):

$$\dot{y}(t) = \hat{F}(t) + \hat{\alpha}(t) u(t) \quad (28)$$

The corresponding adaptive Proportional-Integral controller, or *a*-PI, reads:

$$u(t) = \frac{-\hat{F}(t) + \dot{y}^d(t) + K_P e(t) + K_I \int e(t)}{\hat{\alpha}(t)} \quad (29)$$

The combination of the two equations (28) and (29) gives:

$$\ddot{e}(t) + K_P \dot{e}(t) + K_I e(t) = 0 \quad (30)$$

The tracking condition is therefore easily satisfied by an appropriate choice of K_P and K_I . If $K_I = 0$ in Equation (29), we obtain the adaptive proportional controller, or *a*-P, which turns out until now to be the most useful adaptive controller:

$$u(t) = \frac{-\hat{F}(t) + \dot{y}^d(t) + K_P e(t)}{\hat{\alpha}(t)} \quad (31)$$

4.3 Parameter identification method

According to the algebraic parameter identification developed in (13), (14), where the probabilistic properties of the corrupting noises may be ignored, a simultaneous estimation of both parameters $\hat{F}(t)$ and $\hat{\alpha}(t)$ is proposed. The main idea of the linear integrated filter is to apply a successive integrations on the studied model equation (28) (see e.g. (29)). However, the integrations are carried out over a sliding window of length T .

Assume that $\hat{F}$ and $\hat{\alpha}$ are constant from $t_0 = t - T$ to t in the following ultra-local model:

$$\dot{y}(\tau) = \hat{F} + \hat{\alpha}u(\tau) \quad (32)$$

with $\tau \in [t_0, t]$. In order to conclude that the parameter identification process is independent of any initial conditions, we multiply the previous equation (32) by $(t - t_0)$ as follows:

$$(t - t_0) \dot{y}(\tau) = \hat{F}(t - t_0) + \hat{\alpha}(t - t_0)u(\tau) \quad (33)$$

We once integrate the expression (33) between t_0 and t using the integration formula by parts to get:

$$(t - t_0)y(t) - \int_{t_0}^t y(\tau) d\tau = \hat{F} \frac{(t - t_0)^2}{2} + \hat{\alpha} \int_{t_0}^t (\tau - t_0)u(\tau) d\tau \quad (34)$$

Note that in this case, we succeeded to achieve the initial conditions independence. Considering $T = t - t_0$, the first linear relation between the two parameters $\hat{F}$ and $\hat{\alpha}$ is obtained by the following equation:

$$Ty(t) - \int_{t_0}^t y(\tau) d\tau = \hat{F} \frac{T^2}{2} + \hat{\alpha} \int_{t_0}^t (\tau - t_0)u(\tau) d\tau \quad (35)$$

We integrate once more to obtain a second relation. For this, the equation (34) is rewritten, for $\mu \in [t_0, t]$:

$$(\mu - t_0)y(t) - \int_{t_0}^{\mu} y(\tau) d\tau = \hat{F} \frac{(\mu - t_0)^2}{2} + \hat{\alpha} \int_{t_0}^{\mu} (\tau - t_0)u(\tau) d\tau \quad (36)$$

The integration of (36) between t_0 and t , gives the following expression:

$$\begin{aligned} & \int_{t_0}^t (\mu - t_0)y(\mu) d\mu - \int_{t_0}^t \int_{t_0}^{\mu} y(\tau) d\tau d\mu \\ &= \hat{F} \int_{t_0}^t \frac{(\mu - t_0)^2}{2} d\mu + \hat{\alpha} \int_{t_0}^t \int_{t_0}^{\mu} (\tau - t_0)u(\tau) d\tau d\mu \end{aligned} \quad (37)$$

Taking into account the fact that:

$$\int_{t_0}^t \int_{t_0}^{\mu} f(\tau) d\tau d\mu = \int_{t_0}^t (t - \tau)f(\tau) d\tau \quad (38)$$

the following equation is obtained:

$$\begin{aligned} & \int_{t_0}^t (\tau - t_0) y(\tau) d\tau - \int_{t_0}^t (t - \tau) y(\tau) d\tau \\ &= \hat{F} \frac{(t - t_0)^3}{6} + \hat{\alpha} \int_{t_0}^t (t - \tau) (\tau - t_0) u(\tau) d\tau \end{aligned} \quad (39)$$

Since $t - t_0 = T$, the second linear relation is then obtained between $\hat{F}$ and $\hat{\alpha}$ which is defined by:

$$\begin{aligned} & \int_{t_0}^t (2\tau - t - t_0) y(\tau) d\tau \\ &= \hat{F} \frac{T^3}{6} + \hat{\alpha} \int_{t_0}^t (t - \tau) (\tau - t_0) u(\tau) d\tau \end{aligned} \quad (40)$$

From the two previous relations (35) and (40), we can generate a linear system of equations in the following matrix form:

$$\begin{aligned} & \begin{bmatrix} \frac{T^2}{2} & \int_{t_0}^t (\tau - t_0) u(\tau) d\tau \\ \frac{T^3}{6} & \int_{t_0}^t (t - \tau) (\tau - t_0) u(\tau) d\tau \end{bmatrix} \begin{bmatrix} \hat{F} \\ \hat{\alpha} \end{bmatrix} \\ &= \begin{bmatrix} Ty(t) - \int_{t_0}^t y(\tau) d\tau \\ \int_{t_0}^t (2\tau - 2t + T) y(\tau) d\tau \end{bmatrix} \end{aligned} \quad (41)$$

The unknown parameters $\hat{F}$ and $\hat{\alpha}$ are estimated by solving the linear system of equations (41). Noting:

$$\Delta(t) = \frac{T^2}{6} \int_{t_0}^t (3t - 3\tau - T) (\tau - t_0) u(\tau) d\tau \quad (42)$$

the solution of the linear system (41) is defined as follows:

$$\begin{aligned} & \begin{bmatrix} \hat{F} \\ \hat{\alpha} \end{bmatrix} = \frac{1}{\Delta(t)} \\ & \begin{bmatrix} \int_{t_0}^t (t - \tau) (\tau - t_0) u(\tau) d\tau - \int_{t_0}^t (\tau - t_0) u(\tau) d\tau \\ -\frac{T^3}{6} & \frac{T^2}{2} \\ \begin{bmatrix} Ty(t) - \int_{t_0}^t y(\tau) d\tau \\ \int_{t_0}^t (2\tau - 2t + T) y(\tau) d\tau \end{bmatrix} \end{bmatrix} \end{aligned} \quad (43)$$

The proposed ultra-local control approach will be compared to the integral sliding mode control for the considered coupled tanks system control.

5 Simulation results

The dynamic model of the system has taken from (2), in which area of the orifices $c_{12} = 0.58 \text{ cm}^2$ and $c_2 = 0.24 \text{ cm}^2$ are given. The cross-section area of tank 1 and tank 2 are found to be $S = 208.2 \text{ cm}^2$. The gravitational constant is $g = 981 \text{ cm/s}^2$. The desired trajectory $h_2^d(t)$ is generated to ensure a transition from $h_2^d(t_0) = 5 \text{ cm}$ to $h_2^d(t_f) = 10 \text{ cm}$ with $t_0 = 150 \text{ s}$ and $t_f = 500 \text{ s}$.

The controller parameters used in the case of ISMC simulations are taken to be $\lambda = 0.05$, $K = 80$ and $\varepsilon = 0.15$. For the ULMC approach, the parameters of adaptive controller are chosen $K_P = 30$ and $K_I = 10$. These gains are tuned by a placement of two poles in the functional equation (30) in order to stabilize the tracking error with good dynamics. Moreover, we have chosen the sampling time $T_e = 0.1 \text{ s}$ and the sliding window $T = 5T_e$.

To properly show the robustness of the proposed algebraic approach, a performance comparison with a classical PID controller is implemented. The PID controller parameters, $K_P = 12$, $K_I = 2$ and $K_D = 7$, are settled by applying the Cohen-Coon method.

A centered white noise with variance of 0.001 is added to the system output $h_2(t)$ in the different control approaches. At $t = 550 \text{ s}$, a level water perturbation of 0.5 cm is applied to the output measurement in order to test the robustness of our proposed approach.

The figures 2 and 3 show the simulation results for the two control approaches. Figure 2 shows that the output $h_2(t)$ converges to its desired value in about 120 s. However, the system output converges to the desired trajectory $h_2^d(t)$ in about 40 s in the case of ULMC (see the figure 3). The best performances obtained thanks to the our proposed approach are shown in the figure 3. It is clear that, with the proposed adaptive PI controller, the consequence of the level water perturbation is smaller and rejected faster than the classic PID controller and the ISMC. In the figure 3, we can observe that the tracking error of new α -PI controller converge to zero despite the severe operating conditions.

For the different approaches comparison, the system dynamics is tested in the case of parameter uncertainties. For this, the parameters k_1 and k_2 are decreased by -50% when the time $t > 300 \text{ s}$. In the simulation results shown in the figures 4 and 5, we can see that the effect of parameter uncertainties is more significant in the case of integral sliding mode control. Consequently, the numerical simulation show the superiority of the proposed control technique in terms of trajectory tracking and robustness with respect to external disturbances and parameter uncertainties.

6 Conclusion

The contribution of the paper has allowed the design of a new adaptive PID controller based on the ultra-local model concept which is easily applied for the level water control of coupled tanks system. The proposed control strategy, extended from the algebraic derivation techniques, provides an improvement in terms of robustness and trajectory tracking performances. The most important benefit of this work is the online estimation of the both ultra-local model parameters which allows to obtain a control approach more robust and effective. Due to its properties of robustness, adaptability and simplicity, the ultra-local model control provides outstanding performance with a very short time of implementation. In addition, the new adaptive PID controller is more easier to tune than the classic PID controller.

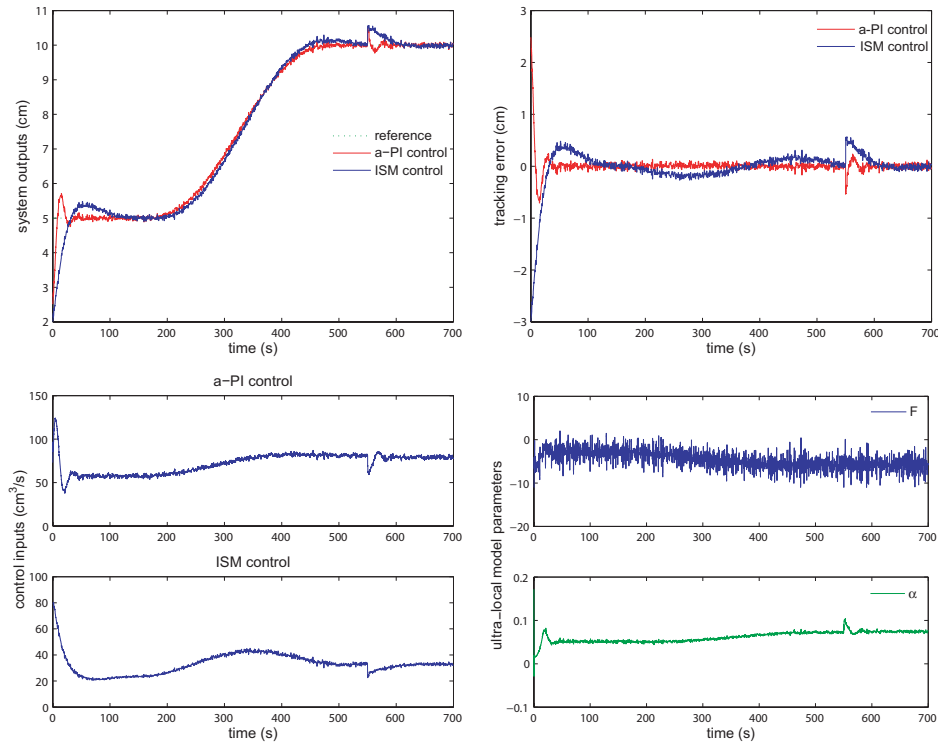


Figure 2 Simulation results of comparison between a-PI control and ISM control.

The numerical simulation results show that the proposed adaptive controller provides a better performance with respect to the integral sliding mode control approach and the classic PID control in the presence of external perturbations and uncertainties parameters. Due to its simplicity implementation, the ultra-local model control appears particularly adapted to industrial environments. It is straightforward to extend the adaptive PID control approaches to some Multiple-Input Multiple-Output systems. An interesting future works consist to compare the ultra-local model control against active disturbance rejection control (ADRC) (16) and some other existing approaches (30), (32).

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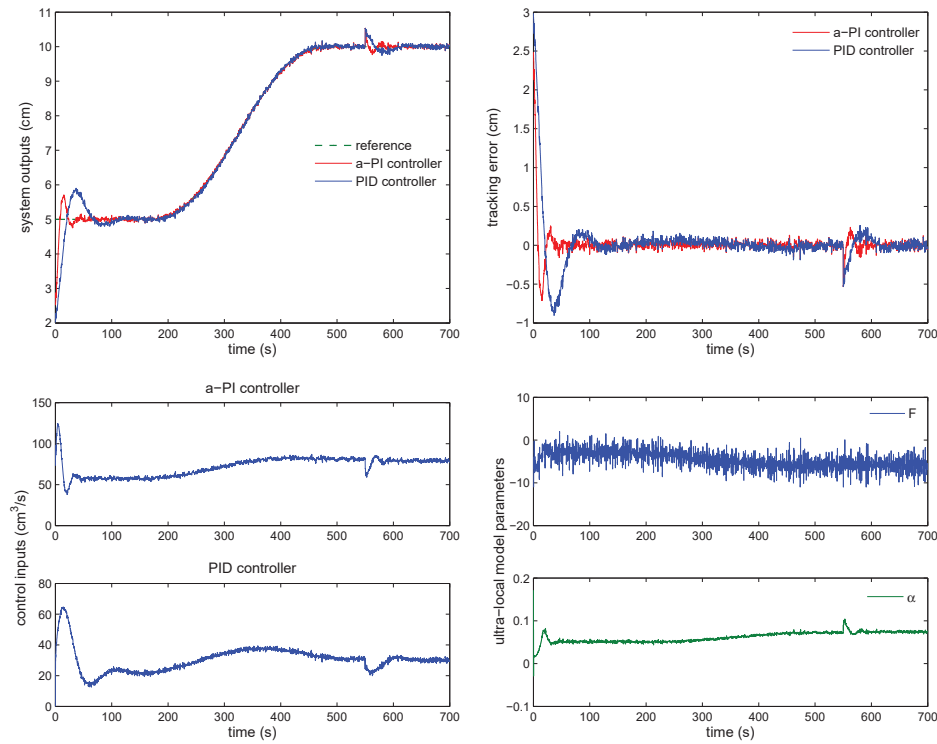


Figure 3 Simulation results of comparison between a-PI controller and PID controller.

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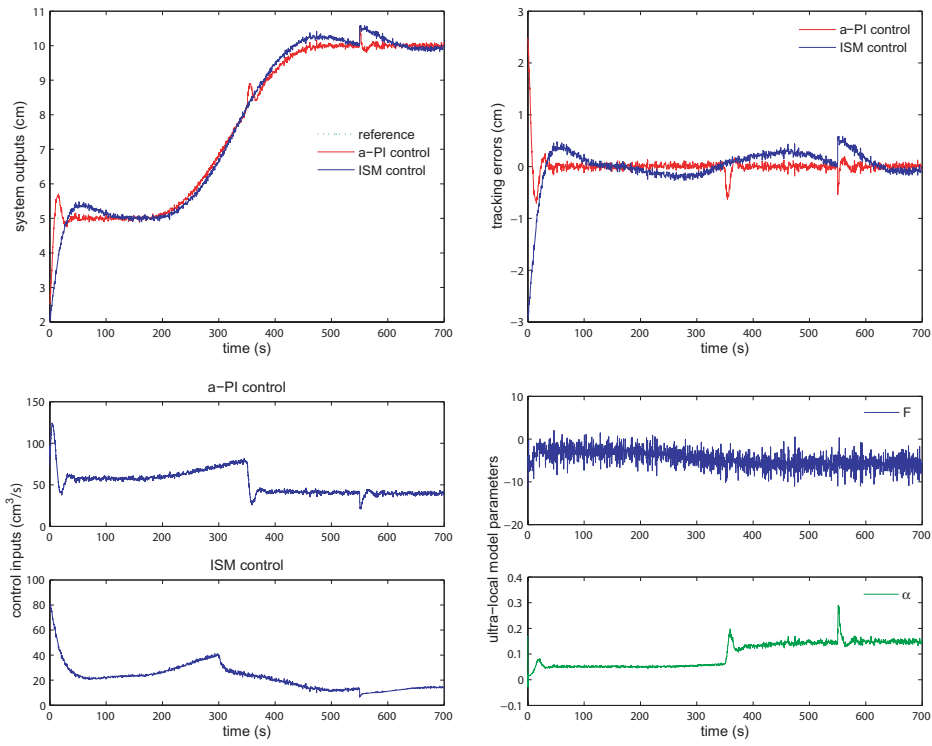


Figure 4 Simulation results of comparison between a-PI control and ISM control - parameter uncertainties by -50% .

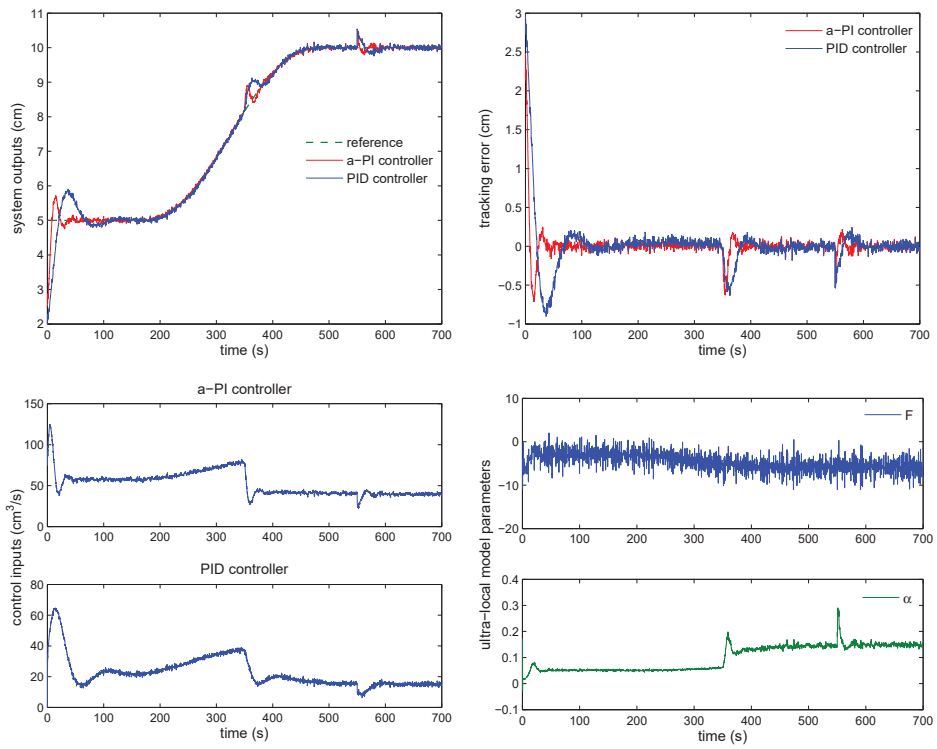


Figure 5 Simulation results of comparison between a-PI controller and PID controller - parameter uncertainties by -50% .

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