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Abstract. Phonation is a natural example of self-sustained oscillations in a nonlinear dynamical system. It results from the controlled nonlinear coupling between the deformable vocal folds and the airflow expired from the lungs through the glottis and the vocal tract. As a proof of concept, we explore the port-Hamiltonian approach to propose a minimal model of the full vocal apparatus. This port-Hamiltonian theory emphasises on the structure of a system, i.e., on the separation between the behaviour of the subsystems and their assembly, with two essential consequences : the modularity of the approach (enabling improvement in modelling of some subsystem independently of the others) and guaranteed power balance.

We here present the minimal model of the vocal apparatus, and compare the time-domain simulations based on the numerical methods designed in the port-Hamiltonian approach, to results on bifurcations obtained by means of continuation methods.

Motivations

The physics of voice production has motivated a wide variety of models, from full-featured numerical ones (mainly based on the finite element methods for vocal folds and finite volume methods for the airflow) to reduced order models focusing on the essential phenomenon underlying the phonation [1]. A large body of work in the latter category relies on the description of the glottal aerodynamics provided by van den Berg in the late 1950 [2]. This work and its extensions are based on experiments made on synthetic or moulded human larynx, i.e. considering airflow in channel with rigid static boundaries. However phonation intrinsically implies the vibration of the vocal folds periodically closing the glottis. Consequently, there is a significant paradox in those simplified models: the vocal folds move as a consequence of the power exchanged with the glottal flow, but the description of the latter assumes that it does not receive or provide any power to the folds. This leads to an inconsistent power balance in the vocal apparatus model.

The historical reasons for the simplified flow modelling are related to the source-filter separation theorised by Fant [3]. Concisely, the vocal tract is supposed to act as a mere wave guide between the larynx and the outer space. In this representation, the larynx is then required to act as an acoustic source, the simplest one being a fluctuating flow rate through the glottis. Thus, simplified models of phonation mainly focused at a physically realistic way to produce the so called *glottal flow signal*. From a fluid-structure interaction perspective, the power imbalance introduces a bias in the analysis of the coupling occurring in the larynx and of the instability leading to voice production.

The objective of the present communication is to present a minimal model of the vocal apparatus that is able to simulate oscillating regime similar to patterns observed *in vivo* while being energetically consistent. The port-Hamiltonian theory provides a valuable framework for this purpose and more generally to the modelling, analysis and control of complex nonlinear dynamical systems.

Port-Hamiltonian systems (PHS)

The port-Hamiltonian theory combines the views of the (geometric) Hamiltonian mechanics and of port-based modelling approach. It emphasises the separation between the behaviour of subsystems and their interconnection. The *lingua franca* of this theory is energy : port-Hamiltonian systems are open passive systems that can store, dissipate and exchange power with their neighbourhood. Energy-storage is defined by the Hamiltonian $H(\mathbf{x})$ as a function of state variables $\mathbf{x}$. This dependency, formulated as the Hamiltonian gradient $\nabla_{\mathbf{x}} H$, expresses the *effort* of the energy-storing component. Conversely, the evolution of this component is described by the *flux* variable $\dot{\mathbf{x}}$. Effort and flux are power dual variables, i.e., they jointly define the *energy flow* $dH/dt = \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}} H(x)$.

Dissipating components are described by dissipation variable $\mathbf{w}$ and their constitutive laws $\mathbf{z}(\mathbf{w})$, such that the dissipated power is $\mathcal{P}_{\text{diss}} = \mathbf{w} \cdot \mathbf{z}(\mathbf{w}) \geq 0$. Finally, external interactions are classically described in terms of inputs $\mathbf{u}$ (also called efforts) and output $\mathbf{y}$ (fluxes) that are dual with respect to the external power: $\mathcal{P}_{\text{ext}} = \mathbf{y} \cdot \mathbf{u}$ (≥ 0 when the system yields power).

The geometric structure (Dirac structure, see [4]) accounting for the interconnection of the subsystems is then described by a matrix S relating efforts to fluxes:

$$\begin{pmatrix} \dot{\mathbf{x}} \\ \mathbf{w} \\ \mathbf{y} \end{pmatrix} = S(\mathbf{x}, \mathbf{w}) \begin{pmatrix} \nabla_{\mathbf{x}} H \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{pmatrix}. \tag{1}$$

When the interconnection is conservative, the power balance writes down as

$$0=\frac{dH}{dt}+\mathcal{P}_{\text{diss}}+\mathcal{P}_{\text{ext}}=\begin{pmatrix}\dot{\mathbf{x}}\\ \mathbf{w}\\ \mathbf{y}\end{pmatrix}\cdot\begin{pmatrix}\nabla_{\mathbf{x}}H\\ \mathbf{z}(\mathbf{w})\\ \mathbf{u}\end{pmatrix}=\begin{pmatrix}\nabla_{\mathbf{x}}H\\ \mathbf{z}(\mathbf{w})\\ \mathbf{u}\end{pmatrix}\cdot S(\mathbf{x},\mathbf{w})\cdot\begin{pmatrix}\nabla_{\mathbf{x}}H\\ \mathbf{z}(\mathbf{w})\\ \mathbf{u}\end{pmatrix}\tag{2}$$

which is ensured when the matrix S is skew-symmetric ($e\cdot S\cdot e=0$ for any effort e).

Minimal port-Hamiltonian model of the vocal apparatus

As depicted in Fig. 1, the vocal apparatus is modelled as the interconnection of vocal folds, glottal flow, vocal tract and a subglottal pressure supply. Every component is designed as minimal, but with an emphasis on the dual pairing on the interconnection ports. For instance, each vocal fold is considered to be a single-d.o.f. oscillator with an elastic cover (as in Ref. [6]), and is submitted to pressure on the upstream and downstream faces, and interacts with the flow at the glottal face. Reciprocally, the glottal flow is modelled as the simplest kinematics of potential incompressible flow of inviscid air preserving the continuity of the normal velocity on the surface of the vocal folds (see details in Ref. [7]). Downstream the glottis, the flow separates from the folds into a jet that spreads and dissipates its kinetic energy into heat. Finally, the vocal tract is represented as seen by the larynx, i.e. by means of its input impedance. We use an analytical modal formulation. When considering a single mode, the vocal tract acts as an Helmholtz resonator providing an acoustic feedback on the larynx.
The full vocal apparatus is obtained by conservative interconnection of the previous subsystems. For the sake of conciseness, the resulting Hamiltonian and matrix S are not reported here but can be efficiently computed using the Python package PyPHS [8] designed for the symbolic manipulation of PHS systems.

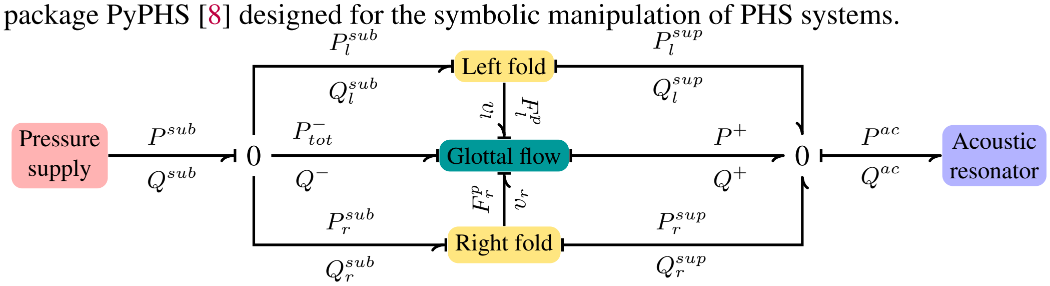

Figure 1: Components of the vocal apparatus. The interconnection takes place via pairs of effort (P) and flux (Q) variables. The 0 connection expresses the equality of efforts and the division of flux. See Ref. [5] for an introduction to bond graphs.

Numerical results

Time-domain simulations are performed using the numerical scheme designed after the principles of the port-Hamiltonian theory [9]. It is shown that it not only preserves power balance, but it also more consistent than standard integrators (such as the Runge-Kutta schemes). The simulations evidence the ability of the model to produce oscillating regimes above some pressure threshold, and also quasi-periodic vibrations when vocal folds are detuned.
The model are also been analysed from the point of view of the bifurcations by means of continuation methods such as the prediction-correction used in the AUTO software or the numerical asymptotic methods of the MANLAB software [10]. They confirme qualitatively the results of the time-domain simulations, notably with the existence of two stable oscillating regimes for detuned folds.

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