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## Bright Luminescence from Indirect and Strongly Bound Excitons in h-BN

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A quantitative analysis of the excitonic luminescence efficiency in hexagonal boron nitride (h-BN) is carried out by cathodoluminescence in the ultraviolet range and compared with zinc oxide and diamond single crystals. A high quantum yield value of  $\sim 50\%$  is found for h-BN at 10 K comparable to that of direct band-gap semiconductors. This bright luminescence at 215 nm remains stable up to room temperature, evidencing the strongly bound character of excitons in bulk h-BN. *Ab initio* calculations of the exciton dispersion confirm the indirect nature of the lowest-energy exciton whose binding energy is found equal to  $300 \pm 50$  meV, in agreement with the thermal stability observed in luminescence. The direct exciton is found at a higher energy but very close to the indirect one, which solves the long debated Stokes shift in bulk h-BN.

DOI:

Hexagonal boron nitride (h-BN) is a wide band-gap semiconductor ( $> 6$  eV) that has recently encountered a tremendous regaining of interest with the emergence of the two-dimensional (2D) crystals family [1,2]. Because of its lattice isostructural to graphene and its excellent dielectric properties, it has been designated as the best insulating material for improving electron mobility in graphene [3] or enhancing intrinsic optical properties of transition metal dichalcogenides (TMDs) [4,5].

In the past decade, the growth of high quality h-BN single crystals has also opened new possibilities for light-emitting devices. Pioneering works by Watanabe *et al.* have revealed the high UV radiative efficiency of h-BN at room temperature [6] and have led to the design of the first h-BN-based light emission device in the deep ultraviolet range [7]. However, the physical mechanisms behind h-BN luminescence are still debated. The main controversy revolves around two apparently contradictory aspects: on the one hand, the highly efficient luminescence (up to  $10^3$  times higher than in diamond at 300 K) and on the other hand, the energy shift between absorption and emission (Stokes shift).

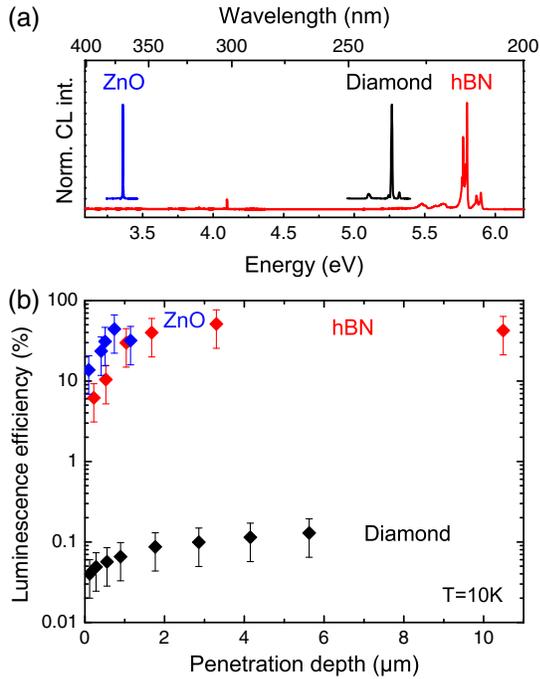
Among the various interpretations proposed in the past, Watanabe *et al.* first came to the conclusion that h-BN luminescence was driven by direct excitonic recombinations [6], and they attributed the Stokes shift to the lifted degeneracy of free exciton levels due to a dynamical lattice distortion [8]. But recently, Cassabois *et al.* attributed the luminescence lines to phonon-assisted transitions from an indirect exciton [9]. The interpretation is based on a weakly bound hydrogenic picture of the exciton, the Wannier-Mott

model, whose limits have already been emphasized for excitons in TMD atomic layers [10–12]. It also does not look pertinent in h-BN where *ab initio* calculations predict a direct exciton close to the Frenkel type [13,14].

Until now, although being widely established, the high luminescence efficiency (LE) of h-BN has never been quantitatively investigated. The classical methods using photoluminescence (PL) combined with integrating spheres [15–17] are not adapted for the deep UV range as required for wide band-gap semiconductors such as h-BN.

In this Letter, we report on a quantitative study of the cathodoluminescence (CL) in h-BN compared to two wide band-gap semiconductors, zinc oxide, and diamond. A careful calibration of our detection setup gives access to the absolute CL intensities in the 200–400 nm range. For the first time, low-temperature measurements (10 K), as a function of excitation depth, provide a value of the internal quantum yield (QY) in a bulk h-BN single crystal. Temperature-dependent CL from 10 K up to 300 K further show the high stability of excitons in bulk h-BN compared to diamond, and this attests of their strong binding energy. These findings are found consistent with *ab initio* calculations performed in this work to establish the dispersion of excitons in h-BN. They have encouraged us to revise the fundamental optical properties of excitons in h-BN, marked by a long debated Stokes shift. A consistent picture describing exciton luminescence and absorption energies is proposed.

For h-BN, all measurements were performed on a single crystal provided by the NIMS (Japan), grown at a high pressure high temperature (HPHT) [18], the highest quality



F1:1 FIG. 1. (a) Cathodoluminescence spectra of h-BN, diamond,  
 F1:2 and ZnO recorded in the 200–400 nm range at 10 K. For clarity,  
 F1:3 the excitonic spectra of ZnO and diamond are upshifted. All  
 F1:4 spectra are corrected from the spectral response of the detection  
 F1:5 system. (b) Luminescence efficiencies of ZnO, h-BN, and  
 F1:6 diamond plotted as a function of electron penetration depth.  
 F1:7 Accelerating voltages from 3 to 30 kV.  $T = 10$  K. The incident  
 F1:8 power  $V_i$  is kept constant to avoid nonlinear effects:  $1 \mu\text{W}$ ,  
 F1:9  $6 \mu\text{W}$ , and  $0.4 \mu\text{W}$  for h-BN, diamond, and ZnO, respectively.

82 source available today. The diamond and ZnO crystals, as  
 83 well as the experimental CL set-up, are described in  
 84 Supplemental Material I [19]. The intensity calibration  
 85 of the CL detection system relies on a deuterium lamp with  
 86 a calibrated spectral irradiance (LOT Oriel 30 W) used as a  
 87 measurement standard within the 200–400 nm wavelength  
 88 range. Once corrected for the spectral response of the setup,  
 89 absolute CL intensities are obtained. The light emission  
 90 power is assessed through a careful evaluation of optical,  
 91 geometrical, and sample parameters. The full LE measure-  
 92 ment procedure, including the absorbed power determina-  
 93 tion, is detailed in Supplemental Material I. Note that, the  
 94 uncertainty related to the absolute LE measurements in the  
 95 deep UV by CL ( $\pm 50\%$ ) is higher than when using  
 96 integrating spheres in the visible range.

97 Prior to any quantitative analysis, an overview of the  
 98 low temperature CL signal is given in the UV range  
 99 (200–400 nm) for h-BN, diamond, and ZnO single crystals  
 100 [Fig. 1(a)]. The luminescence of ZnO is dominated by a  
 101 sharp peak at 3.37 eV, corresponding to the recombinations  
 102 of neutral-donor bound excitons [34]. In contrast, the lumi-  
 103 nescence occurs from free excitons in diamond and h-BN.  
 104 In diamond, it presents a series of lines detected around  
 105 5.25 eV coming from phonon-assisted (TA, TO, LO)

recombinations of indirect free excitons [35,36]. 106  
 Similarly, the UV spectrum of h-BN displays a series of 107  
 sharp lines with a maximum at 5.795 eV, recently attributed 108  
 to recombinations of indirect free excitons [9]. Note that, in 109  
 both samples, the luminescence from deep defects is almost 110  
 undetectable, allowing a proper analysis of the intrinsic 111  
 excitonic features. 112

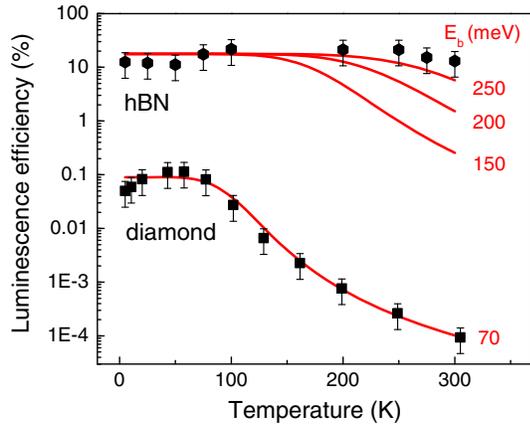
The luminescence efficiencies were measured at increas- 113  
 ing accelerating voltages (3–30 kV). In CL, high-energy 114  
 incident electrons undergo a successive series of elastic and 115  
 inelastic scattering events in the crystal before being 116  
 completely stopped. The penetration depth of electrons 117  
 $R_p$  then increases with the electron beam energy  $V$  118  
 following the empirical relationship of Kanaya *et al.* 119  
 [37],  $R_p(\mu\text{m}) = C.V(\text{kV})^{1.67}$ , where  $C$  is a material-de- 120  
 pendent parameter. 121

Figure 1(b) presents the luminescence efficiencies plot- 122  
 ted as a function of the penetration depth. While at high  $R_p$ , 123  
 the LE saturates in all cases, and one observes a significant 124  
 luminescence quenching at low excitation depths. In  $sp^3$  125  
 semiconductors such as ZnO or diamond, dangling bonds 126  
 ending crystal surfaces are known to introduce nonradiative 127  
 recombination channels that lower the radiative efficiency. 128  
 In 2D crystals with  $sp^2$  hybridization, dangling bonds are 129  
 not expected, but the surface effects still remain poorly 130  
 known. Possible explanations relying on contaminated or 131  
 defective surface [38] or Auger effects [39] can be invoked 132  
 to account for this result. 133

At high penetration depths, the luminescence efficiency 134  
 reaches a constant value called the internal quantum yield 135  
 (QY), which is characteristic of the bulk crystal. Generally 136  
 noted  $\eta$ , it corresponds to the fraction of excitons that 137  
 recombines radiatively inside the crystal. While the LE 138  
 most often depends on the crystal orientation, surface 139  
 terminations, and contaminations, the internal quantum 140  
 yield remains a reference parameter of the bulk material. 141

As expected, the direct band-gap ZnO crystal is found to 142  
 be highly radiative with a QY larger than 50%, in good 143  
 agreement with previous PL analysis [40]. Conversely, the 144  
 internal quantum yield of diamond remains close to 0.1%, 145  
 more than two orders of magnitude lower than in ZnO. 146  
 Indirect excitons formed in diamond require the simulta- 147  
 neous emission of a phonon to recombine radiatively, 148  
 which inevitably lowers the probability of such a process. 149  
 In the case of h-BN, the LE also saturates at high 150  
 penetration depths but it reaches an  $\sim 50\%$  internal quantum 151  
 yield, comparable to that of a direct semiconductor such as 152  
 ZnO. Such a high QY value from phonon-assisted recom- 153  
 binations of indirect excitons far exceeds what is com- 154  
 monly observed for indirect semiconductors. It indicates 155  
 that the nonradiative channels present in the crystal are 156  
 efficiently bypassed by faster radiative recombinations (a 157  
 few hundred picoseconds according to the literature [41]). 158

We then performed temperature-dependence measure- 159  
 ments of the luminescence efficiency. Figure 2 first presents 160

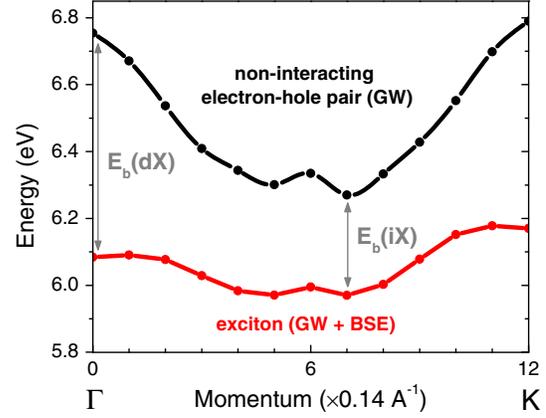


F2:1 FIG. 2. Luminescence efficiency (logarithmic scale) of h-BN  
 F2:2 and diamond as a function of temperature. Excitation parameters:  
 F2:3  $V = 10$  kV,  $i = 0.2$  nA for h-BN and 9.5 nA for diamond. The  
 F2:4 curves (red lines) correspond to the thermal dissociation of  
 F2:5 excitons (See Supplemental Material II) where  $E_b$  is the exciton  
 F2:6 binding energy.

161 the CL results obtained on the two indirect band-gap  
 162 semiconductors, namely h-BN and diamond from 10 to  
 163 300 K. To limit surface effects, the accelerating voltage was  
 164 set at 10 kV. In diamond, the QY is rather stable at low  
 165 temperatures because of the predominance of the exciton  
 166 population over free electrons and holes. Above 60 K, the  
 167 thermal dissociation of excitons occurs and the quasiequilibrium  
 168 between exciton and free carrier populations is  
 169 displaced in favor of free carriers, resulting in a drastic LE  
 170 drop of 3 orders of magnitude. This temperature behavior is  
 171 well described by the classical model for exciton dissociation  
 172 in silicon [27,28] recently implemented in diamond  
 173 [29], with  $E_b = 70$  meV in agreement with the reference  
 174 values of the literature [42,43].

175 The situation is much different in h-BN, where the LE  
 176 remains almost constant over the full temperature range. At  
 177 300 K, the exciton population is still predominate, demonstrating  
 178 that the exciton binding energy in h-BN is much higher than  
 179 in diamond. By applying the same thermal dissociation model  
 180 (see Supplemental Material II), the binding energy of the  
 181 lowest-energy excitons in bulk h-BN is estimated to be larger  
 182 than 250 meV. This lower bound is much higher than the  
 183 previously reported values in the literature (149 meV [6] and  
 184 128 meV [9]).

185 To elucidate this controversy, the *ab initio* Bethe-Salpeter  
 186 equation (BSE) has been solved to obtain the exciton dispersion  
 187  $E_{\text{exc}}(Q)$ , with  $Q$  being the exciton momentum. Quasiparticle  
 188 energies have been computed within the perturbative *GW* method  
 189 and further blue shifted by 0.47 eV on the basis of recent  
 190 electron energy loss spectroscopy experiments [44]. The  
 191 state-dependent corrections of *GW* ensure an accurate  
 192 dispersion at the single-particle level. Details of the  
 193 computational method are given in Supplemental Material III.  
 194 Two different

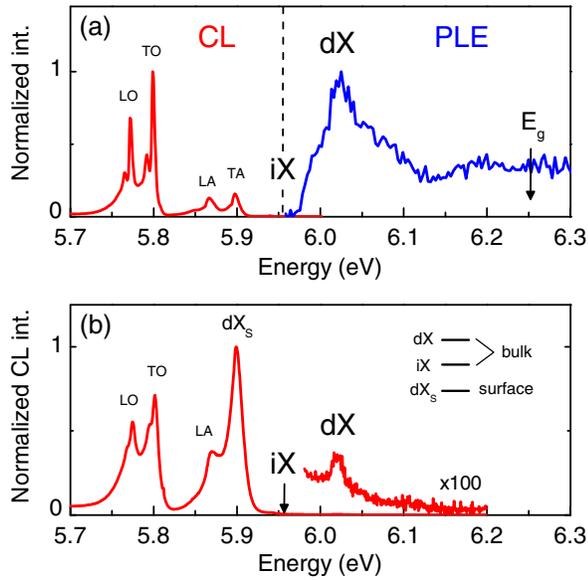


F3:1 FIG. 3. Dispersion of excitons in h-BN compared to the non-  
 F3:2 interacting electron-hole pair along the  $\Gamma$ K direction. The  
 F3:3 vertical arrows indicate the binding energy  $E_b$  for the direct ( $dX$ )  
 F3:4 and indirect ( $iX$ ) excitons. Adapted from Ref. [45].

two-particle dispersion relations can then be obtained. 195  
 First, the dispersion of the noninteracting electron-hole 196  
 pair (*GW*, black curve) has been traced by reporting the 197  
 lowest energy difference between the empty and occupied 198  
 quasiparticle states compatible with the given  $Q = 199$   
 $K_e + K_h$ . Second, the exciton dispersion (*GW*-BSE, red 200  
 curve) results from the solution of the BSE at finite  $Q$ . 201  
 Electron-hole pairs formed with electrons sitting in M at the 202  
 bottom of the conduction band and holes sitting near K at 203  
 the top of the valence band are expected to have a 204  
 momentum close to  $KM = 1/2 \Gamma K$  [9]. That is why we 205  
 report in Fig. 3 both noninteracting *e-h* pair and exciton 206  
 dispersions along the  $\Gamma K$  direction. 207

The independent *e-h* dispersion (*GW* curve) reaches its 208  
 lowest energy near the middle of  $\Gamma K$  and lies 0.5 eV below 209  
 the direct transition ( $\Gamma$ ), consistently with the indirect 210  
 electron band structure (see Supplemental Material III 211  
 and Ref. [14]). In the presence of Coulombic interactions, 212  
 the minimum of the exciton energy (*GW*-BSE curve) is 213  
 also found close to the middle of  $\Gamma K$  at 5.97 eV. The 214  
 exciton dispersion behavior corroborates the attribution of 215  
 the luminescence lines as being phonon-assisted recombinations 216  
 of an indirect exciton [9]. It is also confirmed 217  
 through high-resolution CL spectroscopy presented in 218  
 Supplemental Material I. The value of 5.956 eV is found 219  
 by CL for the no phonon recombination, which accurately 220  
 defines the indirect exciton (*iX*) energy in h-BN. 221

More insights come from the comparison of the exciton 222  
 and free electron-hole pair dispersions (Fig. 3). Strikingly, 223  
 excitonic effects tend to flatten the dispersion of the 224  
 noninteracting electron-hole pair. In other terms, the 225  
 exciton is found to be much heavier than the sum of the 226  
 electron and hole masses, a sum that is assumed to be 227  
 the exciton mass in the Wannier-Mott model. As a first 228  
 consequence, the binding energies of direct and indirect 229  
 excitons are different. The binding energy of the indirect 230



F4:1 FIG. 4. (a) Normalized spectra of CL and PLE recorded at the  
 F4:2 CL maximum (from Ref. [46]) illustrating the Stokes shift  
 F4:3 observed in bulk h-BN crystals. (b) CL spectrum of a thin  
 F4:4 exfoliated h-BN flake showing the radiative recombination of  
 F4:5 nonthermalized direct excitons detected at 6.03 eV. CL and PLE  
 F4:6 spectra taken at 10 K.

231 exciton [ $E_b(iX)$ ] is found equal to  $300 \pm 50$  meV (at  
 232  $Q = (7/12)\Gamma K$ ), consistent with the high thermal stability  
 233 observed experimentally (Fig. 2). Incidentally, from the  
 234 energies found for  $E_b(iX)$  and  $iX$ , the indirect band-gap  
 235 energy in bulk h-BN is refined to  $E_g = 6.25 \pm 0.05$  eV.  
 236 Instead, at  $Q = 0$ , the direct exciton binding energy  
 237  $E_b(dX)$  is found to be equal to 670 meV, close to the  
 238 previously reported values [13,14]). The second conse-  
 239 quence of the exciton dispersion flattening is that the direct  
 240 and indirect excitons are very close in energy. The direct  
 241 exciton lies only  $\sim 100$  meV above the indirect one.

242 This theoretical work sheds a new light on the origin of  
 243 the Stokes shift reported for h-BN between the maxima of  
 244 luminescence and absorption energies [6,46], as illustrated  
 245 in Fig. 4(a), with the CL and photoluminescence excitation  
 246 (PLE) (from Ref. [46]) spectra recorded on the same  
 247 crystal. It has been reported that the sharp PLE signal  
 248 peaking at 6.025 eV would arise from a phonon-assisted  
 249 process [9]. In indirect semiconductors, one should expect a  
 250 mirror symmetry between absorption and luminescence  
 251 around the  $iX$  energy position [47], both in energy and  
 252 intensity. It is, however, not observed in h-BN, and it does  
 253 not apply to interpreting the absorption spectrum.

254 On the other hand, Fig. 4(a) shows that the energy  
 255 difference between the  $iX$  and the PLE maximum energies,  
 256 about 70 meV, is comparable to the theoretical  $\sim 100$  meV  
 257 energy difference between direct and indirect excitons  
 258 (Fig. 3). This lets us propose that the dominant PLE peak  
 259 arises from the direct exciton labeled  $dX$  in Fig. 4(a).

260  $Al_xGa_{1-x}As$  alloys face a similar situation when  $x \sim 0.3$  but  
 261 with weakly-bound excitons ( $E_b \sim 5$  meV) [48]. The  
 262 luminescence of h-BN, which only reveals the lowest  
 263 energy states, is driven by indirect exciton ( $iX$ ) recombi-  
 264 nations, while in the PLE spectrum, being similar to an  
 265 absorption one, the direct exciton ( $dX$ ) is predominate.

266 Even though high resolution absorption data are still  
 267 lacking, all available experiments exhibit a main peak  
 268 around 6.1 eV with a strong absorption coefficient, higher  
 269 than  $10^5$  cm $^{-1}$  [49–51]. Such a behavior is typical of  
 270 optical transitions without phonons (in silicon, for instance,  
 271  $\alpha$  is higher than  $10^5$  cm $^{-1}$  at the direct gap edge [52]).  
 272 Furthermore, recent studies have shown that the absorption  
 273 spectrum of h-BN multilayers is fairly independent of the  
 274 number of layers [53,54]. The direct exciton is theoretically  
 275 described as being quite similar in bulk h-BN and in the  
 276 monolayer, with its energy being weakly influenced by the  
 277 surrounding BN planes [55–57].

278 Finally, CL experiments done on exfoliated h-BN  
 279 samples have also provided precious information. Fig-  
 280 ure 4(b) displays a typical CL spectrum recorded on  
 281 an 80 nm thick h-BN flake, where the bulk contribution of  
 282 indirect excitons (TO, LO, LA, and ZA phonon-assisted  
 283 lines according to the assignments proposed in Ref. [9]) is  
 284 detected together with a sharp peak at 5.90 eV. In a previous  
 285 work [58], we have already shown that when reducing the  
 286 thickness, a progressive vanishing of the bulk luminescence  
 287 lines is accompanied by the emergence of a narrow single  
 288 emission at this energy. This single emission peak tends to  
 289 indicate that the associated luminescence process does not  
 290 involve phonons and originates from a direct optical  
 291 transition. It is further attributed here to surface excitons  
 292 ( $dX_s$ ) with zero momentum and an energy slightly below  
 293 the indirect exciton of bulk h-BN, according to recent  
 294 theoretical works [59]. More interestingly for the present  
 295 discussion, in Fig. 4(b) a weak luminescence peak could be  
 296 detected at 6.03 eV, the same energy as the PLE maximum.  
 297 It is attributed to the recombination of nonthermalized (hot)  
 298 direct excitons ( $dX$ ) from bulk h-BN, observed simulta-  
 299 neously with the TO, LO, LA replica of the indirect one  
 300 ( $iX$ ). Finally, the coexistence of direct and indirect excitons  
 301 with close energies is confirmed as a key to understanding  
 302 the luminescence and absorption properties of h-BN.

303 In summary, the luminescence quantum yield in bulk  
 304 h-BN is quantitatively estimated to be  $\sim 50\%$  as high as in  
 305 direct band-gap semiconductors such as zinc oxide. *Ab initio*  
 306 calculations of the excitonic dispersion have been  
 307 decisive in providing a consistent picture of exciton  
 308 properties in bulk h-BN. The lowest-energy excitons  
 309 involved in luminescence are of an indirect nature, with  
 310 a  $300 \pm 50$  meV binding energy, consistent with the  
 311 stability of the luminescence intensity observed up to room  
 312 temperature. The excitonic dispersion has revealed the  
 313 presence of a direct exciton at a slightly higher energy,  
 314 responsible for the maximum of absorption in bulk h-BN.

315 The long-standing debate on the Stokes Shift in bulk  
 316 h-BN is thus completely elucidated. As a perspective,  
 317 the extremely high efficiency of the phonon-assisted  
 318 luminescence in h-BN, unique for an indirect band-gap  
 319 semiconductor, still needs a better understanding for  
 320 **1** applications with light emitting devices in the deep  
 321 UV range.

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 383 supplemental/10.1103/PhysRevLett.000.000000](http://link.aps.org/supplemental/10.1103/PhysRevLett.000.000000) for further  
 384 information on the samples and the CL setup, description of  
 385 the luminescence efficiency measurement procedure and the  
 386 analysis of thermal dissociation of excitons, as well as  
 387 details regarding the *ab initio* calculations, which includes  
 388 Refs. [20–33]. 389  
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# Supplemental Material

## SECTION I.

### A. Samples and experimental setup

The cathodoluminescence setup combines an optical detection system (Horiba Jobin Yvon SA) and a JEOL7001F field-emission gun scanning electron microscope (FEG-SEM) equipped with a Faraday cup for the beam current measurement. Samples are mounted on a GATAN cryostat SEM-stage to cool them at temperatures from 300 K down to 5 K thanks to cold-finger cryostat with liquid helium. The temperature reported in this work is measured on the sample holder. The CL signal is collected by a parabolic mirror and focused with mirror optics on the open entrance slit of a 55 cm-focal length monochromator. The all-mirror optical setup provides an achromatic focusing with a high spectral sensitivity down to 190 nm. A nitrogen-cooled charge-coupled detector (CCD) silicon camera is used to record CL spectra. The spectral calibration is performed using a Hg lamp.

The diamond and ZnO single crystals used in our experiments are of high quality and commercially available: Element6 (Electronic Grade CVD, impurities  $<10^{14}$  at/cm<sup>3</sup>, dislocation density  $<10^5$  /cm<sup>2</sup>) and CrysTec GmbH (impurities  $\sim 10^{17}$  at/cm<sup>3</sup>) respectively. Note that the hBN crystal is less pure, with carbon and oxygen impurities detected by secondary ion mass spectroscopy in the  $10^{18}$  at/cm<sup>3</sup> range (see Ref. [18] in the main text). The luminescence from deep defects near 4.1 eV remains several orders of magnitude weaker than exciton ones. A 200-1000 nm spectrum of the same sample is presented in our previous work [1].

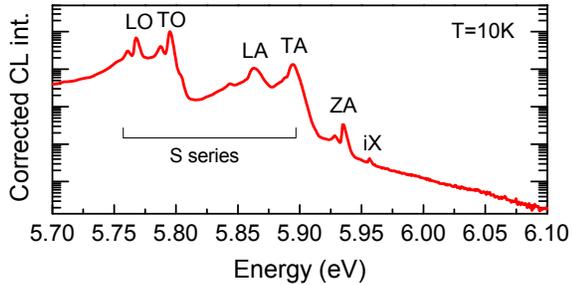


FIG. S1. High resolution CL spectrum of a HPHT-grown hBN crystal recorded in the near band edge region (logarithmic scale).

Figure S1 displays a high-resolution CL spectrum of the hBN single crystal at 10 K, showing the same emission energies as those obtained with two-photon PL ex-

citation [2]. The series of peaks at 5.768(S4), 5.795(S3), 5.864(S2), 5.894(S1) and 5.935 eV initially reported as the S-series corresponds to phonon-assisted recombinations of the indirect exciton as proposed in Ref. [2]. Importantly, we observe that the radiative recombination without phonon (labeled *iX*) occurs at 5.956 eV which, accurately defines the indirect exciton energy in hBN.

### B. Measuring the luminescence efficiency by cathodoluminescence

One of the fundamental differences between CL and PL is that, whereas one absorbed photon generates one coherent electron hole (e-h) pair in PL, one single electron generates hundreds of incoherent e-h pairs in CL. The generation rate  $G$  (*i.e.*, the number of e-h pairs generated in the crystal per unit of time) is given by:

$$G = \frac{Vi(1-f)}{\langle E \rangle e} \quad (1)$$

where  $V$  is the accelerating voltage of incident electrons,  $i$  the beam current,  $e$  the electronic charge and  $f$  the backscattering factor.  $\langle E \rangle$  is the average energy required for the formation of an electronhole pair in the crystal. Here, we assume that  $\langle E \rangle$  is equal to  $3E_g$  [3]. This empirical relation is known to account for the electron-hole formation energy with a typical uncertainty of 25% (see for instance the measured value of  $\langle E \rangle$  in diamond in Ref. [4]). Given the further uncertainties on  $V$  and  $i$  measured with a Faraday cup, the generation rate is evaluated at  $\pm 30$  %.

Under continuous excitation, the steady state population of excitons inside the crystal is equal to  $G\tau$ , with  $\tau$  the exciton lifetime. The total photon flux emitted inside the crystal is equal to  $G\tau/\tau_{\text{rad}}$ ,  $\tau_{\text{rad}}$  being the radiative lifetime of excitons. The ratio of the light emission over the generation rate defines here the luminescence efficiency  $\text{LE} = \tau/\tau_{\text{rad}}$ .

However, in a luminescence experiment, photons are collected outside the crystal so that corrections have to be made to determine the luminescence efficiency ( $\tau/\tau_{\text{rad}}$ ) from absolute CL intensities. One has to consider how a photon escapes the crystal (light extraction efficiency) as well as instrumental parameters. The flux of photons collected by the detector from the specimen is given by the following equation assuming an isotropic light emission [5]:

$$I_{\text{CL}} = \frac{G\tau}{\tau_{\text{rad}}} \cdot (\Omega/4\pi) F_A F_R F_I \quad (2)$$

Where:

- $\Omega$  is the solid angle of collection of the detection system limited by the parabolic mirror aperture. In this work, it is equal to 3.9 steradians which corresponds to a  $\Omega/4\pi = 18\%$  collection of the emitted light outside the crystal.
- $F_R$  is the light extraction factor which corrects internal reflections and refraction at the specimen surface.  $F_R$  is an analytic function of the refractive index  $n$  [6]. We took  $n(\text{hBN}) = 3.95$  at 215 nm [7],  $n(\text{diamond}) = 2.67$  at 236 nm [8] and  $n(\text{ZnO}) = 2.26$  at 372 nm [8].
- $F_A$  accounts for the luminescence re-absorption along the escape path of photons. It is neglected in semiconductors exhibiting large Stoke shifts between light absorption and emission, such as diamond and hBN. Though re-absorption is significant for a direct bandgap semiconductors such as ZnO, it was also neglected in this work. Note this assumption results in underestimating the luminescence efficiency of ZnO.
- $F_I$  is the response of the optical detection system, calibrated with a reference deuterium lamp of known irradiance.
- $I_{\text{CL}}$  is obtained by integrating the absolute CL signal in the 200-400 nm range.

Considering the whole CL procedure, the absolute LE values are considered to be measured with a 50% uncertainty.

## SECTION II.

### Thermal dissociation of excitons

With above bandgap excitations, a part of the generated electron-hole pairs bounds under exciton quasiparticles such as  $e + h \rightleftharpoons X$ . This quasi-equilibrium depends on the exciton binding energy and the temperature as stated by the action mass law [9]:

$$n^*(T) = n_{eh}^2/n_X = (\mu k_B T / 2\pi \hbar^2)^{3/2} \exp(-E_b/k_B T) \quad (3)$$

where  $n_{eh}$  and  $n_X$  are respectively the concentrations of electron hole pairs and excitons,  $\mu$  a factor including electron, hole and exciton effective masses,  $E_b$  the exciton binding energy respectively,  $k_B$  and  $\hbar$  the Boltzmann and Planck constants. Considering the total concentration of injected carriers, free or bound as excitons,  $n = n_{eh} + n_X$ , the fraction of excitons in a crystal  $f = n_X/n$  is:

$$f = 1 + \frac{n^*}{2n} - \sqrt{\left(\frac{n^*}{2n}\right)^2 + \frac{n^*}{n}} \quad (4)$$

$f$  depends on the temperature  $T$  via  $n^*$ , but also on the injection level  $n$ . The equilibrium then displaces in favor of excitons at low temperatures and high injection levels. This simple model accurately describes exciton and free electron-hole pair concentrations measured with pump-probe techniques in silicon [9,10]. Implemented more recently for diamond, the thermal dissociation model successfully accounts for its luminescence properties [11].

As free electron-hole pairs present a much weaker radiative recombination probability compared to excitons, the temperature dependence of the diamond luminescence efficiency reflects the fraction of excitons present in the crystal. The experimental curve  $\text{LE}(T)$  is then fitted using the above expression for  $f$ , assuming no significant dependence of  $n$  as a function of the temperature [12]. With  $\mu = 0.076$  as taken in [11], a correct fit of experimental data is obtained with  $E_b = 70$  meV and  $n = 2 \times 10^{13} \text{ cm}^{-3}$ . The order of magnitude found for  $n$  appears correct, being below the formation threshold of polyexcitons ( $\sim 3 \times 10^{13} \text{ cm}^{-3}$ ) [13] consistently with their absence of detection in the diamond CL spectrum. As mentioned in the body text, the value found for  $E_b$  also well agrees with the standard 70-80 meV exciton binding energy determined experimentally in diamond. As a summary, the PL quenching of three orders of magnitude observed in diamond at room temperature is well fitted by the thermal dissociation of its weakly bounded excitons. For hBN we assumed  $\mu = 0.5$  and  $n = 2 \times 10^{14} \text{ cm}^{-3}$  considering a much shorter diffusion range of excitons in hBN than in diamond [12]. The curves of  $f$  plotted in the Fig. 2 show the increase of  $E_b$  required to describe the almost constant luminescence efficiency observed over the 5-300 K temperature range in hBN. Such an analysis thus only provides a lower bound for the exciton binding energy in hBN. The present analysis indicates it is larger than 250 meV.

The corresponding CL spectra of hBN recorded at different temperatures are provided below (Fig.SII). The stability of the luminescence efficiency as a function of the temperature can be seen directly on this figure, where the integrated CL intensity appears almost constant.

## SECTION III.

### Computational details

The simulated hBN has lattice parameters  $a = 2.5$  Å and  $c/a = 2.6$  [14]. The Kohn-Sham system and the GW corrections have been computed with the ABINIT simulation package (a plane-wave code [15]). Norm-conserving Troullier-Martins pseudopotentials have been used for both atomic species. DFT energies and wave

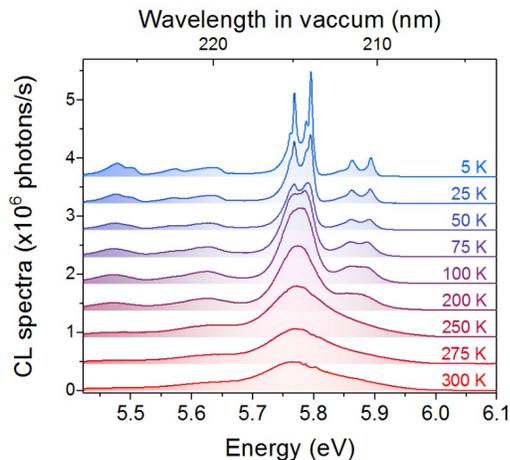


FIG. II. CL spectra of the HPHT hBN single crystal as a function of temperature from 5 K to 300 K.  $V=10$  kV and  $i=0.2$  nA. Spectra are shifted for clarity.

functions have been obtained within the local density approximation (LDA) to the exchange-correlation potential, using a plane-wave cut-off energy of 30 Ha and sampling the Brillouin zone with a  $8 \times 8 \times 4$   $\Gamma$ -centered k-point grid.

The GW quasiparticle corrections have been obtained within the  $G_0W_0$  approach. They have been computed on all points of a  $6 \times 6 \times 4$   $\Gamma$ -centered grid. A cut-off energy of 30 Ha defines the matrix dimension and the basis of wave functions to represent the exchange part of the self-energy. The correlation part of the self-energy has been computed including 600 bands and the same cut-off energy as for the exchange part. To model the dielectric function, the contour deformation method has been used, computing the dielectric function up to 60 eV, summing over 600 bands and with a matrix dimension of 6.8 Ha. Finally, as explained in Ref. [16], an additional shift of 0.47 eV of the empty levels (scissor operator) has been introduced to match the EELS measurements reported recently by Schuster *et al.* [17]. This discrepancy has to be ascribed to lack of self-consistency in the  $G_0W_0$  scheme. However this GW+Scissor scheme is preferable to a simple scissor operator because the GW self-energy is state-dependent, hence it takes into account quasiparticle effects on the dispersion of the bands and not only on their position. This is most relevant when investigating the exciton dispersion.

In Figure SIII.1, we report the LDA and the shifted GW band structure. The latter has been obtained by making a cubic interpolation on the appropriate k-point grid of the quasiparticle corrections computed on the  $6 \times 6 \times 4$  grid.

The dielectric function  $\epsilon(Q, \omega)$  has been calculated using the EXC [18] code on a  $36 \times 36 \times 4$   $\Gamma$ -centered k-point

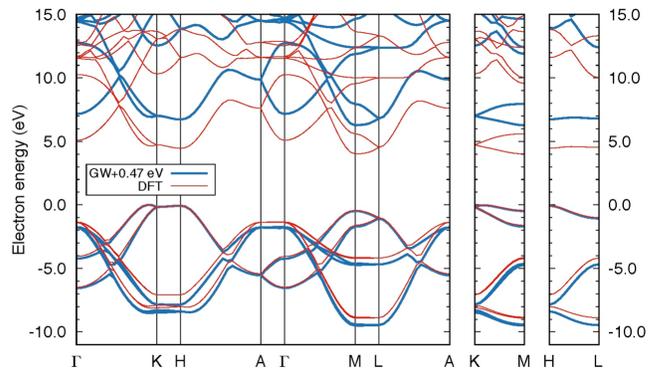


FIG. SIII.1. Electron band structure of hBN: LDA (thin red line) and  $G_0W_0 + 0.47$  eV scissor (thick blue line).

grid using the shifted GW energies (properly interpolated as explained above). The GW-BSE has been solved in the Tamm-Dancoff approximation. Six valence bands and three conduction bands have been included in the calculation, and a cut-off energy of 360 eV for both the matrix dimension and the wave function basis has been used. The static dielectric matrix entering in the BSE kernel has been computed within the random phase approximation with local fields, including 350 bands and with cut-off energies of 120 eV and 200 eV for the matrix dimension and the wave function basis respectively. Within these parameters, the energies of the first excitons are converged within 0.01 eV.

In Figure SIII.2 we report the energies of non-interacting electron-hole pair (black line) defined as:

$$E_{NI}(Q) = \min[E_C(k+Q) - E_V(k)] \quad (5)$$

for all  $k$  in the full Brillouin zone and for  $Q$  along  $\Gamma K$ .  $E_C$  and  $E_V$  are respectively the shifted GW energies of empty and conduction states. Also we report the exciton dispersion (in red), solution of the BSE (thick line = lowest laying exciton, thinner lines = higher energy excitons). The energy axis on the left includes the scissor operator of 0.47 eV, while the energy axis on the right is without the shift (only GW+BSE).

We point out that the lowest-energy direct exciton (at  $\Gamma$ ) is doubly degenerate and optically dark, the closest bright exciton is about 80 meV above. The calculated values may vary depending on the approximation used. In order to give an estimate of the uncertainty we compared our data to the appendix of Ref. [16] and to other GW-BSE calculations [19]. The calculation uncertainty is estimated to be  $\pm 50$  meV for the difference between the lowest indirect and direct excitons, as well as for the indirect exciton binding energy.

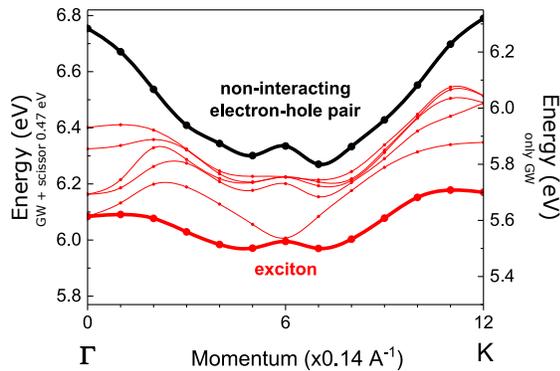


FIG. SIII.2. Dispersion of excitations for  $Q$  along  $\Gamma K$ . Non-interacting electron-hole pair (black line) and exciton dispersion (red lines) of the first six excitons (the lowest energy one reported with a thicker line). Energy axis are GW+scissor (on the left) and GW only (on the right).

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