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Workforce Scheduling Linear Programming Formulation

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Abstract: This paper introduces a linear programming formulation for a ternary-integration Workforce Scheduling and Routing Problem that incorporates scheduling of tasks, assigning of workers to the tasks according to their skills and the definition of the workers' trips. Each task has a time window, and is related to a customer who has a preference list of the workers. Each worker has a cost, a preference list of tasks and a working time window. The objective is to perform the tasks and simultaneously minimizing the number of unassigned tasks, the traveling distance, the workers' cost, and maximizing the customers and workers preference satisfaction.

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Keywords: Workforce Scheduling, Routing, Linear Programming Formulation, Integrated Problem

1. INTRODUCTION

This paper presents an investigation into the application of a mixed-integer linear programming approach to tackle a Workforce Scheduling and Routing Problem (WSRP). WSRP refers to matching workers with tasks that are required by customers. The assignment of the workers with skills to a series of tasks at different locations must be achieved according to the required skills to perform the tasks and of the workers' skills and the workers' wishes concerning the geographical location where they want (or do not want) to work. Both customers and workers have time windows defining the task period availability in which tasks have to be performed and defining the workers' time availability in which worker wish to work. The WSRP resolution requires tackling, in a proper coordination, the assignment of tasks to workers, the scheduling of the tasks and the routing of the workers to travel between the locations of the tasks with the objective to maximize the number of performed tasks first, to minimize Quality of Service of the workers second (that includes the respect of the worker's working time window and geographical working regions), to maximize the preference satisfaction between customers and workers thirdly and to minimize the cost (which includes the transport and the cost for a worker to perform a task) fourthly.

The WSRP definition is given in Section 2, Section 3 provides the MILP formulation and Section 4 is devoted to numerical experiments.

2. WORKFORCE SCHEDULING PROBLEM

2.1 Definition of the WSRP and related work

Workforce Scheduling and Routing Problems combine vehicle routing and scheduling problems with time-window and additional temporal constraints including, but not limited to, pairwise synchronization and pairwise temporal precedence. WSRP encompasses problems encountered in Home Health Care (HHC) logistics where demands include transportation of drugs/medical devices between the depot and patients' home, delivering drugs from hospital to patients or blood samples from patients to the laboratory. A high quality of service should have to be defined considering both customers (or patients) and vehicles (health care managers) assuming transportation patterns constantly optimized, and flexible organizations, while minimizing costs. To propose flexibility and efficiency, a strong cooperation between scheduling and routing must be warranted. Home Health Care planning problems differ from the WSRP by constraints and objective functions (Castillo-Salazar et al., 2016).

(Pinheiro et al., 2016) proposed a variable neighbourhood search for solving the WSRP, incorporating two novel heuristics. (Algethami and Landa-Silva, 2017) introduced an adaptive genetic algorithm that uses a diversity-based adaptive parameter control method to solve the WSRP with an indirect representation of the solution.

(Misir et al., 2010) defines the main characteristics of HHC in two sets which can be used to the WSRP, and including but are not limited to:

For worker,

- A working time window $[TW_{inf}^{w}, TW_{sup}^{w}]$, a worker w is available only in that period. The definition of the working time window can vary in different scenarios: the traveling time is considered as working time; a worker can be forced to violate his working time windows if it is necessary to perform a task. In this paper, the traveling time is not considered as working time and a worker is allowed to

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work over his time window if necessary but this overtime is penalized in the objective function.

- Departure and finishing locations: are the locations where workers begin and finish their trips. These locations may be a unique depot (Eveborn et al., 2006) or several locations in the case assuming that each worker may start from their own home (Castillo-Salazar et al., 2016). Different scenarios are possible, for example, some company's policy can enforce workers to start their working time at the main office but they are allowed to return home directly after the last task performed.
- Skills: each worker has different skills.
- Teaming: some tasks may require several workers to be performed, and can start only when all workers are together.
- Available regions: workers have preference regions to work and despised regions.

For task,

- Time window $[e_j, l_j]$ to perform a task j: if a worker arrives before the time window, the task cannot begin and a waiting-time is incurred (Akjiratikarl et al., 2007).
- Skill requirements: a task necessitates one or several specific skills. In order to perform a task, a worker must have at least one required skill by the task (Cordeau et al., 2010).
- Processing time is the time to perform a task. The processing time is often assumed to be a fixed duration but it could be worker dependent or stochastic.
- Location: each task has a location and so a transportation time has to be taken into account from one location to another one.

For the WSRP, the following characteristics have to be considered:

- Each customer has a preference list between the workers who can perform the required task.
- Each task is included into an agreement between the customer and the company, this contract also includes a list of workers who are allowed to perform the task.

2.2 Problem description

This section describes the Workforce Scheduling and Routing Problem with time-dependent activities constraints. (de Armas et al., 2015) introduces a formalization of a Ship Routing and Scheduling Problem with Time Window that can be tuned to WSRP. We define a graph $G = (V, E)$, where V is the set of nodes, and E the set of edges. $V = T \cup D \cup D'$ is composed of the set D of departure locations of the workers (the workers can start their trip from their home, or from a depot), the set D' of finishing locations of the workers (and similar remarks hold for the departure locations and for the finishing location of the trips), and the set T of tasks characterized by their processing time and their location. A task $j \in T$ requires to be achieved by r_j workers; if $r_j > 1$, the task is represented by r_j nodes in the graph. An edge between two nodes represents the trip of a worker between both tasks' location: an edge between two tasks meaning that both tasks are performed by the same worker. Transportation time between two tasks i and j is $dist_{i,j}$.

$W = \{w_1, w_i, \dots w_{|W|}\}$ is the set of available workers. For a task j and a worker w, there are three types of preferences: satisfaction worker-customer pairing: ps_j^w; worker's satisfaction regions: pa_j^w; and the customer's satisfaction skills: pw_j^w. These three preferences are aggregated in $\rho_j^w = (pa_j^w + pw_j^w + ps_j^w)$. Moreover, p_j^w is the cost of the worker w performing the task j according to the agreement between the customer and the company. Both workers and tasks have time window: a task j must be started during its time window $[e_i, l_i]$ (but can be finished over the time window if the processing time is too long); and a worker w can work only during his working time window $[TW_{inf}^w, TW_{sup}^w]$: the traveling time is not considered as working time in this paper.

Table 1. Objectives and Constraints in the proposed model

Objective	Constraints	Preferences
Traveling cost	Skill requirements	Worker Time Windows
Payment cost	Contract	Worker available regions
Preferences (skills, regions, workers)	Task time window	Number of required workers by a task
Unassigned tasks		

Table 1 introduces the objectives, constraints and preferences to the WSRP in this paper. The objective is to simultaneously minimize: the traveling cost of the workers, the payment cost from the customers, the number of unassigned tasks; and maximize the preferences (of the workers' preferred regions, customers' preferred skills, and preferred matching worker-task).

The objective is to minimize a weighted function considering four parts:

1) Minimization of the traveling cost and the working cost of the workers,

2) Maximization of the preferences (of the workers' preferred regions, customers' preferred skills, and preferred matching worker-task). This objective models the Quality of Services for both customers and workers.

3) Minimization of the total number of tasks assigned to workers with a process finishing time exceeding the workers' time window, plus the total number of tasks assigned to one worker and that tasks are located into a region that does not fit with the worker's preferences.

4) Minimization of the number of workers required by a task and who has not been assigned to the task.

The constraints are the required skills, the respect of the contract between customers and workers (a worker who does not appear on the contract cannot work for the customer) and the time windows of the tasks.

Let us note that part 2, part 3 and part 4 of the objective function define a satisfaction score, evaluating a degree of satisfaction for preferences that encompass both preference of tasks (and by consequence of customers) and preferences of workers.

3. WORKFORCE SCHEDULING AND ROUTING PROBLEM

3.1 Model description

The objective of the tackled WSRP is to assign customers' tasks to workers and to define the workers' trip.
Let us remind that $W = \{w_1, w_i, \dots w_{|W|}\}$ is the set of workers; $T = \{T_1, T_i, \dots T_{|T|}\}$ is the set of tasks to be performed; and some tasks can require more than one worker.

A solution to the WSRP requires all tasks to be assigned while satisfying some requirements:

- Assignment of tasks must be achieved considering workers skills;
- Assignment of workers to tasks must be achieved considering customers preferences;
- Assignment of workers to tasks must be achieved considering the worker-customer compatibility matrix (a customer cannot be assigned to a worker due to legacy constraints).
- Time-conflicts must be avoided to obtain workers' planning with no overlapping in time.

A high quality solution for a WSRP should have both low operational costs and a penalty cost referring to high quality of service for customers and workers.

In this section, we introduce a MILP that deals with the WSRP. The proposed MILP simultaneously considers the following set of constraints:

1. the assignment of tasks to workers to define trip for workers;

2. the assignment of workers to tasks considering the number of workers required by each task

3. the assignment of tasks depending on the workers' working time window that differ for each worker according to their individual contract; and the satisfaction of the workers' preference regions;

4. the respect of the contracts between workers and customers;

5. the departure/arrival time of the workers at the task location depending on the transportation time and the finishing time of the previous task;

6. the starting time of each task must comply with the task time window and the arrival time of the workers.

3.2 Notations for parameters

W	set of workers.
T	set of tasks.
$dist_{i,j}$	distance from task i to task j.
p_j^w	worker w's cost to perform j.
Dur_j	processing time of the task j
r_j	number of workers required to achieve the task j.
pw_j^w	level of satisfaction ($pw_j^w \in [0,1]$) when worker w is assigned to task j.
pa_j^w	level of satisfaction ($pa_j^w \in [0,1]$) when worker w is assigned to task j considering the regional preferences (task j is located into one region k).
ps_j^w	level of skill satisfaction ($ps_j^w \in [0,1]$) with $ps_j^w = Max\ (ps_j^i)$ when the task j is assigned to worker w.
ρ_j^w	$\rho_j^w = (pa_j^w + pw_j^w + ps_j^w)\ \forall\ w \in W, j \in T$ is the quality of service.
$[e_j\ , l_j]$	time window of task j.
$[TW_{inf}^w\ , TW_{sup}^w]$	working time window of worker w.

3.3 Notations for decision variables

Binary variables:

$$x_{i,j}^w \quad \begin{cases} =1 \text{ if worker } w \text{ moves from task } i \text{ to } j \\ =0 \text{ otherwise} \end{cases}$$

$$x_{0,j}^w \quad \begin{cases} =1 \text{ if worker } w \text{ moves from departure} \\ \quad\quad \text{location to task } i \\ =0 \text{ otherwise} \end{cases}$$

$$x_{i,0}^w \quad \begin{cases} =1 \text{ if worker } w \text{ moves from task } i \\ \quad\quad \text{to finishing location } 0 \\ =0 \text{ otherwise} \end{cases}$$

$$\psi_j^w \quad \begin{cases} =1 \text{ if the worker } w \text{ is assigned to a task } j \\ \quad \text{situated outside } w\text{'s preference region} \\ =0 \text{ otherwise} \end{cases}$$

$$\theta_j^w \quad \begin{cases} =1 \text{ if time window violation occurs when task } j \\ \quad\quad \text{is assigned to worker } w \\ =0 \text{ otherwise} \end{cases}$$

Continuous variables:

y_j	number of workers not available to achieve the task j.
t_j	starting time of task j.
d_j^w	departure time of worker w from task j.
a_j^w	arrival time of worker w to task j.

3.4 Linear Programming

$$Min\ f(S) = \lambda_1 \sum_{w=1}^{W} \sum_{i=0}^{T} \sum_{j=1}^{T} (\,dist_{i,j} + p_j^w) x_{i,j}^w + \lambda_2 \sum_{j=1}^{T} \left(3r_j - \sum_{i=0}^{T} \sum_{w=1}^{W} \rho_j^w\, x_{i,j}^w \right) + \lambda_3 \sum_{j=1}^{T} \sum_{w=1}^{W} (\psi_j^w + \theta_j^w) + \lambda_4 \sum_{j=1}^{T} y_j$$

s.t.

$\sum_{j=0}^{T} x_{i,j}^{w} \leq 1$	$\forall\, i = 0..\lvert T\rvert,\ w = 1..\lvert W\rvert$	(1.1)
$\sum_{i=0}^{T} x_{i,j}^{w} \leq 1$	$\forall\, j = 0..\lvert T\rvert,\ w = 1..\lvert W\rvert$	(1.2)
$\sum_{i=0}^{T} x_{i,j}^{w} = \sum_{u=0}^{T} x_{j,u}^{w}$	$\forall\, j = 0..\lvert T\rvert,\ w = 1..\lvert W\rvert$	(1.3)
$\sum_{w=1}^{W}\sum_{i=0}^{T} x_{i,j}^{w} + y_j = r_j$	$\forall\, j = 1..\lvert T\rvert$	(2)
$\psi_j^w + M.pa_j^w \geq \sum_{i=0}^{T} x_{i,j}^{w}$	$\forall\, w = 1..\lvert W\rvert,\ j = 1..\lvert T\rvert$	(3)
$M.\theta_j \geq t_j + Dur_j - TW_{sup}^{w} + \left(\sum_{i=0}^{T} x_{i,j}^{w} - 1\right).M$	$\forall\, w = 1..\lvert W\rvert,\ j = 1..\lvert T\rvert$	(4.1)
$M.\theta_j \geq TW_{inf}^{w} - t_j + \left(\sum_{i=0}^{T} x_{i,j}^{w} - 1\right).M$	$\forall\, w = 1..\lvert W\rvert,\ j = 1..\lvert T\rvert$	(4.2)
$\sum_{i=0}^{T} x_{i,j}^{w} \leq c_j^w$	$\forall j = 1..\lvert T\rvert,\ w = 1..\lvert W\rvert$	(5)
$d_j^w \geq \left(t_j + Dur_j\right) + \left(\sum_{i=0}^{T} x_{i,j}^{w} - 1\right).M$	$\forall w = 1..\lvert W\rvert,\ j = 1..\lvert T\rvert$	(6.1)
$a_j^w \geq \left(d_i^w + dist_{i,j}\right) + \left(x_{ij}^{w} - 1\right).M$	$\forall w = 1..\lvert W\rvert,\ i = 0..\lvert T\rvert, j = 1..\lvert T\rvert$	(6.2)
$t_j \geq a_j^w$	$\forall w = 1..\lvert W\rvert,\ j = 1..\lvert T\rvert$	(7.1)
$t_j \geq e_j$	$\forall\, j = 1..\lvert T\rvert$	(7.2)
$t_j \leq l_j$	$\forall\, j = 1..\lvert T\rvert$	(7.3)
$x_{ij}^{w}, \theta_j^w, \psi_j^w \in \{0,1\}, y_j \in \mathbb{N},\ t_i, a_j^w, d_j^w \in \mathbb{R}$	$\forall w = 1..\lvert W\rvert, i = 0..\lvert T\rvert, j = 1..\lvert T\rvert$	(8)

Constraints can be divided into six sets: each set of equations represents one type system constraints.

1) The assignment of tasks to workers to define trips (constraints 1.1, 1.2 and 1.3)

The trip of a worker w from task i to task j is defined by $x_{i,j}^{w} = 1$ (task 0 is the starting location and task $|T|$ is the finishing location). Hence, constraints 1.1, 1.2 and 1.3 ensure that for a worker w: from a task i a worker w can travel to at most one other task (constraint 1.1); to a task j a worker can travel from at most one task (1.2); and if w is assigned to a task j after task i then $x_{i,j}^{w} = 1$ and (1.3) can be rewritten as $\sum_{u=0}^{T} x_{j,u}^{w} = 1$ meaning that w must travel to another task (or to the finishing location) and cannot stay in this position.

2) The assignment of workers to tasks considering the number of workers required by each task (constraint 2)

The number of assigned workers to task j must comply with r_j i.e. with the number of required workers to achieved task j (constraint 2). y_j is the number of missing workers to achieved task j and it is defined by: $y_j = r_j - \sum_{w=1}^{W}\sum_{i=0}^{T} x_{i,j}^{w}$. If a task j requires 3 workers ($r_j = 3$), but only one worker w_1 is assigned to j so there exists one and only one $x_{i,j}^{w} = 1$ and (2) can be rewritten as $\sum_{w=1}^{W}\sum_{i=0}^{T} x_{i,j}^{w} + y_j = 1 + y_i = 3$. By consequence, $y_i = 2$ that is the number of missing workers to perform the task.

3) The assignment of tasks depending of the workers' preferences (constraints 3, 4.1. and 4.2)

Ideally, a worker should be assigned to tasks in his available geographical regions. However preference region violations are possible and have to be measured to define the third part of the objective function. In the constraint 3, if $pa_j^w = 0$, the worker w is assigned to task j which is located into a non wished region, constraint 3 can be rewritten as, $\psi_j^w \geq \sum_{i=0}^{T} x_{i,j}^{w}$ implying $\psi_j^w \geq 1$.

When both tasks i and j are assigned to worker w, $x_{i,j}^{w} = 1$ and the constraints 4.1:

$$M.\theta_j \geq t_j + Dur_j - TW_{sup}^{w} + \left(\sum_{i=0}^{T} x_{i,j}^{w} - 1\right).M \qquad (4.1.)$$

$$M.\theta_j \geq TW_{inf}^{w} - t_j + \left(\sum_{i=0}^{T} x_{i,j}^{w} - 1\right).M \qquad (4.2.)$$

can be rewritten as:

$$M.\theta_j \geq t_j + Dur_j - TW_{sup}^{w} \qquad (4.1.)$$

$$M.\theta_j \geq TW_{inf}^{w} - t_j \qquad (4.2.)$$

Constraint (4.1) ensures that if the completion time $(t_j + Dur_j)$ of task j exceeds TW_{sup}^{w}, the binary variable θ_j is set to 1. Since $t_j + Dur_j - TW_{sup}^{w} > 0$, the constraint (4.1.) is rewritten as $M.\theta_j \geq t_j + Dur_j - TW_{sup}^{w} > 0$ and ensures that $\theta_j = 1$. Constraint (4.2.) ensures that $\theta_j = 1$, if $t_j < TW_{inf}^{w}$, i.e. when $TW_{inf}^{w} - t_j > 0$, meaning that worker w has to perform task j before the end of his working time window.

4) The assignment of tasks to workers considering the legacy constraints meaning that a task can be assigned to authorized workers (constraints 5) only

In the scenarios tackled in this paper, a worker can achieve a task requires by a customer if and only if the worker has been included in the customer's contract (5), in this case $c_j^w = 1$, otherwise $c_j^w = 0$ and $\sum_{i=0}^{T} x_{i,j}^{w} < 0$ that enforces $x_{i,j}^{w} = 0$.

5) The departure and arrival time of the workers to the tasks (constraints 6.1 and 6.2.)

If a worker w is assigned to a task j after a task i, so $x_{i,j}^{w} = 1$ then constraint (6.1) can be rewritten as $d_j^w \geq \left(t_j + Dur_j\right)$ implying that the departure time d_j^w of worker w from task j

must be greater than the starting time t_j of j plus the processing time Dur_j of j. If $x_{i,j}^w = 0$, (6.1) can be rewritten as: $d_j^w \geq -M$ and this constraint holds.

Constraint (6.2) ensures that if a worker w is assigned to task j after task i ($x_{i,j}^w = 1$) so $a_j^w \geq \left(d_{s_i}^w + dist_{i,j}\right)$ must hold, implying that the arrival time a_j^w of w on the task j is after his departure time d_i^w from task i plus the distance between the tasks i and j. If $x_{i,j}^w = 0$, (6.2) can be rewritten as: $a_j^w \geq -M$ and this constraint holds.

6) The starting times of each task according to their time windows and arrival time of the required workers (constraints 7.1. and 7.2.)

The starting time t_j of task j must be after the arrival time a_j^w of all the required workers w to perform the task (7.1). Moreover, a task j has a time window $[e_j, l_j]$ and can start only after the beginning e_j of its time window (7.2) and before the end of its time window (7.3).

<u>The objective function</u>

The objective function to be minimized involves four criteria which are balanced by four weights $(\lambda_1, \dots, \lambda_4)$ corresponding to priority levels. The associated weights to each criteria clearly reflect the difference between the priority levels and are defined in (Algethami and Landa-Silva, 2017).

Since one of the most important objectives of the service is to perform as many tasks as possible, the highest priority in the objective function is given to minimize unassigned tasks through weight λ_4.

Since, in practical case (in particular in HHC, see (Rasmussen et al., 2012) and (Laesanklang and Landa-Silva, 2017)), some workers must accept to perform some tasks that are over their working time window or outside their available geographical region; the second highest priority in the objective function is to minimize the number of workers time availability violations and the number of working region preference violations, through weight λ_3.

Weight λ_2 gives priority to minimize the preferences penalties: preferred workers-customers pairing, workers preferred regions and customers preferred skills. The degree of satisfaction of these preferences when worker w is assigned to task j is given by $\rho_j^k = pa_j^w + pw_j^w + ps_j^w$ which has a value in the range $[0, 3]$ since the satisfaction of the three types of preferences for each assignment has value $pa_j^w, pw_j^w, ps_j^w \in [0,1]^3$: a value of 1 means that w is fully satisfied, a value 0 meaning w is not at all satisfied. The satisfaction level is reverted to penalty by subtracting it from the maximal satisfaction score, which is $3r_j$ for each task j. These values are given by (Pinheiro et al., 2016).

Lastly, through weight λ_1, the lowest priority is given to minimize the monetary cost that represents the operational cost in terms of total travel cost $dist_{i,j}$ and worker's salary p_j^w.

4. NUMERICAL EXPERIMENTS

4.1 WSRP instances

The experiment has been achieved considering instances provided by (Pinheiro et al., 2016) and (Laesanklang and Landa-Silva, 2017) from real-world HHC scenarios in the United Kingdom.

These instances offer the data for six different scenarios and seven different planning periods resulting in 42 instances. The instances are classified into two datasets: small one and large one. The smallest instances are those referred to as WSRP-A and WSRP-B, whereas the largest instances are those referred to as WSRP-C, WSRP-D, WSRP-E and WSRP-F, each dataset contains seven instances from 01 to 07.

4.2 Experiment conditions

The experiments have been performed under Windows 7 on a 3.60 GHz Intel Core i7-4790 computer with 16.0 GB RAM, which is equivalent to 2671 MFlops according to Dongarra (http://www.roylongbottom.org.uk/linpackresults.htm). The computer used by (Laesanklang and Landa-Silva, 2017) has the same number of MFlops.

The linear programs have been solved by CPLEX 12.7 with the parameter "set threads 1": in this case the CPU times of CPLEX is equal to the User Time.

4.3 Results

Table 2 introduces: the best solution from (Laesanklang and Landa-Silva, 2017), with the objective value and the computational time; the optimal solutions found by CPLEX and the CPU time of CPLEX to find the optimal solutions. Let us note that the data of the instance C-02 are incoherent and so the instance is impossible to solve.

Table 2. Linear programming results

			(Algethami and Landa-Silva, 2017)		**Proposed MILP**	
	$\|T\|$	$\|W\|$	S*	t (sec)	S*	t (sec)
A-01	32	23	3.49	7.00	3.119	2.11
A-02	31	22	2.49	8.00	1.608	2.18
A-03	38	22	3.00	14.00	1.978	5.23
A-04	28	19	1.42	5.00	0.521	1.87
A-05	13	19	2.42	1.00	1.574	0.21
A-06	28	21	3.55	5.00	2.516	1.28
A-07	13	21	3.71	1.00	2.559	0.11
B-01	36	25	1.70	21.00	0.294	7.06
B-02	12	25	1.75	2.00	0.684	0.07
B-03	69	34	1.72	6003.00	0.229	82.47
B-04	30	34	2.07	25.00	0.429	6.50
B-05	61	32	1.82	585.00	0.355	46.21
B-06	57	32	1.62	184.00	0.257	38.11
B-07	61	32	1.79	300.00	0.262	47.71
C-01	177	1037	114.21	301.32	113.406	8959.79
C-02						
C-03	150	1077	103.52	550.29	103.449	1775.47
C-04	32	979	11.15	90.00	11.129	23.71
C-05	29	821	12.44	55.00	12.427	7.51
C-06	158	816	140.44	323.52	140.319	615.81
C-07	6	349	4.30	1.00	4.304	0.17
AVG			**20.93**	**424.11**	**20.07**	**581.179**

The average value of the objective of the MILP resolution is 20.07 in 581.179 seconds and all instances are proved to be optimal, while the average value of the (Laesanklang and Landa-Silva, 2017) is 20.93 in 424.11 seconds, but the solutions are not proved optimal.

When the instances have less than 40 tasks, the linear formulation is solved in few seconds whatever the numbers of workers. CPLEX solves these instances in less than 15 seconds although the numbers of workers can be high: 821 workers for instance C-05. The instance C-04 has 32 tasks but required a longer time to be solved (23 seconds), due to a huge number of workers: 979. The instances with more than 100 tasks have a long CPU time up to four hours for instance C-01 due to a huge number of variables, in particular $x_{i,j}^{w}$ since $w \in [0;\ 1037]$ and $i, j \in [0; 177]^2$.

Table 3 provides the details of the optimal solutions found by CPLEX. For convenience, we define:

- $z_1 = \sum_{w=1}^{W} \sum_{i=0}^{T} \sum_{j=1}^{T} \left(dist_{i,j} + p_j^w \right) x_{i,j}^w$
- $z_2 = \sum_{i=0}^{T} \sum_{w=1}^{W} \rho_j^w \, x_{i,j}^w$
- $z_3 = \sum_{j=1}^{T} \sum_{w=1}^{W} \left(\psi_j^w + \theta_j^w \right)$
- $z_4 = \sum_{j=1}^{T} y_j$

Let us note that very few instances have a $z_4 > 1$ (i.e. some unassigned tasks) which is heavily penalized in the objective function because it is the priority criteria to minimized thanks to a λ_4 bigger than the others λ.

Table 3. Details of the solutions

	S*	z_1	z_2	z_3	z_4
A-01	3.119	555.60	60.38	4	0
A-02	1.608	529.95	73.62	2	0
A-03	1.978	639.78	96.21	3	0
A-04	0.521	407.90	69.92	0	0
A-05	1.574	237.25	25.15	1	0
A-06	2.516	511.73	70.08	4	0
A-07	2.559	247.71	24.33	3	0
B-01	0.294	951.24	98.16	0	0
B-02	0.684	318.43	28.48	0	0
B-03	0.229	1235.32	197.3	0	0
B-04	0.429	378.64	77.68	0	0
B-05	0.355	1153.75	164.56	0	0
B-06	0.257	917.87	156.85	0	0
B-07	0.262	1000.82	167.56	0	0
C-01	113.406	43797.19	167.88	136	1
C-02					
C-03	103.449	349828.82	155.39	128	1
C-04	11.129	4625.08	28.56	18	0
C-05	12.427	3304.08	21.23	21	0
C-06	140.319	38755.94	181.46	119	2
C-07	4.304	1408.63	7.99	5	0

5. CONCLUDING REMARKS

This paper introduces a linear formulation for a Workforce Scheduling and Routing Problem which has been benchmarked on literature instances. The linear formulation is able to solve instances up to 177 workers and 1077 tasks. For the smallest instances up to about 40 tasks, the linear programming solves them in few seconds, and the biggest instances require several hours.

Our researches are now directed on resolution of the biggest instances with more than 400 tasks up to 1500 and a meta-heuristic may be considered.

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