# Autoregressive Parameter Estimation for Equalizing Vibrotactile Systems

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*Musical performance, involving highly physical and cognitive expertise, can benefit from vibrotactile feedback systems offering independent control of amplitude and frequency of vibration over a wide frequency bandwidth. For advanced musical use, it is essential to perform the characterization of the amplifiers and actuators involved, as well as the equalization of their overall frequency response characteristics, a step typically implemented with the help of manually configured parametric equalizers. This paper proposes an autoregressive method that automatically estimates minimum-phase filter parameters, which by design, remain stable upon inversion. This approach is shown to offer a less heuristic approach to equalization. We demonstrate this method with an example implementation and discuss the degree of equalization achieved using it.*

## Introduction

Haptic feedback has become commonplace in digital devices, particularly in the form of vibration, using different types of vibrating actuators (i.e. vibrotactile feedback). However, current systems are predominantly based on Eccentric Rotating Mass (ERM) motors, or more recently, Linear Resonant Actuators (LRAs), which result in systems with a limited frequency response range or exhibiting a coupled control of the amplitude and frequency of vibration. A plethora of such systems are available, including general tools such as the *TECHTILE Toolkit*, which enable prototyping and testing of vibrotactile stimuli [19].

Vibrotactile feedback has also been a popular area of interest for musical applications [22], implemented as either Digital Musical Instruments (DMIs) or general-purpose haptic interfaces [5]. Musical interaction involves highly developed physical and cognitive expertise [20], potentially benefiting from large bandwidth [8, 3], and decoupled control of the amplitude and frequency of vibration [6, 17]. This is made possible through the selection of specific actuators typically excited by Alternating Current (AC) input signals, and therefore requiring an additional amplifier to drive them [4, 16]. The implementation of some such systems has previously relied on commercial high-fidelity amplifiers such as *Bryston 2B-LP*[3, 4], or *Yamaha P2700*[21], which are bulky (8 kg, 24 kg respectively), and expensive (costing approx. 2500 US$). Though providing excellent response, such implementations can hinder portability and potentially lead to impractical implementations [14].

Vibrotactile actuators and their driving amplifiers have been characterized in terms of their frequency and amplitude responses, frequency resolution [16], and frequency content [4], primarily focusing on the bandwidth of 40-1000 Hz, where humans perceive vibration [26]. In the characterization of the frequency content of such systems, some low-cost (<10 US$) amplifier boards were found to have a low-fidelity output, introducing harmonic distortion into the signal [4]. Such harmonic distortion can influence the results of vibrotactile experiments [15, 24], and is commonly quantified in electro-mechanical vibrating systems in the form of Total Harmonic Distortion (THD) calculations [12]. Alongside the amplifiers, the actuators also commonly possess intrinsic non-linearities and frequency characteristics that can influence the accurate reproduction of an intended vibration signal [21].

Interfaces intended for musical applications require high accuracy, especially where psycho-physical experiments are concerned [18]. This is typically achieved through the characterization of the system and equalization of the vibrotactile output. The characterization of vibrotactile systems also helps provide verified data, which in-turn allows for refinement and better interpretation of experimental results [21]. While some of the existing toolkits address portability and simplicity [19, 9], they are not characterized and do not offer insights on factors like fidelity and frequency response, or lack an equalized output. An implementation of such a toolkit is being addressed more elaborately in the first author's master's thesis [2], where one of the primary objectives is to equip the New Interfaces for Musical Expression (NIME) and other Do-It-Yourself (DIY) music communities with a portable, affordable advanced haptic toolkit that would aid the exploration of more detailed haptic responses, particularly in music related haptic research.

In this paper, we focus on the equalization of the frequency response of the vibrotactile display system, which is often observed to have some resonant structure. This resonant structure has previously been equalized through manually configured parametric equalizers [6, 16, 21], which compensates the linear component in the system's frequency response. This form of equalization has been shown to improve frequency discriminability of the system [16], offer a more accurate reproduction of an intended vibrotactile output [6], and is often key to musical applications displaying simultaneous vibration frequencies [3]. Some alternative systems also attempt to correlate the output and input of a vibrotactile system to produce a "perceptually transparent" rendering of tactile sensations, by encoding the relation between the input control voltage of an ERM and the perceived intensity of vibration, in the form of a function [23]. However, this approach still relies on ERMs, where one and only one input parameter can exist, either perceived magnitude of vibration, or frequency.

Here, we use an auto-regressive system identification technique to estimate the frequency response of the vibrotactile system in the form of a parametric model. The technique models the system in the form of a minimum-phase filter that guarantees stability upon inversion, as opposed to manually configuring filter parameters. In addition to demonstrating the technique, we evaluate the degree of equalization achieved by measuring spectral flatness of the actuator's equalized response.

## Methodology

This section discusses issues on actuator selection and measurement, followed by the theory of the autoregressive Yule-Walker method being proposed for the equalization step.

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### Actuators: Selection, and Measurement

Various types of actuator technologies are available for producing vibrotactile feedback, each offering pros and cons to different applications. LRAs are optimized to provide high energy output over a very narrow bandwidth, resulting in fixed frequency implementations [4], while ERMs alter the frequency of vibration with a changing of amplitude. However, these technologies do offer lower power consumption [4]. Voice Coil Actuators (VCAs) and Piezoelectric actuators, on the other hand, offer independent control of amplitude and frequency [16] but require an amplifier, while Piezo actuators require additional voltage biasing circuitry [21].

Once an actuator is selected, its frequency characteristics are measured by observing the vibration output for an input sine signal of varying frequency [16] or from transfer function calculations obtained by inputting a logarithmic sine-sweep [4]. The vibration output of the actuator is typically observed with the help of a low-mass accelerometer affixed to the actuator [4, 21], which in turn is also sometimes mounted to a heavy rigid mass to decouple the measurement from structural vibrations [1].

### Autoregressive Parameter Identification

Kay and Marple [11] surveyed and compared various methods to estimate the Power Spectral Density (PSD) of a process. Among these, deterministic and stochastic processes found in practice are said to be well approximated as white noise filtered by a rational transfer function system, that relates an input sequence $\{x_n\}$ to an output sequence $\{y_n\}$ by the linear difference equation:

$$y_n = \sum_{l=0}^{q} b_l x_{n-l} - \sum_{k=1}^{p} a_k y_{n-k}$$

The transfer function derived from the above equation is the one of a linear filter (ARMA model), consisting of an Autoregressive (AR) part, and a Moving Average (MA) part, which writes:

$$H(z) = \frac{B(z)}{A(z)}$$

where,

$$B(z) = \sum_{l=0}^{q} b_l z^{-l} \qquad \text{(Z-transform of the MA part)}$$

$$A(z) = 1 + \sum_{k=1}^{p} a_k z^{-k} \qquad \text{(Z-transform of the AR part)}$$

For a discrete-time system, evenly sampled at $\Delta t$ seconds, driven by a zero-mean white noise of variance $\sigma^2$, the PSD of the ARMA model output is expressed as [11]:

$$P_{ARMA}(f) = P_y(f) = \sigma^2\, \Delta t \left| \frac{B(e^{j2\pi f \Delta t})}{A(e^{j2\pi f \Delta t})} \right|^2$$

An ARMA process $y_n$ is entirely determined by parameters $a_k$, $b_l$ and $\sigma^2$, where $a_0 = b_0 = 1$. When all $b_l = 0$ but $b_0$ $(= 1)$, the model reduces to an all-pole (AR)system of order $p$, which PSD simplifies as:

$$P_{AR}(f) = \frac{\sigma^2 \Delta t}{\left|1 + \sum_{k=1}^{p} a_k e^{-j2\pi f k \Delta t}\right|^2} \qquad (1)$$

The estimation of the PSD consists then in estimating $\{a_1, a_2, \cdots, a_p\}$ and $\sigma^2$ [11].

### Autoregressive Yule-Walker Estimation

The Yule-Walker equations [29, 27] establish a relation between the AR parameters $\{a_1, a_2, \cdots, a_p\}$, $\sigma^2$ and the autocorrelation sequence of $y_n$. This relation is as follows [11]:

$$R_{yy}(k) = \begin{cases} -\sum_{l=1}^{p} a_l R_{yy}(k-l), & \forall \quad k > 0, \\ -\sum_{l=1}^{p} a_l R_{yy}(-l) + \sigma^2, & \text{for} \quad k = 0. \end{cases} \qquad (2)$$

By selecting $p$ equations for $k > 0$ in Eq. (2), we can solve for $\{a_1, a_2, \cdots, a_p\}$, and then compute $\sigma^2$ for $k = 0$. This can be expressed in matrix form as shown in Eq. (3), whose solution is found with $p+1$ estimated autocorrelation lags where $R_{yy}(-m) = R^*_{yy}(m)$ [11].

$$\begin{bmatrix} R_{yy}(0) & R_{yy}(-1) & \cdots & R_{yy}(-p) \\ R_{yy}(1) & R_{yy}(0) & \cdots & R_{yy}(-p+1) \\ \vdots & & & \vdots \\ R_{yy}(p & R_{yy}(p-1) & \cdots & R_{yy}(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_p \end{bmatrix} = \begin{bmatrix} \sigma^2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (3)$$

The linear system in Eq. (3) can be solved efficiently by using the *Levinson-Durbin* algorithm, which recursively computes the parameter sets $\{a_{1m}, a_{2m}, \cdots, a_{pm}\}$, for $m = \{1, 2, \cdots, p\}$ where $p$ is the order of the AR process [11].

Parameters $\{a_{11}, \cdots, a_{pp}\}$ can be interpreted as reflection coefficients, namely $\{K_1, K_2, \cdots, K_p\}$. It appears that the necessary and sufficient stability condition of the AR model, which requires all the poles of $A(z)$ to lie within the unit circle $z = e^{j2\pi f \Delta t}$, is simply that all $|K_k| \leq 1$, for $k = 1, 2, \cdots, p$. The resulting AR model is proven to be stable and minimum-phase, i.e. whose inverse is also stable [25] [1]. Notice that the inverse filter $A(z)$ is a Finite Impulse Response (FIR) filter.

## Implementation

To demonstrate this method, we measure a voice coil actuator, characterize its frequency response to design its inverse filter, and then investigate the same actuator's response, when the inverse filter is applied. We now present the apparatus and methodology used to perform these measurements.

### Measurement Apparatus

The VCA chosen for this demonstration is a *Haptuator Redesigned TL-002-14R* [2] by *Tactile Labs* with impedance = 6 Ω, bandwidth = 50-500 Hz, and weighing 11 g. A lightweight (0.2 g) single-axis accelerometer, PCB Piezotronics 352C23, is affixed to the shell of the Haptuator, using an instant glue[3] that is recommended by the manufacturer for adhesive mounting. A PCB Piezotronics 482B11 line conditioner is used to supply the biasing current required for the accelerometer's operation, and for amplifying its output. The accelerometer's signals are recorded in MATLAB by a National Instruments USB-4431 data acquisition unit. The actuator is also fixed to a rigid mass (stone pestle) weighing approx. 897 g, placed on a carpeted floor. The accelerometer cable is tucked under the pestle to create a relief, and the axis of the accelerometer is aligned with the axis of the VCA's vibrating element. This apparatus is illustrated in Figure 1.

[1] Link: CCRMA, J. O. Smith: Minimum Phase Definition.
[2] Link: TactileLabs, HaptuatorRedesignSpec_v1.0.pdf
[3] Loctite 454, debonded with Acetone

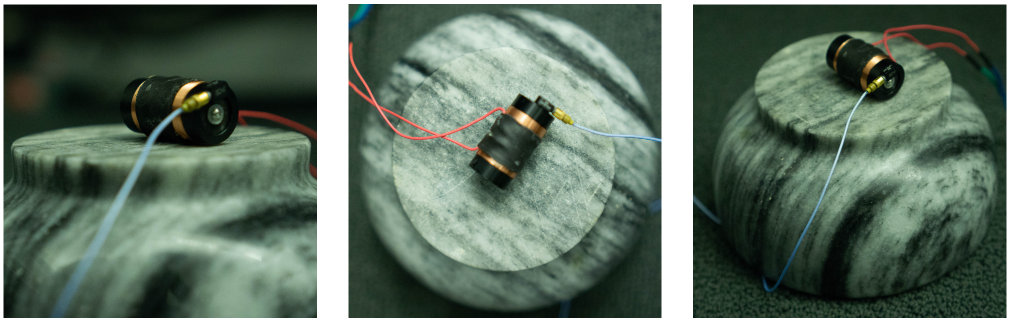


*Figure 1: Haptuator Redesigned, mounted onto a stone pestle, with accelerometer affixed to its shell.*

White noise signals of 1 second length are generated in MATLAB, low-pass filtered at 1 kHz, and fed to the actuator with the help of an RME Fireface UC Audio Interface, whose output is amplified by a *Bryston 2B-LP* reference amplifier. The output of the Bryston is set to output a white noise signal at 1 $V_{rms}$. The signals are generated and sampled at 48 kHz.

### Actuator Measurements

The recordings of the actuator being driven with white noise are repeated 5 times. The PSDs of each of the 5 recordings are computed and then averaged in order to reduce the variance of the PSD estimator, mitigating the effects of noise on the parameter estimation process. The averaged PSD is shown in Figure 2. The magnitude spectrum is obtained as the square root of the PSD and then multiplied with a random phase between $[0, 2\pi]$ to make it possible to perform an Inverse Fourier transform operation. The result is then Inverse Fourier transformed into a time series vector, whose autocorrelation function is used to compute the autoregressive Yule-Walker parametric model.

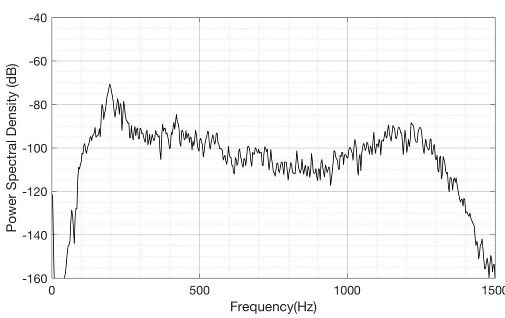



*Figure 2: Welch PSD estimate averaged over 5 measurements. Welch estimates are used for smoother visualization.*

### Parametric Model

The bandwidth requirement of vibrotactile systems has been known to extend up to 1 kHz [26] and sometimes higher among the hearing-impaired demographic [28]. In this case, we aim to assess the actuator up to 1.5 kHz, and as per the data-sheet[2] of the Haptuator Redesigned, driving the actuator above its rated 500 Hz isn't expected to damage the actuator, but rather, render the signal audible. In order to ensure that the Yule-Walker estimation is focused within this bandwidth, we down-sample the measured data to a sample rate of 3 kHz after applying a 10$^{th}$ order Type-I Chebyshev filter as an anti-aliasing stage.

One of the noted remarks regarding the use of autoregressive parameter estimation methods is that the number of parameters is user-defined. As a result, while the accuracy of the estimate tends to increase with an increase in the number of parameters, it becomes hard to discern how many parameters are sufficient to estimate the model adequately [11]. In this case, a minimum of 4 poles was required to model the two resonant features, observed in the actuator's PSD estimate, shown in Figure 2.

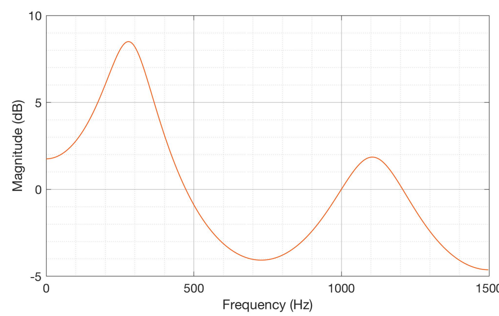



*Figure 3: Autoregressive Yule-Walker estimate using 4 poles*

Figure 3 shows the model obtained with a 4-pole estimate, and Figure 4 shows the accuracy in modelling the overall gain characteristics of the actuator's frequency characteristics, with increasing AR order.

## Results and Discussion

In this section, we present the results of the modelling and inverse filtering process. The model estimated by a 12-pole Yule-Walker analysis is selected heuristically, and its inverse model is derived. The frequency characteristics of the resulting inverse filter are shown in Figure 5.

The measurements are then repeated by applying the inverse filter to pre-emphasize the white noise signal, before feeding it to the actuator. It should be noted that the parametric model was designed for a sampling rate of 3 kHz. For this reason, the white noise signal is down-sampled to 3 kHz before applying the inverse filter, and then up-sampled back to 48 kHz, to be able to operate at audio rate. Figure 6 shows the Welch PSD estimate of the Haptuator, measured with the inverse filter applied, and averaged over 5 measurements.

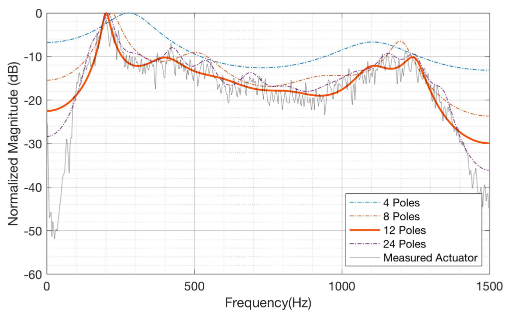



*Figure 4: Estimated models show a visible improvement in the accuracy of modelling of the overall gain structure of the response, for increasing pole orders. The magnitude responses of the estimated models are normalized to their peak values. The magnitude response of the actuator, derived from the Welch PSD estimate, shown in Figure 2, and normalized to its peak value. The model estimated with 12 poles is the selected model.*

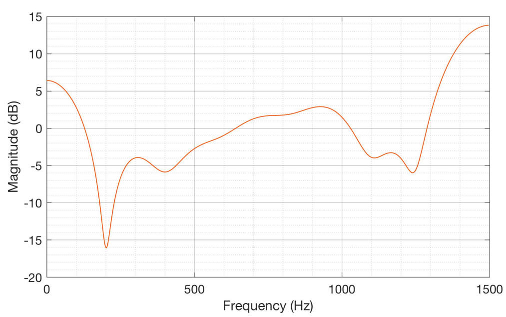



*Figure 5: Inverse filter derived from the 12-pole Yule-Walker estimate, chosen heuristically.*

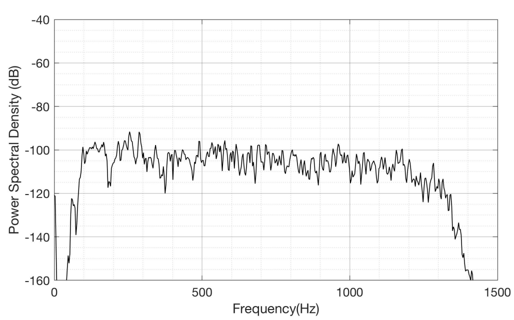



*Figure 6: Welch PSD estimate of the actuator measurement, with inverse filter applied.*

### Estimation of Spectral Flatness

The most elementary assessment of spectral flatness of the inverse filtering process can be done visually, as shown in Figure 7.

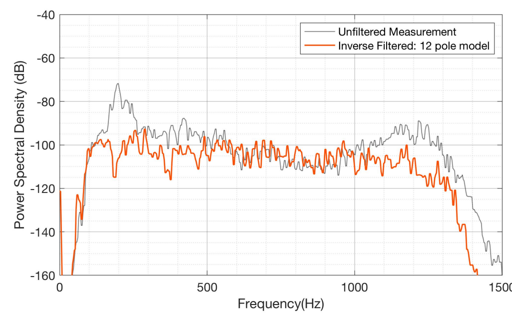



*Figure 7: PSD of the inverse filtered result, superimposed on the original PSD estimate. PSD estimates are smoothed with a 3rd order median filter.*

The degree of spectral flatness can also be estimated as the ratio of geometric mean to the arithmetic mean of the PSD estimate [7]. Additionally, the reflection coefficients $K_p$ of the estimated model decay to a minimum over a certain number of poles. Increasing the order of the model beyond this point has a minimal effect on the overall estimation and would be regarded as over-fitting poles to the model. The spectral flatness estimate in Figure 8a and the decay of reflection coefficients 8b suggest a minimum of 8 poles and a maximum order of 32 poles, to model this device.

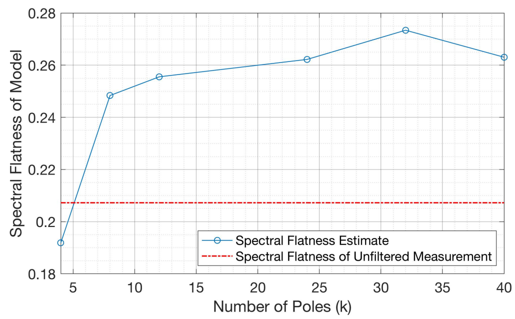
### Discussion

In this type of equalization method, there is a need to operate at a sampling rate much lower than audio rates, in order to restrict the estimation of parameters to within the bandwidth of vibrotactile perception. Traditional Digital to Analog Converters (DACs) are designed to operate at audio sampling rates (44.1 kHz, 48 kHz, or higher). We are often required to operate at audio rates to generate vibrotactile feedback, where amplitude and frequency can be independently controlled and generated through computer programs. There is a fairly extreme sample rate conversion stage (48 kHz to 3 kHz or less), which begins to play a role in the implementation of parameter estimation methods for equalizing vibrotactile systems.

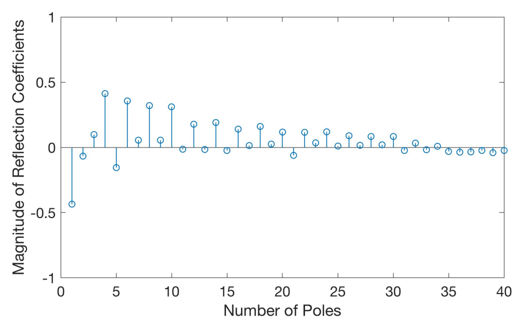
## Conclusion

The equalization of the frequency characteristics in vibrotactile systems has been previously approached using parametric equalizers that are manually configured to compensate the frequency characteristics obtained through measurements [16, 4, 21]. In this paper, we introduce and evaluate the use of the autoregressive Yule-Walker method as a less heuristic alternative.

The measurement of a VCA response is obtained using a lightweight accelerometer and an inverse filter is obtained using a 12-pole Yule-Walker estimate. This inverse filter is applied to equalize the VCA response, by filtering the AC signal that is used to drive it. A spectral flatness estimate of the original and inverse filtered measurements are obtained to help quantify the equalization achieved by the inverse filter. For a 12-pole inverse filter, the spectral flatness of the VCA increased from 0.20 (unfiltered) to 0.25, indicating a 25% improvement in the overall spectral




*(a) Increase in Spectral Flatness Estimate of Inverse Filtered model, increasing up to 32 poles, and then showing a decrease.*




*(b) Reflection Coefficients decay to zero after 32 poles.*

*Figure 8: Evaluating the accuracy of fit.*

flatness. However it is important to note that this is a relative assessment i.e., the response of the unfiltered VCA may already be flat to some extent.

The spectral flatness estimate is also obtained for inverse filters of higher orders. While a higher pole order is observed to yield a higher accuracy in modelling the actuator's frequency characteristics, there is a marginal improvement in the spectral flatness estimate for inverse filters estimated with 12 poles and higher. This suggests that a lower pole order could sufficiently equalize the frequency characteristics of the VCA. However, a minimum of 4 poles was found necessary to model at least the two resonances in the VCA measurement. Therefore, the minimum order required would remain determined by the actuator's frequency profile.

The result of the Yule-Walker estimate is always a minimum-phase filter, that models the frequency characteristics of the VCA. The inverse of this filter is always a stable FIR filter. This eliminates the issue of filter stability, and simplifies the equalization process to a choice of selecting the filter order. Another advantage of this method is that it relies solely on the output characteristics of the Device Under Test (DUT), as long as the input is a zero-mean white noise.

## Future Work

The technique discussed in this paper predominantly accounts for the vibration of the the actuator in its primary axis of vibration. An interesting next step would be to analyze the vibration of actuators using triaxial accelerometers.

The Yule-Walker method is but one among a collection of parameter identification methods, each having pros and cons of their own. In real-time applications, a higher order model would introduce latency into the process, which may be detrimental to scenarios like musical performance, where latency is ideally kept to a minimum [21], typically not exceeding 25 ms for tactile feedback [10]. Therefore, techniques that can achieve a similar or higher accuracy of equalization with a lower order model would be worth investigating.

The method demonstrated in this paper assumes a linear system. Therefore, we only equalize the linear component in the actuator's response. Vibrating transducers can exhibit non-linear operation [12], which might be addressed by methods that can also account for non-linearity [13].

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