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Vitaly Kuyukov

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Quantum tunneling approach of noncommutative geometry

Kuyukov Vitaly Petrovich

vitalik.kayukov@mail.ru

SFU, Krasnoyarsk, Russia, 650041

Hawking and Beckenstein's theory of the thermodynamics of black holes indicates that there is a connection between quantum information and gravity. In general, their result is called the holographic principle. According to it, the entropy of a black hole is proportional to the area of the sphere of the event horizon. In this paper, noncommutative geometry is generalized using the holographic principle. Under certain assumptions, it is possible to obtain results from this synthesis regarding the geometry of the Minkowski space-time. To do this, we consider two main provisions for the generalization of noncommutative geometry.

1. The operators of noncommutative geometry are generalized, in which the coordinates of space and the canonical coordinates do not commute with each other.

$$x_k = (x, y, z)$$

$$\{x_k, y_k\} = i \frac{l_p^2}{\pi}$$

$$l_p^2 = \frac{Gh}{c^3}$$

2. Coordinates of space become operators.

$$x_k \Psi = i \frac{l_p^2}{\pi} \frac{\partial \Psi}{\partial y_k}$$

So the derivative takes the canonical coordinates of noncommutative geometry.

From this synthesis of the holographic principle with the Hilbert space of states of noncommutative geometry, we obtain the concept of a complex vector on a plane with coordinates.

$$z_k = x_k + i y_k$$

This allows you to enter a direct distance in the phase space of non-commutative geometry

$$s^2 = \tilde{z}^k z_k = (x^k - i y^k)(x_k + i y_k) = x^k x_k + y^k y_k$$

Thus, we will assume that the distance in the phase space of non-commutative geometry will be a measure of the definition of a new type of geometry, in which the space coordinates and the canonical coordinates have the same physical dimension.

1. Quantum tunneling of noncommutative geometry

In the wave function, Euclidean gravity is considered as the complete quantum state of the Universe in the Euclidean form (Hawking) as a form of tunneling.

$$\Psi_H = \int e^{-I\{dg\}}$$

In this case, the coordinates of space are not just numbers, but the internal characteristics of the quantum state of the Universe, that is, operators.

$$x_k \Psi = i \frac{l_p^2}{\pi} \frac{\partial \Psi}{\partial y_k}$$

If the wave function is a real number, then tunneling takes place. Let the canonical coordinates have imaginary values (spatial coordinates are real numbers).

$$y_k = i q_k$$

We introduce the entropy of a tunnel junction as the natural logarithm of the probability of the wave function

$$S = \ln |\Psi|^2$$

Let the tunneling be isotropic in all directions of space

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r dq_r = \frac{l_p^2}{\pi} \frac{d\Psi}{\Psi}$$

Hence the tunneling parameter

$$q_r = \frac{l_p^2}{2\pi} \int \frac{dS}{r}$$

Such a definition makes it possible to understand the occurrence of a pseudo-Euclidean distance of noncommutative geometry.

$$s^2 = x^k x_k + y^k y_k = r^2 - q_r^2 = r^2 - \left(\frac{l_p^2}{2\pi} \int \frac{dS}{r} \right)^2$$

This structure already differs sharply from the Euclidean definition. It contains special structures of cones, which are divided into specific areas for the phase space of noncommutative geometry. This definition of the pseudo-Euclidean interval, if the entropy of tunneling is isotropic non-commutative geometry, is determined on the surface boundary.

2. Pseudo-Euclidean interval of noncommutative geometry and the structure of space-time.

In the last chapter, a definition of the pseudo-Euclidean interval in the pseudo-Euclidean phase space of non-commutative geometry is obtained, in which quantum tunneling passes through the boundary. And there are special structures, cones. Minkosky spacetime has the same structure.

$$s^2 = r^2 - (c t)^2$$

Here we will consider this as not a coincidence, but a fundamental consequence of the geometry of the Minkowski space-time from the definition of a holographic interval.

$$s^2 = r^2 - \left(\frac{l_p^2}{2\pi} \int \frac{dS}{r} \right)^2 = r^2 - (c t)^2$$

From here, the definition of time is obtained, as the ratio of the entropy at the boundary of the sphere to its radius. This is the definition of arising time. Where the entropy at the boundary of the sphere should be considered as entropy of entanglement between the boundary of the sphere and the point inside, where the moment of time is determined. In general, the resulting time will be as a closed surface integral

$$t = \frac{Gh}{2\pi c^4} \int_A \frac{dS}{r}$$

In this form, you can come to the general formula for any closed arbitrary surface. In this formula, time is determined at a certain point, where a closed surface is taken around through the integral of entropy of entanglement on a given surface. In this case, the differential form of the space-time interval will be

$$ds^2 = dr^2 - \left(\frac{l_p^2}{2\pi} d \int \frac{dS}{r} \right)^2$$

This is the general definition of the new differential interval of space-time. If the time counts equally uniformly in all points of space. The definition of arising time can be given as the ratio of the sphere entropy to the radius.

$$t = \frac{Gh}{2\pi c^4} \frac{S(r)}{r}$$

3. Invariance of the definition of arising time.

Consider a simplified version for the sphere. In a moving system, the radius of the sphere intersects the coordinates and is determined through the Lorentz transformations. If we assume that the entropy of a given spherical surface is the same everywhere in any reference system, that is, the invariant

$$\frac{Gh}{2\pi c^4} dS = (r - v t) dt$$

Then time in the moving frame will have the following definition

$$t^| = \frac{Gh}{2\pi c^4} \int_A \frac{dS}{r^|} = \int \frac{(r - v t)}{r^|} dt = \int \sqrt{1 - \frac{v^2}{c^2}} dt$$

In general, a time dilation formula is obtained for a moving frame of reference. From this it follows that the definition of “surface” time is invariant with respect to the Lorentz transformations. This proof was for a spherical surface, but it is possible to show the validity of arbitrary closed surfaces with the help of special mathematical methods.

Conclusion

In general, the main new position of noncommutative geometry in the synthesis with the holographic principle gives a promising start. First, a new concept of distance in the phase space of non-commutative geometry is obtained. In the second, we can consider noncommutative geometry as a pre-geometry for the resulting space-time when considering the quantum tunneling approach. Thirdly, the arising time takes place in noncommutative geometry and this largely explains the necessity of introducing time as an imaginary coordinate for Minkowski space-time.

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