Physics of Mind: Hidden Symmetries of Brain Space

Benoy Chakraverty

Physics of Mind: Hidden Symmetries of Brain Space

Benoy Chakraverty*

February 3, 2019

Laboratoire d'Etudes des Propriétés Electroniques des Solides, C.N.R.S, 38042, Cedex 9,Grenoble, France[1]

31/01/2019

Abstract

In this article we shall address the physics involved in the Mind process . We shall show that our brain specifically our cortical brain which is the seat of information field that can be taken as a Higgs field and that carries a minimum symmetry SU(3) of a three-level information field . It is a Hilbert space of three complex wave functions. The projective Hilbert space from the brain constitute our *mental space*. We can think of the neurons as the broken gauge symmetry of the cortical brain. In this sense, the three dimensional network of neurons is a gauge fiber - bundle.We will show that the three states of brain characterised by the state of *Consciousness*, (with the state of *deep sleep*) and the state of *REM sleep* are two aspects of only two possible gauge symmetry breaking of cortical brain. The relevance of functional resonance magnetic spectroscopy of brain with respect to our conclusions will be shown.

Part I

Introduction

The main function of human brain is to help us comprehend the world that surrounds us so that we can have a reasonable description of the events of the world as well as an adequate representation so that we can survive in it. This representation is nothing but a transcription of the excitations as they stream in from the outside world into bits and pieces of information thanks to our preprogrammed genetic blue-print. In the brain cells, astrocytes (which are electrically insensitive) and neurons (which are electrically sensitive) , the

*Emeritus Directeur de recherche, C.N.R.S & Retired

[1]Restructured as Institut Louis Neel, C.N.R.S, Grenoble

cortex region has the fundamental task of producing mental representations out of a brain space. As the pixels of information, called qubits or the quantum bits of information pile up, our mind reorganises it all in some recognisable pattern , rebooting its own architecture continually to encounter the dynamics of the world and sets up what we can only call its incessant mental activity. Whether a Mind space was there to start with , just like whether electromagnetic space preceded electromagnetic waves may well be a chicken and egg question, futile in the very least. But what is for certain is that the wealth of information that brain gathers so meticulously and hoards so preciously would be quite meaningless if mind cannot be called in to give it a sense or direction and elaborate strategies of survival. And yet it is the *reality* of mind that we often seem to forget. This is based on the fact that none of the mind processes can be measured or "seen" and in the end it remains totally private . The physics of this process, if there is one, is very frustrating to physicists. For physics has to do with $'all'$ reality in nature and to do so one has introduced the concept of Hermitian operators boosted with a local gauge invariance principle so as to to ensure total objectivity of any measurement . This space is the Hilbert space ([1]) where hermitian operators act on complex functions called wave functions, also called basis vectors, which give eigen values that are measurable. Every machine man has invented is a hermitian machine, conceived to perform a precise measurable task . How can something that is felt to be real and yet remain not-measurable? In this paper we will try to give beginning of some sort of answer, with the irrational belief that physics must hold a key to that locked black box which we call our mind.

This paper has three basic sections: a section concentrating on cortical brain, its Hilbert space and its symmetry elements , a section devoted to conscious mind and the concomitant gauge symmetry breaking and finally the piece devoted to two states of sleep: REM sleep and deep sleep. We will endeavour to show that each of these three state of cortical brain carries a different signature of gauge symmetry breaking. There is a fundamental structural and mathematical unity between these three spaces that cohere to make our brain the extraordinary theater of phenomena that we know it to be. In the end, Mind turns out to be our primal reality.

The paper in order to facilitate its readability cover these topics in the following order:

- Cortex or our brain as a Hilbert space of complex information carrying wave functions : Brain as a Higgs boson
- Neurons are seen as broken symmetry state of the Higgs boson
- Mind space is shown to be a projective Hilbert space
- Conscious state of Brain as a coherent state vector in the coset space
- Rem (rapid eye movement sleep) and Deep sleep states:Two distinct symmetry breaking channels
- Signatures of Consciousness from functional resonance spectroscopy

- Memory states are derived from massless gauge photons

Part II
General Symmetry considerations

1 Some Elemental Topology and aspects of Homotopy

We need to define a few terms so as not to be confused by what we mean by the word topology . Topology has to do with spaces and relationship between them . Topology permits us to define and differentiate geometric nature of one space from another ([2]) . It also tells us how one can go from between two spaces so that one can form a map of one space or another . To give one simple example an ordinary three dimensional sphere is topologically different from a torus . While sphere is compact and connected, a torus is not . Such spaces are characterised by their *genus* which is the number of holes or disconnection the enclosed space contains . A sphere has a genus number zero while a torus has one . These numbers are topological invariants and can be derived from Gauss-Bonnet theorem, called also Euler number or Chern number.

The simplest space is our daily Euclidean space of different dimensions x,y,z etc, that one calls flat Euclidean space space, $\mathbb{E}$. Its dimensionality one or two or three or n is given by the notation $\mathbb{E}^{\mathrm{n}}$. The space of real scalar objects a,b,c are known as a real space manifold $\mathbb{R}$. Next in degree of complexity, one also defines complex numbers of the form $\mathbf{a}+\mathbf{ib}$, where i is the imaginary quantity i=$\sqrt{-1}$. A space made of such numbers is called a complex space $\mathbb{C}$. This is a space, for example of quantum wave functions ψ of Schrodinger equation and the number n of such wave functions or dimensions are denoted by $\mathbb{C}^{\mathrm{n}}$. This has the alternative denotation known as a Hilbert space, with some added requirements on wave functions.Thus $\mathbb{C}^{\mathrm{n}}$ has 2n real dimensions of the space $\mathbb{R}^{2\mathrm{n}}$. We also introduce at this point topological notation of a sphere . What physicists call an ordinary three-dimensional sphere, , topologists call it a 2-sphere, basically a two-dimensional spherical surface denoted by the symbol $\mathbb{S}^2$. A four dimensional sphere will be a 3-sphere $\mathbb{S}^3$. What it is really is that a physicist is talking of the volume of the sphere, while a topologist is talking of the surface of the same sphere. In this paper we shall continually go from one space or notation to another and this should not cause confusion. It should be noted that the topological spheres are always taken to be unit spheres, with a radius $\mathbf{r}$ =1.

One defines a 'round' sphere $\mathbb{S}^{\mathrm{n}}$ by its equation

$$r.r=\sum_{n=0}^{n}\left(x^0\right)^2+\left(x^1\right)^2+\cdots+\left(x^n\right)^2=1 \tag{1}$$

We adopt the topological system of notation that a sphere $\mathbb{S}^{\mathrm{n}}$ is a n-dimensional *manifold* that is embedded in n+1 flat Euclidean space $\mathbb{E}^{\mathrm{n}+1}$. Thus our ordinary sphere designated as $\mathbb{S}^2$ is embedded in 3 Euclidean dimensions, X,Y, Z, $\mathbb{E}^3$ of our ordinary space .

Homotopy is a fundamental relationship between two spaces which tells us whether one space can be continuously deformed into another without tearing or glueing. To go back to our torus, one can do so and transform it into a teacup. Both have just one hole and is characterised by the same homotopy class . To a topologist a doughnut and a teacup with just one handle is one and the same! The important message is that you cannot continuously go from one homotopy class to another . This is also the central idea behind mapping. The invariance that one has to respect is Poincaré index that allows or does not allow such mapping. Let us give a few examples.

The simplest map that comes to mind is a $1 \rightarrow 1$ geographical map of earth's surface to a point on a piece of paper. Every piece of painting or a drawing on a paper or canvas of some object is a map, but not necessarily one to one. Going from three to two dimension is the essence of this mapping. The missing dimension is subtly hidden in the perspective chosen, which is called 'affine connection' in topology. From the few lines here we can understand that there are two kinds of mapping: one to one and many to one. A simple example of the later is the function value of $\cos\theta$ by any reverse mapping or imaging of $\cos^{-1}\theta$ from its value, say 1(e.g $\cos\theta = 1$for all $\theta = 0, \pm n\pi$, with n=1,2,...∞). A true inverse function does not exist. A more substantial example will be a mapping or image y(x), meaning a *field* y, for a set of points x belonging to some spatial domain X . Let us denote by Y, the manifold of possible values for y. A manifold is a sphere $\mathbb{S}^{\mathrm{n}}$. For example let Y=$\mathbb{S}^1$, a circle , if y is a two component vector of fixed length or take Y=$\mathbb{S}^2$ a 2-sphere, if y is a three component vector of fixed length etc. Let us begin with X and use a one dimensional domain interval by $0 \leq x \leq 2\pi$. Consider a field distribution y(x). Hence at each spatial point $x_i \in X$,there is an image point or map y(x_i) in Y. As x varies from 0 to 2π,the image point or field value traces out a curve in Y starting at y(0) and finishing at y(2π). let us limit ourselves to a periodic boundary condition where y(0)=y(2π) $= y_0$. This the mapping from many to one. One calls y_0, the base point , returning at the base after a spatial voyage in X. A compact way to write this is y(∂X) $= y_0$, where ∂Xdenotes the boundary points of X. In our particular case, the image point traces out a closed curve pegged at y_0 . In all configurations that we shall consider there will exist such image loop and *vice versa.* A central motivation of our paper will be *to treat our mind as an image space* . A topological study of the system will reduce to the study of the corresponding image loop in Y ([3]).

There are two basic homotopy rules one has:

- Any two loops belonging to the same homotopy class can be continuously deformed into each other. Thus these set of loops are indistguishable from one another and can continually change from one to the other.

- Loops belonging to different classes cannot be continuously distorted to each other . These sets of images will remain strictly separate and distinguishable.

These classes are called homotopy classes and will lead to Poincaré invariants. Let us take one simple example, taken from Shankar (ref). Let the field space be Y= $\mathbb{E}^2$-$\{0,0\}$ which is an Euclidean space from which the origin is missing ., a restricted space where the null vector is excluded. Let y_0 be any point in this space . It is quite clear that any loop not enclosing the origin can be collapsed to y_0 , but those which do enclose the origin cannot be. These latter loops can be further classified by an integer **m** , which will tell us how many times a given loop is going around the origin . One calls the loops indexed by different values of **m** as non homotopic. Consider the set $\left\{y^0, y^1, y^2....y^\infty\right\}$ where y^m stands for all loops that encircles $\{0,0\}$ point **m**-times. The index m is also known as winding number, a measure of how often one space (x) winds around another space (y) and can be given the definition

$$m = \frac{1}{2\pi} \int_0^{2\pi} \left(\frac{dy}{dx}\right) dx \tag{2}$$

Such a set of y^m constitutes a group , obeying group axioms. The first homotopy group or Poincaré's fundamental group is denoted by $\mathit{\Pi}_1$(Y) where the subscript 1 is indicating that X acquires the form of a closed loop (which is one dimensional, hence the subscript 1) which is topologically $\mathbb{S}^1$, and is a 1-sphere. The integer m indicating the class of loops is represented by an infinite series of additive integers $\mathbb{Z}$ and expressed as

$$\mathit{\Pi}_1(\mathbb{E}^2 - \{0,0\}) = \mathit{\Pi}_1\left(\mathbb{S}^1\right) = \mathbb{Z}_\infty \tag{3}$$

The infinite Abelian group of electromagnetism falls in the same class given by

$$\mathit{\Pi}_1\left(U_1\right) = \mathbb{Z}_\infty$$

The gauge symmetry group U(1) is non compact not simply connected space so that Bohm-Aharanov effect can occur. It can be similarly shown that $\mathit{\Pi}_2\left(\mathbb{S}^2\right)=$ $\mathbb{Z}_\infty$. Here the subscript 2 is saying that the X-space is a 2-sphere going around a Y which is taken to be another 2-sphere. This is second homotopy group of Poincaré. The integer m= 1 map also called identity map where X covers Y as the skin of an orange covers the orange in $\mathbb{S}^2$. The general Poincaré rules of wrapping one topological space (e.g : configuration space X) around another (e.g: field space Y) are given by

$$\begin{aligned} \mathit{\Pi}_n(\mathbb{S}^m) &= \mathbb{Z}_\infty, \text{for n = m} \\ &= I, \text{ for } n \prec m \end{aligned} \tag{4}$$

where I is the identity (not the same as identity map) element .

Let us go back briefly to quantum mechanics where one works with *complex vector space*, called Hilbert space . Like real vector space, its dimension is given by the number of its basis vectors, each one of which is a complex function. A space with one complex element is known as $\mathbb{C}^1$, one with two complex elements is $\mathbb{C}^2$ etc. For example one has

$$\mathbb{C}^1 = \left(\mathrm{x}^0 + \mathrm{ix}^1\right)$$

$$\mathbb{C}^2 = \begin{pmatrix} x^0 + ix^1 \\ x^2 + ix^3 \end{pmatrix}$$

Real space $\mathbb{R}$ is embedded in the higher dimensional space $\mathbb{C}$ just as real numbers are embedded in complex numbers.We note that the complex vector space $\mathbb{C}^1$ is the wave function space of the Schrodinger wave function ψ. Such a space may be carrying a paricle of electrical charge , q. The space $\mathbb{C}^2$ designates a so-called spinor wave function carrying spin or iso-spin σ.

It is important that we introduce here the internal symmetry that concerns us most, that of Lie group. Such a group has a number of elements, call them $\boldsymbol{E}$. To qualify as a member of Lie group, we have to have

$$\frac{\mathrm{d}\boldsymbol{E}}{\mathrm{d}\theta} = i\boldsymbol{E} \tag{5}$$

For such a group, any element can be written in the form

$$\boldsymbol{E(\theta_1, \theta_2, \cdots, \theta_{\mathrm{n}}) = exp\left(\sum_{i=1}^{n} i\theta_i F_i\right)}$$

The quantities F_i are the generators of the group and the θ_i are the parameters of the group. They are a set of i real numbers that are needed to specify a particular element of the group. Note that the number of generators and parameters are the same . There is one generator for each parameter . We now illustrate though several examples.

The group U(1) is the set of all one dimensional complex unitary matrices. The group has one generator F=1 and one parameter, θ.It is given by

$$E(\theta) = e^{-i\theta F} = e^{-i\theta}$$

The group element produces a complex phase change of the wave function. Since the space is $\mathbb{C}^1$, one calls it U(1) symmetry. The group elements commute

$$E(\theta_1)E(\theta_2) = E(\theta_2)E(\theta_1)$$

Such groups are called Abelian.

This is also the inner symmetry of the Schrodinger wave function ψ given by its invariance to rotation in its own charge space, where the scalar electrical charge q is the generator of rotation θ

$$\psi' = \psi exp - iq\theta \tag{6}$$

If we take the space $\mathbb{C}^2$, then writing for the spinor wave function ξ, we will write it as as two row one column complex vectors

$$\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} \tag{7}$$

The unitary transformation of the wave function from one wave function to another is known as U(2).

It has four generators and four parameters,

$$E(\theta_1, \theta_2, \theta_3, \theta_4) = e^{-i\frac{\theta_j F_j}{2}}$$

The generators F_j are given by the four 2x2 Pauli matrices, $\sigma_0, \sigma_1, \sigma_2, \sigma_3$ .we will the invariance of the spinor wave function to internal rotational symmetry in the internal spin space of σ given byIf we require that the matrix be unitary $U(2)U(2)^{\dagger}=1$ and We shall have the generators

$$F_0 = \sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \ F_1 = \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, F_2 = \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , F_3 = \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{8}$$

Since the generators do not commute with each other, this is a non-Abelian group. In case of SU(2) symmetry, one has got an unit determinant, it will remove one parameter and one generator and the symmetry is called the SU(2), the special unitary matrix. The symmetry operation on the spinor wave function as

$$\xi = \xi' = E\xi = exp\left(i\frac{\sigma}{2}\theta\right)\xi = SU(2)\xi$$

The structure of the SU(2) group is defined by its Lie Algebra, with F^0 generator missing and is given by the three remaining generators F_1 , F_2 , F_3 whose commutators are given by

$$[F_i, F_j] = i\epsilon_{ijk}F_k$$

$$[\sigma_i, \sigma_j] = i\epsilon_{ijk}\sigma_k$$

Here ϵ_{ijk} are the Levi-Civita symbol. Just as U(1) symmetry operates in the charge space q of the particle with wave functionψ,the SU(2) symmetry operation is taking place in the spin space of the spinor wave function ξ .

We begin by accepting that a general information field is a field with N-complex wave functions of SU(N) symmetry and may be represented by a scalar field $\Phi(x)$ in its adjoint representaion . An adjoint representation is where number of vectors is of same dimension as the number of generators , which is N^2-1 . The generators T_a form a Lie group of continous *local* rotation in the symmetry space and is given by operators of the form

$$\Omega(x) = exp - iT_k\alpha_k(x) \tag{9}$$

The $\alpha's$ are the rotation angles.The generators T_k of the Lie group form a vector space and has the commutation relationship

$$[T_k, T_l] = if^{klm}T_m \tag{10}$$

Here $f^{k,l,m}$ are the structure constants, completely antisymmetric and the generators do not commute.

2 Brain space as an information field space with SU(3) symmetry

A "classical" Maxwellian electron has only electrical charge and is describable by one complex wave function . Such an object has got just one energy level. When we go to a "Pauli" electron , it gets an extra degree of freedom in the form of an internal quantum number called spin, Pauli spin $\sigma = \frac{1}{2.}$. Now we need two complex functions or two wave functions to describe it, one for Info-spin 'up' and another one for Info-spin 'down'- a two level system. If we translate the vocabulary to quantum unit of information then our Pauli electron is just a *qubit* of information . Our brain can be thought at least just to be that, a store house of quantum information bits which will then qualify it to have a basic symmetry U(2) , that of two complex functions. But we have to add to it , the part that arrives from the outside world (including from our own body), which is in the charge sector . As a result it is an Infospin space as well as a charge space that has symmetry U(1) . Hence we must add a third complex function to the two already existing so that it becomes a composite of three wave functions or a three level system. This we shall express as

$$\Psi = \begin{pmatrix} \psi_a \\ \psi_b \\ \psi_c \end{pmatrix} \tag{11}$$

Thus we will take our Brain as a *homogenous Hilbert space* of three complex functions , a $\mathbb{C}^3$- space. We can also think of the three wave functions ψ_i as complex coefficients of three basis vectors

$$\mid a \succ = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\mid b \succ = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\mid c \succ = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

This way we can write our Brain information field as a wave function

$$\mid \Psi \succ = \psi_a \mid a \succ + \psi_b \mid b \succ + \psi_c \mid c \succ \tag{12}$$

With respect to quark analogy each of the three basis vectors can be considered as *quons* .

These information carrying particles like quarks are info-doublets, u and d and an info singlet s , will have fractional charges. The analogy of these three wave functions with quarks (see:Zuber& Itzkyson ibid) must be remembered. If we characterize the inner symmetry by traceless Hermitian matrices , this will be SU(3).While Pauli spin σ has three symmetry elements or generators, of the SU(2) symmetry given by the three Pauli spin 2x2 matrices, $\sigma_{1,}\sigma_2, \sigma_3$, the SU(3) symmetry has equivalently eight Gell-Mann ([4]) the 3x3 traceless $\lambda-$matrices or generators (an SU(N) matrix has N^2 - 1 symmetry elements which is 8 for N=3). These symmetry elements span an eight dimensional space . These are given in figure 1:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \ \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \ \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \ \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Figure 1: Gell-Mann matrices

We begin by accepting that a general information field is a field with N-complex wave functions of SU(N_m) symmetry and may be represented by a scalar field $\Phi(x)$ in its adjoint representation . An adjoint representation is where number of vectors is of same dimension as the number of generators , which is N^2-1 . The generators T_a form a Lie group of continuous *local* rotation in the symmetry space and is given by operators of the form

$$\Omega(x) = exp - iT_k\alpha_k(x) \tag{13}$$

The $\alpha' s$ are the rotation angles.The generators T_k of the Lie group form a vector space and has the commutation relationship

$$[T_k, T_l] = if^{klm}T_m \tag{14}$$

Here $f^{k,l,m}$ are the structure constants, completely antisymmetric. The information scalar field in the adjoint representation of SU(N_m) is expressed as

$$\Phi(x) = \sum_a \phi_a(x)T_a \tag{15}$$

The T_a's are $N\times N$ matrices of the SU(N_m) group. Because we are considering only local rotation in space-time, the information field $\Phi(x)$ is also a Higgs field. We note that Higg's field is a spin zero field, the most basic singlet representation one can have while the gauge field $\mathscr{A}_\mu$(x) is a spin 1 field like our photons, but unlike photons some of them can carry charge and are often called gauge photons. It has been demonstrated ([5]) that any field in the adjoint representation can act as an effective Higgs field. Orientation of the Higg's vector $\Phi(x)$ in the quon space (color space in quantum color dynamics) at a given space-time point x defines a gauge . In effect, for our our cognitive brain , it tells us to which direction our mind is pointing to , it gives us the possible polarisation of our mind. We will come back to this point later on. The basic idea at this point is that objective nature of any information requires it to be independent of our perspective, of our *mental gauge*. This is built in the simplest gauge-invariant Lagrangian one can write for our field $\Phi(x)$.There are two essential ingredients in such a Lagrangian: One has to introduce a gauge potential $\mathscr{A}_\mu^{\mathrm{i}}$ to counteract any local space-time variation of the information field $\Phi(x)$ in the form of $\frac{\partial \Phi}{\partial x}$ as it completes its trajectory. We call this gauge potential our awareness potential A. One introduces also a self-interaction term between $\Phi(x)$ fields, negotiated by the gauge photons. A typical phenomenological Lagrangian which is standard in the literature ([6]) is written as

$$\mathscr{L} = (D_\mu \phi_{\mathrm{i}})\,(D_\mu \phi_{\mathrm{i}}) - \mathrm{m}^2 \phi_{\mathrm{i}}^* \phi_{\mathrm{i}} - \lambda\,(\phi_{\mathrm{i}}^* \phi_{\mathrm{i}})^2 - \frac{1}{4}\mathscr{M}_{\mu\nu}\mathscr{M}^{\mu\nu} \tag{16}$$

There are four sets of terms in this Lagrangian that one should explain . Note the subscript $\boldsymbol{i}$ is the index of a particular component of the Higgs field vector. The first term is a gradient energy of variation in space and in time: the space-time point μ is three of space and one of time where one does not admit cross-terms. Written in terms of covariant derivative defined as

$$D_\mu \phi_{\mathrm{i}} = \partial_\mu \phi_i + g f_{ijk} A_\mu^j \phi_k \tag{17}$$

The second and third terms of the Lagrangian are quadratic and quartic self-interaction terms with parameters m & λ which can be positive or negative and the last term gives the field energy term , coming from the gauge field tensor, an energy stored in the mind space. It is the price brain pays to make the information objective. When it is present, it appears to our *mind as a stress*. For electro-magnetic gauge it is the term $\left(E^2 + B^2\right)$, the electromagnetic field energy. It is present only in the conscious brain state and absent when we are in deep sleep state . Hence our sense of deep relaxation as we come out of deep sleep, but we will come to these aspects in the next sections.These field tensors are also non-Abelian and do not commute mutually unlike electromagnetic fields which are Abelian. Here $\mu\nu$ are physical space-time indices and g is the coupling constants that connect the gauge potential to quon information field $\Phi(x)$. The superscript on $\mathscr{M}$ − fields will go from 1 to 8 . The A-gauge potentials are vector bosons with spin=1 .

When one minimises the Lagrangian with respect to ϕ_i to set $\frac{\partial \mathrm{L}}{\partial \phi_i}$=0 at some ϕ_i=ϕ_0, one may find that ϕ_0 does not have the full symmetry G of the Lagrangian but some lower symmetry H$\prec G$ such that

$$H\phi_s = \phi_s$$

This is known as the phenomenon of spontaneous symmetry breaking. The Lagrangian or Hamiltonian retains the full symmetry G of SU(N) but the lowest energy state ϕ_0 does not. It is invariant under H $\in G$. We also have

$$G\phi_s = \phi_s'$$

We see that the broken energy state is not invariant under the action of the parent gauge symmetry. To understand what is happening in the symmetry breaking process, we need to look at the *coset space* defined by $\frac{\boldsymbol{G}}{\boldsymbol{H}}$ which gives us those symmetry elements *not involved* or consumed by symmetry breaking. We look at two distinctly different scenarios whose essential conclusions we will summarize here : different text books exist to give us details we need ([7]).

1. If from the very start we consider , not local rotation as in the expression of 9, but a ***global rotation***, given by

   $$\Phi' = \Phi exp - i\alpha \tag{18}$$

   where the angle α is applied at the same time at all space points , then the Lagrangian above remains essentially the same except that the gradient term does not need to be corrected by the vector potential and the covariant derivative will be just our ordinary derivative . In such a case no gauge potential need be invoked, symmetry breaking from G$\rightarrow H$ will be entirely manifested on the information field wave function . No mind processes are involved in the description since we are have ascribed mind to be a gauge field. The coset space $\frac{\boldsymbol{G}}{\boldsymbol{H}}$ now signifies the number of *massless* bosons or *Goldstone modes* that the broken symmetry state has and are available for excitation if some external energy is brought in the system. Being massless, one can have excitation beginning at zero energy upwards in a continuous spectrum. The H symmetry elements are locked in ϕ_0 and are *massive*. This is also known as a Goldstone mechanism of symmetry breaking.

2. Now let us go back to the scenario of ***local rotation*** we started with. This is known as the Higgs mechanism of gauge symmetry breaking ([8]). Now the gauge symmetry G is fully involved from the very beginning to give the objective character to the information field in order to make it frame independent. The symmetry that would *have to be broken is the gauge symmetry*. The coset space $\frac{\boldsymbol{G}}{\boldsymbol{H}}$ remains the same but tells us a different story . The H-gauge bosons or symmetry elements involved in the broken symmetry state are the *free* or *massless gauge fields* while the coset space are housing $\frac{\boldsymbol{G}}{\boldsymbol{H}}$ massive gauge fields . All the Goldstone modes have disappeared . One says that $\frac{\boldsymbol{G}}{\boldsymbol{H}}$ gauge bosons have become heavy by eating $\frac{\boldsymbol{G}}{\boldsymbol{H}}$ Goldstone modes. This is the situation when some of the mind-elements will be missing from mentation processes. The mind processes we are conscious of are only with those gauge elements that are free , massless to be excited like ordinary photons and only these will be of cognitive relevance.

Let us look more carefully into the Higgs boson state Φ as a state where full gauge symmetry is unbroken . Does this imply that Φ is invariant under G?. If we write for a symmetry element, the unitary transformation

$$U = expiQ^a\alpha_a$$

where the commutation relationships between charge (generators Q of unitary transformation) are

$$\left[Q^a, Q^b\right] = C^{abc}Q^c$$

then true definition of invariance of the wave function to a symmetry operation is

$$Q^a \mid \Phi \succ = 0$$

This is seen from the operation

$$U \mid \Phi \succ = expiQ^a\alpha_a \mid \Phi \succ = (1 + iQ^a\alpha_a + \cdots) \mid \Phi \succ = 1 \mid \Phi \succ + i\alpha_a Q^\alpha \mid \Phi \succ$$

We immediately see that only if $Q^\alpha \mid \Phi \succ = 0$ that one can fulfill the criterion of full gauge invariance of the wave function. This full gauge invariance redefines Φ as our vacuum state $\mid 0 \succ$ because

$$G \mid 0 \succ = \mid 0 \succ \tag{19}$$

2.1 Topological symmetry space of a single Neuron: A Symmetry Breaking of vacuum state

Vacuum brain space $\mid 0 \succ$ was taken in the previous section as the space of Higgs boson carrying the full gauge symmetry $\boldsymbol{G}$, a SU(3) symmetry of a three level biological system. It is considered as a *homogeneous* field space of information field. If there is self-interaction, the vacuum state can be unstable and can lower its energy spontaneously by breaking into some lower symmetry state. In the case of our brain this happens in the physical space of a neuron cell. This is a spontaneous gauge symmetry breaking phenomenon all over again . There are only two possible symmetry breaking patterns possible . This can be seen by diagonalising the matrix representation of Φ and to begin with we take the full symmetry group U(3) of the three quon wave functions of Ψ. One diagonalises it with ([9])

$$\Phi_D = \Omega(x)\Phi\,\Omega^\dagger(x) = \begin{pmatrix} \eta_1 & 0 & 0 \\ 0 & \eta_2 & 0 \\ 0 & 0 & \eta_3 \end{pmatrix} \tag{20}$$

The eigen values give us the symmetry breaking patterns.

There are only two possibilities. Either all the Eigen values are non-degenerate and we have $\eta_1 \neq \eta_2 \neq \eta_3$; then the symmetry breaking pattern is

$$\frac{U(3)}{U(1) \times U(1) \times U(1)} = \frac{SU(3)}{U(1) \times U(1)} \tag{21}$$

This probable state of the neuron, we have called A-symmetry breaking. We will show later that that the U(1)$\times U(1)$ gauge space is not a stable symmetry pattern . In fact this symmetry broken state gives, we think the *Dream State*. The other possibility will be called B-symmetry breaking pattern . In this situation , two of the Eigen values will be equal and degenerate, so that we may take $\eta_1 = \eta_2 \neq \eta_3$. This gives the symmetry breaking

$$\frac{U(3)}{U(2) \times U(1)} = \frac{SU(3)}{SU(2) \times U(1)} \tag{22}$$

The B-symmetry breaking is a stable symmetry and gives the neuron the state of lowest energy. This is the ground state configuration, the *Sleep State*. We have shown (in the appendix $\boldsymbol{A}$) that the symmetry of a single neuron is that of a space that carries information and electrical charge both at the same time . The neuron space is globally of U(2) symmetry but *locally* we can envision a separation of spin and charge which will give a symmetry SU(2) $\times U(1)$ that we designate by $\boldsymbol{H}$.

This symmetry breaking is accompanied by a *confinement phenomenon* : the quons will be confined within a region the size of a neuron tube of radius$\sim$ $\boldsymbol{r}$.This localisation of the broken symmetry state costs a localization energy of the wave function and it can only do so if it can recuperate through symmetry breaking of Higg's Φ a condensation energy. If the energy balance is negative then and then only we shall have locally a confinement of the new phase in the the neuron tube as a localised wave function ϕ_s. Without the symmetry breaking this would have been impossible.

2.2 Brain as a gauge fiber bundle of neurons

Brain can be thought as a three dimensional net work of neurons interconnected through synaptic junctions; It can also be thought of as a gauge fiber bundle.

If we consider a single neuron as a gauge fiber, then our brain carrying the Higgs can be considered as a fiber bundle of symmetry G and the two symmetries are related by the well-known topological product expression ([10])

$$\boldsymbol{G} = \boldsymbol{H} \times \mathbb{M} \tag{23}$$

This expression is very important from our point of view. It says that *locally* (and we are interested in gauge covariant *local* fluctuation of Higgs boson amplitudes) our brain is a geometrical product of the neuron symmetry $\boldsymbol{H}$ acting on a base space-manifold $\mathbb{M}$. We shall define this base space manifold as the *conscious mental space* , which will be shown to be also a projective Hilbert space . $\boldsymbol{G}$ has eight symmetry elements, $\boldsymbol{H}$ has four elements from U(2) or SU(2)$\times U(1)$ symmetry and hence $\mathbb{M}$ is a manifold space of dimension four; it is a $\mathbb{S}^4$ compact surface and has five euclidean dimension $\mathbb{E}^5$. This just happens to be the sensorial space dimension of our five senses, as we mentioned before!

$\mathbb{M}$ is the definition of the coset space

$$\mathbb{M} = \frac{\boldsymbol{G}}{\boldsymbol{H}}$$

When the vacuum state $\mid 0 \succ$ spontaneously breaks to $\boldsymbol{H}$, we can write the corresponding set of states as $\mid \phi_s \succ$, taken to be a collective state so that

$$\boldsymbol{H} \mid \boldsymbol{\phi_s} \succ = \mid \boldsymbol{\phi_s} \succ$$

Thus the set of states $\mid \phi_s \succ$ is invariant to the symmetry operation $\boldsymbol{H} \in \boldsymbol{G}$. Since the energy of this state, because the symmetry of the vacuum is broken *spontaneously*, $\mathrm{E}_{\mid \boldsymbol{S} \succ} \prec E_{\mid O \succ}$; we take the state $\mid S \succ$ to be the ground state of the cognitive brain. This is the *Sleep state*. Because by definition coset space is *empty of states* (the Higgs mechanism has emptied the coset space of the Goldstone bosons) , see the schematic figure below:

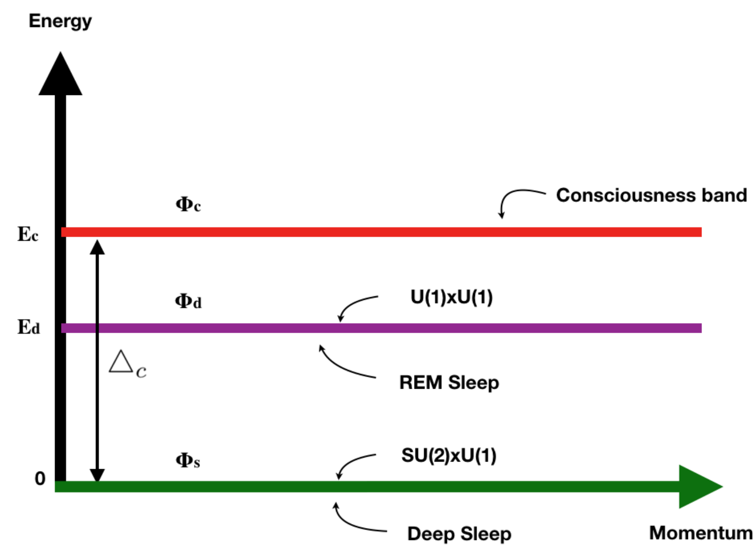


Figure 2: Higgs symmetry breaking

we will have the empty coset band (like a conduction band) separated by an energy gap $\triangle_C$ from the filled $\boldsymbol{S}-$band which is filled with S state particles. The energy gap $\triangle_C$ is a pure symmetry driven gap . We shall identify the gap by calling it the *consciousness gap*, a minimum energy necessary to fire a neuron across the synapse ($\sim$ 100mev), the so called rest potential of a typical neuron ([24]). States in the coset band $\mid \boldsymbol{\phi_c} \succ$ are excited states out of ground state $\mid \phi_s \succ$ through the action of gauge symmetry elements in $\boldsymbol{H}$ which contain a set of Non-Hermitian operators $E_\alpha^\pm$ (akin to raising and lowering operators $\mathrm{S}^\pm$ in the spin operators), we have

$$\boldsymbol{E_\alpha^+} \mid \boldsymbol{\phi_s} \succ = \mid \boldsymbol{\phi_c} \succ \tag{24}$$

There is no continuum of states available until we hit the C-band to get the ϕ_c.We shall show in the next section why we call the coset band the consciousness band . For the instant only argument we have forwarded is that the Euclidean dimension of the coset space reflects the dimension of the five distinct sensorial functions that is involved in cognition and mentation whenever we are conscious.

3 Coset Space as a Projective Hilbert space: $\mathbb{M}$ manifold as our Conscious mental space

Mind is defined as the image place, a many to one mapping from brain space $\boldsymbol{H} \rightarrow \mathbb{M}$. There can be no reverse image . This means in very plain language that measuring Functional magnetic resonance spectroscopic intensities in patches of the cortical brain in a scanner will not give us slightest indications of the representations occurring in the mental space . Yet, this vast image space is our only reality. By image we mean all that is mental . The euclidean dimension of $\mathbb{M}$ is five, it is $\mathbb{E}^5$. This is exactly the five senses that human beings possess (vision, audition, olfactory, touch and gustative). This number is entirely determined by the symmetry of the gauge fiber (our neuron) . $\mathbb{M}$ from the topological aspects of the relationship is

$$\boldsymbol{G} = \boldsymbol{H} \times \mathbb{M} \tag{25}$$

- $\mathbb{M}$ is a *projective Hilbert space* , (Bengtsson, ibid) called $\mathbb{CP}^2$ projected from the Hilbert space of brain which is a $\mathbb{C}^3$. In our specific case the projective space is $\mathbb{CP}^2$. The superscript 2 signifies that in $\mathbb{M}$-space the number of wave functions is two and not 3 as in the brain-space. If the Hilbert space were $\mathbb{C}^2$, the projective Hilbert space would have been $\mathbb{CP}^1$, which is an ordinary 3-dimensional sphere or $\mathbb{S}^2$.
- Quantum mechanically it is called a ray space, a map of many equivalent rays from cognitive brain to one in the mental $\mathbb{M}$. An open circuit in the brain-space will trace out a *closed* curve in the $\mathbb{M}$-space . Physicists call it a Berry phase ([11]). One also calls it a *parameter space* ([12]) . This may be one reason why our mental alters profoundly with chemical parameters like neurotransmitters.
- It is also a Coherent-Spin state space ([14]), a point to which we shall come back in the next section.

3.0.1 Nature of $\mathbb{M}$-space:projective Hilbert space

When we take the $\mathbb{C}^3$ Hilbert space $\mathscr{H}$ of our brain and consider a subset of vectors in the space of $\left\{\mathbb{C}^3 - (0,0,0)\right\}$, we are essentially by excluding the null vector $\mathbf{0}$ at the origin, creating a different Hilbert space $\mathscr{H}^\star$ which is characterised by an *equivalent* class of Hilbert space 'vectors' given by

$$Z' = cZ \quad \in \mathscr{H}^\star \tag{26}$$

$$here\, c\, is\, complex \& \succ 0$$

The action matrix $\boldsymbol{c}$ has SU(3) symmetry and the fact that it is positive and non-zero makes the complex Z's as *nonadditive* quantities . These cannot really be considered as vectors (they are more like points that cannot have the usual vector additivity rules since null vector is not in the group) and are known as equivalent rays ([15]). The equivalence comes from the fact that for ($\mid Z \mid= 1$ and c$\sim$ $expi\theta$, a phase factor) the two states Z and Z' are physically exactly the same state, they map to the same ray and this is true only if $\boldsymbol{c} \neq \mathbf{0}.$. Actually a pure state corresponds to an entire equivalent class of vectors and there can be more than one equivalent class..This is usually done in such a way that the vectors are normalized to unit length but differ by their phase factors.The space of pure states for N-Hilbert space vectors is the projective Hilbert space $\mathbb{CP}^{\mathrm{n}}$ where $\boldsymbol{n}$ is N-1.The new Hilbert space $\mathscr{H}^\star$of symmetry SU(3) is the *principal fiber bundle* corresponding to our brain space over the manifold $\mathbb{M}$. Neuron space $\mathscr{N}$ taken as a gauge symmetry U(2) comes from that assumed fora single neuron, taken as a two level system . If $\mathscr{H}$ has the dimension $\mathbb{C}^3$, the projective place $\mathbb{M}$ will have dimension $\mathbb{C}^2$ which is called $\mathbb{CP}^2$. This is given by the quotient or coset space

$$\frac{brain}{neuron} = \frac{SU(3)}{U(2)}. = \frac{SU(3)}{SU(2) \times U(1)} = \mathbb{M} \equiv \mathbb{CP}^2$$

The bare symmetry of the fiber is U(2) but should be viewed as a *neuron fiber*

$$U(2) \approx SU(2) \times U(1) \tag{27}$$

This decomposition is very apt description of the incoming signals into the neuron where it will live both as a qubit SU(2) and as a charge U(1). The general linear and homogeneous subspace $\mathbb{CP}^{\mathrm{n}}$ are of major importance topologically . They are defined as the *images* of the complex space $\mathbb{C}^{\mathrm{n}+1}$ under the natural map from the vector space to the projective space. It in this sense our $\mathbb{M}$ - space is a manifold where the actions and dynamics of information in our cortical brain the $\mathbb{B}$ space is faithfully mapped as images in $\mathbb{M}$. Hence we call it *our mental space*.

An infinite number of possible motions along the curves $\boldsymbol{C}$ in the Hilbert space $\mathscr{H}$ of cortical brain will project to a given *closed* curve $\tilde{\boldsymbol{C}}$ in the $\mathbb{M}$ space. The curves $\boldsymbol{C}$ in the $\mathbb{B}$-space are lines along which a piece of information is being transported on a curved surface by parallel transport and suffers a net angular displacement of its vector such that the direction with which it started is quite different at the same point where it comes back to. (see fig).

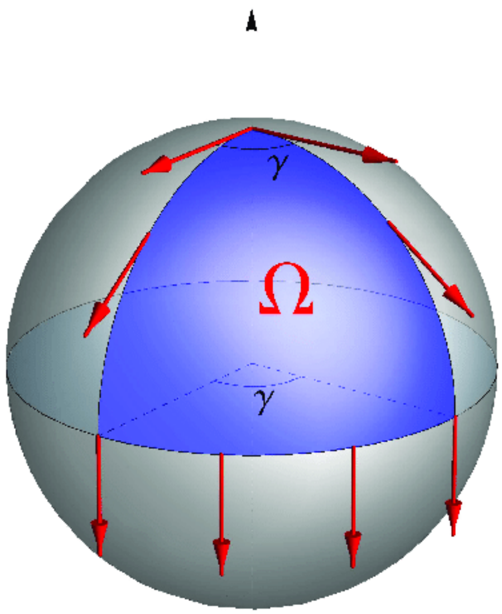


Figure 3: Berry phase

The angle difference γ (called anholonomy) in the figure gives rise to the dynamic Berry phase([11]) .This is what we think confers *meaning* to the information pixel. This is the curve $\boldsymbol{C}$ in the $\mathbb{B}$-space (a spherical surface is used for the sake of illustration only) but in the next figure we see what the curve is like in the projected $\mathbb{M}$ space, $\widetilde{\boldsymbol{C}}$.

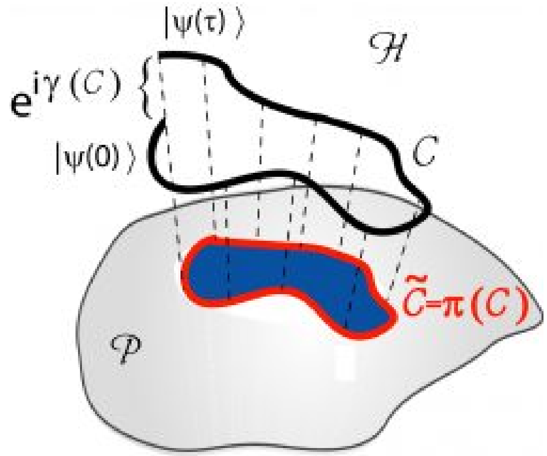


Figure 4: Projective Hilbert space

It is after the closed circuit in the $\mathbb{M}$ (here given a generic name $\mathscr{P}$ to indicate that it is a projective space)that one sees it as an accumulation of Berry phase, $\gamma(C)$. In the picture of projective Hilbert space, as Aharonov & Anandan had shown ([12]) one does not need to evoke an adiabatic path over which our information pixel travels. The *linear subspaces* of $\mathbb{CP}^{\mathrm{n}}$ are of major importance. Not only these are the *images* from $\mathbb{C}^{n+1}$ which is the original Hilbert space of our brain-space, the theater of all actions but it is also a natural map from the vector space to the projective space which we consider to be our mind space. It is also interesting to observe that topologically $\mathbb{CP}^{\mathrm{n}}$ is like $\mathbb{C}^{\mathrm{n}}$ except for $\mathbb{CP}^{\mathrm{n}-1}$ which is attached at 'infinity'!

$\mathbb{CP}^1$ is the 2-sphere $\mathbb{S}^2$, our ordinary 3-dimensional sphere, the familiar ball also known as Poincaré-Bloch sphere in our euclidean $\mathbb{E}^3$. Our proposed mental space $\mathbb{M}$ is $\mathbb{CP}^2$ is a manifold 4-sphere $\mathbb{S}^4$. We have called it our sensorial dimensions, an equivalent Euclidean space $\mathbb{E}^5$-space. Any surface of manifolds greater than $\mathbb{S}^2$, e.g $\mathbb{S}^3$, $\mathbb{S}^4$ and so forth , of respective Euclidean dimensions four , five etc are quite impossible to visualise . But this can be done in a completely different way, which we owe to Majorana ([13]) called Majorana representation , the so called stellar representation of spin $\boldsymbol{S}$. To appreciate Majorana construction, let us go back to the Bloch-Poincaré sphere, the 2-sphere $\mathbb{S}^2$.

The 2-sphere $\mathbb{S}^2$ is the parameter space for a single spin $\boldsymbol{S} = \frac{\mathbf{1}}{\mathbf{2}}$ object, like a qubit or any equivalent two-level system . The 4-sphere $\mathbb{S}^4$ is the *sensorial parameter space of our mind* – two spin $\frac{1}{2}$ objects with a symmetric spin combination of $\boldsymbol{S} = \mathbf{1}$, a three level system. Any point on the $\mathbb{S}^2$ surface is a state of the qubit. If we take an unit sphere, radius $|\, \boldsymbol{r} \,|= 1$, and enquire about the state of the spin $\frac{1}{2}$ particle (is it up or down), we observe that along every radial direction (with its θ, φ) the spin is 'up' along that direction . Therefore space of the spin state is isomorphic to the set of all directions in space–our ordinary physical space and hence to $\mathbb{S}^2 = \mathbb{CP}^1$. This is what gives the great strength to Bloch sphere . A point on the surface $\mathbb{S}^2$ can be written in terms of its wave function or probability of the point being an up or down info-spin

$$\mid \Omega \succ = \mid \theta, \varphi \succ = cos\frac{\theta}{2} \mid\uparrow\succ + sin\frac{\theta}{2} expi\varphi \mid\downarrow\succ \tag{28}$$

In terms of spin raising and lowering operators $\boldsymbol{S}^{+} \& \boldsymbol{S}^{-}$, the state $\mid \theta\varphi \succ$ can be expressed as $exp\ (\frac{\theta}{2}e^{-i\varphi}\hat{S^{+}} - \frac{\theta}{2}e^{i\varphi}\hat{S^{-}})$,which is the displacement operator that would act on a reference state $\mid S, -s \succ$ where $\boldsymbol{S}$ is the spin quantum number and $\boldsymbol{s}$ is its lowest diagonal element to give the state $\mid \theta\varphi \succ$.The coordinate of the point i on the sphere can be written as a complex number z_i (subscript i, indicates the spin in question, not to be confused with i, which is the usual imaginary quantity)

$$complex\ number\ z_i = tan\frac{\theta_i}{2} expi\varphi_i$$

Majorana showed that when one has a system of not just one spin $\frac{1}{2}$ but $\boldsymbol{n}$ spin $\frac{1}{2}$ system, corresponding to a projective space $\mathbb{CP}^{\mathbf{n}}$, the vectors in $\mathbb{C}^{n+1}$ are in one to one correspondence with the set of a n-th degree polynomial

$$w(z) = Z^0(z - z_1)(z - z_2)......(z - z_n) \tag{29}$$

Here Z^0 is a reference vector. The complex roots of the equation w(z) =0 for z_i=z_1, z_2 ... z_n will give us positions of n-*unordered* spins or n-stars on the $\mathbb{S}^2$ Bloch sphere . This is the stellar representation . It is now to be considered as a celestial or Majorana sphere . The major *tour de force* of Majorana was to demonstrate how the same $\mathbb{S}^2$ or our ordinary three dimensional sphere can house not just one star (one spin $\frac{1}{2}$) but any spin S. And each star can move in its proper orbit of closed curve and accumulate an independent Berry phase (ref: Ganezarek et al[16]). One can write the recursion topological product space

$$\mathbb{CP}^1 = \mathbb{S}^2$$

$$\mathbb{CP}^2 = \frac{\mathbb{S}^2 \times \mathbb{S}^2}{s_2}$$

$$\mathbb{CP}^n = \frac{\mathbb{S}^2 \times \mathbb{S}^2 \times .... \times \mathbb{S}^2}{s_n}$$

Here $\boldsymbol{s_n}$ is the symmetry group of permutation of n-objects. Thus $\mathbb{CP}^2$ looks like two three dimensional spheres stitched together.

3.1 Coherent Spin State

The Majorana stellar representation also works when the spins are *ordered*. This is the reason why we can obtain n-spin coherent state on the $\mathbb{S}^2$- sphere. Each coherent state is then a constellation of blazing 'stars' in the night sky. We will work with $\mathbb{S}^2$- sphere, whose surface is the whole parameter space of a spin S=$\frac{1}{2}$ object.

Spin states of maximal projection along some direction of space are called spin-coherent states and are in some respects most "classical" available . For any spin S, the spin coherent states can be drawn on a 2-sphere, thanks to Majorana representation . For a single neuron , with two levels the projective Hilbert space is a $\mathbb{CP}^1$ or a 2-sphere $\mathbb{S}^2$. This is the world of qubit. Thus we can call the Majorana celestial sphere., also as the *cognition sphere*.

We can define the spin coherent state as the highest value of the spin operator $\hat{S}$ written as

$$\prec \Omega \mid \hat{S} \mid \Omega \succ = S\boldsymbol{n} \tag{30}$$

Here $\boldsymbol{n}$ is the unit vector specified by Ω.The most interesting property of the spin coherent state vector Ω sweeping out the surface of the Bloch-Majorana sphere $\mathbb{S}^2$ is that its topological structure naturally carries Dirac's magnetic monopole ([8]), placed at the very center of the sphere . The unitary representation of $\mid \theta, \varphi \succ as$ exp $(\frac{\theta}{2}e - i\varphi \hat{S^+} - \frac{\theta}{2}e^{i\varphi}\hat{S^-})$ is the coset representation of the manifold $\frac{SU(2)}{U(1)} = \mathbb{S}^2 \equiv \mathbb{CP}^1$. In this representation SU(2) is fiber bundle of the principal fiber group U(1) . Physically we can think of it as an electrical charge moving over the surface $\mathbb{S}^2$ and interacting with a magnetic charge at the origin of the sphere . Here the quantum state is embedded in a topologically non-trivial space which is at the origin of magnetic monopole (ref: Yang & Wu). In our case of information carrying qubits in the neuron , the "magnetic charge" at the center of the sphere is to be read or interpreted as a *mental charge* of quantised strength 2S=1, to make the "Dirac string" invisible . From the spin coherent state $\mid \Omega \succ$ we can also derive expression for the Monopole gauge potential, $\mathscr{A}$(x). This is also called a gauge potential for the simple reason that it makes the trajectory of the information particle gauge invariant or covariant, as we see in the next section.

3.2 Spin coherent State & SU(2) gauge field : Abelian monopole

Let us take two vectors, a state vector $\mid 1 \succ = \mid \psi(x) \succ$ and a second vector which we take to be its derivative or tangent $\mid 2 \succ = \mid \partial_\mu \psi(x) \succ$. On a flat euclidean surface we shall have $\prec 1 \mid 2 \succ = 0$. These two vectors are orthogonal. But this not so in a curved surface . This orthogonality is required if we want to do parallel transport of a vector along any curve on a curved space (fig 4), which leads to the Berry angle γ as we have seen earlier. The orthogonality between the two vectors can be achieved through a Gram-Schmidt procedure.

We write the vector $\mid 1' \succ = \mid 1 \succ$ and write $\mid 2' \succ = \mid D_\mu \psi \succ$, which is a covariant derivative. To achieve orthogonality between 1' & 2' vectors we write

$$\mid D_\mu \psi \succ = \mid \partial_\mu \psi(x) \succ - \mid \psi(x) \succ \prec \psi(x) \mid \partial_\mu \psi(x) \succ \tag{31}$$

This results in

$$\prec \psi(x) \mid D_\mu \psi \succ = 0$$

We have obtained $\prec \psi(x) \mid D_\mu \psi(x) \succ = 0$. To get this,We have simply subtracted out of $\mid 2 \succ$ its projection on $\mid 1 \succ$. The gauge covariant derivative can also be written as

$$\mid D_\mu \psi \succ = \mid \partial_\mu \psi(x) \succ - i\mathscr{A}_\mu(\mathrm{x}) \mid \psi(x) \succ$$

Here $\mathscr{A}_\mu(\mathrm{x})$ is a grass root definition of vector potential

$$\mathscr{A}_\mu(\mathrm{x}) = \prec \psi(x) \mid \partial_\mu \psi(x) \succ \tag{32}$$

This is a perfect analog of the electromagnetic vector potential but our parameter space $\boldsymbol{x}$ is not the ordinary three dimensional space . In the case of our spin coherent state, $\boldsymbol{x}$ is the whole $\mathbb{S}^2-$ surface and the state vector ψ is designated $\mid \Omega \succ$ and μ indicates the three orthogonal axes $(\boldsymbol{n}, e_x, e_y)$ on the sphere. The Berry's phase factor along a closed path $\tilde{\mathrm{C}}$ on $\mathbb{S}^2$ along a path is given by (this is the analog of Bohm-Aharanov phase with electromagnetic vector potential $\boldsymbol{A}$)

$$expi\gamma = expi \oint_{\tilde{C}} dx^\mu \mathscr{A}_\mu(\mathrm{x}) \tag{33}$$

Like the electromagnetic vector potential (Maxwell vector potential) the SU(2) vector potential is Abelian but the major difference between the two is that while the former is defined on a flat Euclidean metric, the latter is defined on the curved $\mathbb{S}^2$ surface. This makes the emergence of magnetic (mental in our case) monopole possible, as we mentioned earlier, a topologically non-trivial space .

We write the gauge invariant quantity

$$\prec D_\mu \psi \mid D_\nu \psi \succ = \frac{1}{2} \left[\mathscr{G}_{\mu\nu}(x) + i\mathscr{F}_{\mu\nu}(\mathrm{x}) \right] \tag{34}$$

The first term on the right hand side is the real, symmetric Fubini-Study metric of distance on the curved sphere but the second term is real antisymmetric Berry curvature of the gauge field space , written explicitly as

$$\mathscr{F}_{\mu\nu}(\mathrm{x}) = \partial_\mu \mathscr{A}_\nu(\mathrm{x}) - \partial_\nu \mathscr{A}_\mu(\mathrm{x}) \tag{35}$$

This term is the equivalent of electric and magnetic field tensors of Maxwell's electromagnetic fields. Now we can see why there is a real difference of SU(2) vector potential $\mathscr{A}(\mathrm{x})$ from that of Maxwell's vector potential A(x) . SU(2) curvature , because of its special topology has a conserved topological charge $\mathcal{Q}$ given by the surface integral

$$\iint_{\mathbb{S}^2} \mathscr{F}_{\mu\nu}(\mathrm{x}) \mathrm{dx}^\mu \wedge \mathrm{dx}^\nu = 2\pi \mathcal{Q} \tag{36}$$

The topological charge can easily be obtained by transforming the Stokes surface integral into a Gaussian volume integral. This charge is the first Chern number. In the physical language this is magnetic (*mental*) charge and is seen as the derivative of the magnetic field, just as electric charge is the derivative of electrical field . This term is totally absent from Maxwell's equation and a source of asymmetry of Maxwell's equation . It exists in the SU(2) gauge field because of its topology. Our SU(2) field arises because of information qubit in the neuron, the Chern number $\mathcal{Q}$ will be the measure of *mental charge* located at the origin which is the center of the three dimensional sphere. Origin of this charge is *purement topological* and *is not associated with field of any 'mind' particle*. From this center , a "mental field tensor" $\mathscr{F}_{\mu\nu}(\mathrm{x})$ will radiate out in the mental space. The charge corresponds to a singularity (Dirac monopole) in the $\mathscr{F}_{\mu\nu}(\mathrm{x})$.

An important property of the spin coherent state is that $\mid \Omega \equiv \theta, \varphi \succ$ state is embedded in a topologically non-trivial geometrical phase, the two dimensional Majorana-Bloch sphere $\mathbb{S}^2$. Let us rewrite expression spin coherent state coordinates on the Bloch sphere as

$$\mid \Omega \succ = \mid \theta, \varphi \succ = \left(cos\frac{\theta}{2} + sin\frac{\theta}{2} exp - i\varphi S^{+} \right) \mid \downarrow \succ \tag{37}$$

In this expression the spin coherent states are *one to one correspondence* to the points of $\mathbb{S}^2$ except for the north pole where all values of the point correspond to the same point . It is also the point where there is a singularity in defining the Dirac vector potential . Because of the inherent ambiguity in this description, we can try an alternative expression

$$\mid \Omega' \succ = \mid \theta, \varphi \succ' = \left(cos\frac{\theta}{2} expi\varphi + sin\frac{\theta}{2} S^{+} \right) \mid \downarrow \succ \tag{38}$$

once again from this south pole difficulty we face the same set of problems and the two sets of coherent states are to be related by a phase factor

$$\mid \Omega \succ = \mid \Omega' \succ exp - i\varphi \tag{39}$$

These two wave functions are physically identical and in order to be so, we need

$$exp - i\varphi = 1$$

Geometrically these two coherent states define the two non-singular 'patches ' of $\mathbb{S}^2$ coming in from north and south pole and overlapping in the equatorial region. The phase factor between the two patches is the gauge degree of freedom h given by

$$h = expi\varphi \subset U(1) \tag{40}$$

This is the Abelian or electromagnetic U(1) gauge degree of freedom of the Dirac monopole situated at the center of the sphere carrying an unit monopole charge $\boldsymbol{g}$. This is a scalar topological charge. Vector potentials and field tensors we have defined so far are also Abelian, gauge invariant, observable and mutually commuting.

3.3 SU(3) coherent states, non-Abelian monopole

What do the SU(3) coherent states look like ? The Majorana description of the two spins, S=1 on $\mathbb{CP}^1$ surface of the Bloch sphere $\mathbb{S}^2$ is inadequate . We want coherent state vector $\mid \xi \succ$ to be in a one-to-one correspondence to points on the $\mathbb{CP}^2$ surface which is $\mathbb{S}^4$. For this is the sensorial space because of which which we live and our coherent state vector (Conscious mind vector) is scanning each and every one of these points of this surface as it moves in time.

Coherent states cannot be generated in a finite dimensional Hilbert space . Both the U(2) symmetry space of the neuron system and SU(3) symmetry based Cortical brain are finite dimensional Hilbert space . To generate SU(2) coherent state ([14]) one had to use the the non-Hermitian S^+, S^-*operators* of the Cartan Algebra that operating on a reference state , say $|\downarrow\succ$ create an infinite tower of excited states which will constitute our coherent spin state. The basic idea behind the projective Hilbert space is exactly the same — to be able to have a compact infinite tower of excited states on the the projective space we will need SU(3) non-Hermitian operators . In order to illustrate, let us go back to the spin coherent state of SU(2) symmetry. Its projective Hilbert space is $\mathbb{CP}^1$ which is precisely the coset space manifold

$$\frac{SU(2)}{U(1)} = \mathbb{S}^2$$

The unitary representation $exp\ (\frac{\theta}{2}e^{-i\varphi}\hat{S^+} - \frac{\theta}{2}e^{i\varphi}\hat{S^-})$ is the coset representaion $\frac{SU(2}{U(1)}$ and is a displacement operator that by acting on a reference spin state would create the three dimensional coherent state vector $\mid \Omega \succ$ sweeping out over the $\mathbb{S}^2$ surface .

A generalised coherent state can always be generated by the same procedure ([17]): find the coset space for the symmetry in question, use it to write down the displacement operator ξ, operate on the reference state chosen and generate the coherent state . Our Hilbert space of the brain is taken to be of SU(3) symmetry, of a three level system denoted by $\boldsymbol{G}$. We write the neuron space symmetry of two complex wave functions of the two level system as U(2)= SU(2)$\times U(1)$ and we denote it by $\boldsymbol{H}$. The coset space is

$$\frac{\boldsymbol{G}}{\boldsymbol{H}} = \mathbb{M} \equiv \mathbb{S}^4 \tag{41}$$

To determine the required displacement operator, we go back to the eight Gell-Mann symmetry operators λ_i enumerated above and investigate the required Cartan basis. (See Appendix B). Consider a set of operators $\{\lambda_i\}$ closed under commutation

$$[\lambda_i, \lambda_j] = \lambda_i\lambda_j - \lambda_j\lambda_i = \sum 2i\, f_{ijk}\lambda_k$$

The $\{\lambda_i\}$ *span*a algebra $\mathfrak{g}$ which are elements of the covering group $\boldsymbol{G}$. It is more convenient to write the $\{\lambda_i\}$ in terms of standard Cartan basis $\{H_i, E_\alpha, E_{-\alpha} = E_\alpha^\dagger\}$:

$$[H_i, H_j] = 0, \qquad [H_i, E_\alpha] = \alpha_i E_\alpha$$

$$[E_\alpha, E_{-\alpha}] = \alpha^i H_i, \qquad [E_\alpha, E_\beta] = N_{\alpha\beta} E_{\alpha+\beta} \tag{42}$$

The H-operators are the commuting diagonal operators which are $\lambda_3 \& \lambda_8$ matrices, the E_α are the raising and lowering operators in SU(3) analogous to $\mathrm{S}^+ \&$ S^- raising and lowering operators in spin SU(2) system. The SU(3) generalized coherent state is generated by an element $\mathfrak{g} \in G$ acting on the fixed state $\mid \phi_0 \succ$, our vacuum state

$$\mid \mathfrak{g} \succ_G = \mathfrak{g} \mid \phi_0 \succ$$

We note that the group element $\mathfrak{g}$ acting on the reference state $\mid \phi_0 \succ$ gives us a new state $\mid \mathfrak{g} \succ_G \neq \mid \phi_0 \succ$. The reference state or the vacuum state $\mid \phi_0 \succ$ is *not* invariant under the full group G. However it is invariant under the maximum subgroup H of G such that a group element $\mathfrak{h}$ of H gives (ref: Ryder)

$$\mathfrak{h} \mid \phi_0 \succ = \mid \phi_0 \succ$$

In such a case one says that there has been a spontaneous symmetry breaking from the parent symmetry G or SU(3) $\rightarrow$ H or U(2) . The coset space manifold $\frac{G}{H}$ is just another way of looking at the projective Hilbert space which we have adopted as our mental space $\mathbb{S}^4$. Let us take an operator $\mathfrak{m}$ in the coset space defined by $\mathfrak{g}$=$\mathfrak{m}\mathfrak{h}$, then we can write

$$\mathfrak{m} = \mathscr{D}_{\mathrm{G}}(\xi) = \exp\left\{\sum_{\alpha \succ 0} (\xi \mathrm{E}_\alpha - \xi^* \mathrm{E}_{-\alpha})\right\} \in \frac{\mathrm{G}}{\mathrm{H}} \tag{43}$$

Let us denote the coset space manifold by ξ which can also be considered as the parameter space .This leads to the SU(3) displacement operator which will give us a new set of coherent states by operating on the vacuum state which we write as

$$\mid \xi \succ = D_{\mathrm{G}}(\xi) \mid \phi_0 \succ expi\vartheta = \Phi(\xi) expi\vartheta \tag{44}$$

Here a phase ϑ is restored to the wave function. The Wave function $\Phi(\xi)$ is the coherent state

$$\mid \Phi(\xi) \succ = D_{\mathrm{G}}(\xi) \mid \phi_0 \succ \tag{45}$$

The set of generalised coherent states satisfy the normalisation

$$\int \mid \Phi(\xi) \succ \prec \Phi(\xi) \mid d\Gamma(\xi) = \boldsymbol{I}$$

$\mathrm{d}\Gamma(\xi)$ is the surface element or the $\frac{G}{H}$ closet space.These SU(3) coherent states $\mid \Phi(\xi) \succ$ are in one-to-one correspondence to the points on the coset space $\frac{G}{H}$. The $coherent - state\ vector$ is eight dimensional $\mathbb{E}^8$ and is roaming in time over the mental space or parameter space (sensorial space) $\mathbb{S}^4$. We can call this *coherent state vector* as the true *conscious mental order parameter*. We can write the time evolution of the system as a generalised Berry's angle over the $\xi - space$,using the expression of gauge potential $\mathscr{A}$ or its basic definition one form ω given earlier 32

$$\omega\left[\frac{G}{H}\right] = \int_{\Gamma(\xi) \in \frac{G}{H}} \prec \Phi(\xi) \mid d\Phi(\xi) \succ = \oint_c \mathscr{A} \bullet \mathrm{d}\hat{\xi} \tag{46}$$

$\hat{\xi}$ is an unit vector defined in the $\frac{G}{H}$ parameter space. When the rank of the group is larger than 1, as is our case for SU(3) , the gauge potential is non-Abelian ([17]). Now it is simple to write down the field tensor or gauge connection as

$$\mathscr{F} \equiv \prec \mathrm{d}\Phi(\xi) \mid \mathrm{d}\Phi(\xi) \succ = \sum_{\alpha\alpha'} \omega_{\alpha\alpha'} \mathrm{d}\xi_{\alpha} \wedge \mathrm{d}\xi_{\alpha'} \tag{47}$$

Here the 2-form Berry curvature $\omega_{\alpha\alpha'}$ is given by

$$\omega_{\alpha\alpha'} = \left[\prec \frac{\partial\Phi(\xi)}{\partial\xi_{\alpha}} \mid \frac{\partial\Phi(\xi)}{\partial\xi_{\alpha'}} \succ - \prec \frac{\partial\Phi(\xi)}{\partial\xi_{\alpha'}} \mid \frac{\partial\Phi(\xi)}{\partial\xi_{\alpha}} \succ \right]$$

4 Dynamics and Emergence of Consciousness : Brain's *Default Mode*

To see the , let us define a Green function as an evolution operator in time in the SU(3) coherent states

$$\mathscr{G}(\mathrm{t_f}, \mathrm{t_0}) = \prec \Phi'(\xi) \, |\mathbf{Texp} \left\{ -\mathbf{i} \int_{\mathbf{t_0}}^{\mathbf{t_f}} \boldsymbol{\mathcal{H}}(\mathbf{t})\mathbf{dt} \right\} \mid \boldsymbol{\Phi}(\boldsymbol{\xi}) \succ \tag{48}$$

Here $\boldsymbol{T}$ is the time ordering operator of a causal, retarded green's function and $\mathcal{H}(\mathrm{t})$ is the Hamilton operator. We can rewrite the path integral as action integral in the form

$$\mathscr{G}(\mathrm{t_f}, \mathrm{t_0}) = \int_{\mathrm{t_0}}^{\mathrm{t_f}} [\mathrm{d}\Gamma(\xi(\mathrm{t}))] \exp\left\{ \mathrm{i}\boldsymbol{S}(\xi(\mathrm{t}) \right\}$$

Here $\mathcal{S}[\xi(t)]$ is the action expressible as

$$\mathcal{S}\left[\xi(t)\right] = \int_{t_0}^{t_f} dt \left\{ \prec \Phi(\xi(\mathrm{t}) \mid \mathrm{i}\frac{\mathrm{d}}{\mathrm{dt}} \mid \Phi\left(\xi(\mathrm{t}\right) \succ - \prec \Phi\left(\xi(\mathrm{t})\right) \mid \mathcal{H}(\mathrm{t}) \mid \Phi(\xi(\mathrm{t}) \succ \right\} \tag{49}$$

Here the first on the R.H.S is purely geometric giving us the Berry trajectory while the second term is the usual evolution of the coherent state from one time to another and will give([18])

$$\mathscr{G}(\varpi, p) = \frac{1}{\varpi + \mathrm{i}\delta - \xi_{\mathrm{p}} - \Sigma(\mathrm{p}, \varpi)} \tag{50}$$

This expression is just Fourier transformation from of equation48 time and physical space to energy and momentum space. We want to emphasise that writing the coherent state Green's function in this way is vital if we are to understand the myriads of non-destructive experiments that Neuro-physicists do to probe the mental state of brain $in - vivo$. This is the functional resonance spectroscopy and this is best of what we can do today . But before coming to that , let us clarify the implications of this Green's function. The term ε_p

is the single particle energy , more specifically $\varepsilon_p^0 = \epsilon_p - \mu_p$ where ϵ_p is the bare single particle measured from μ_p the chemical potential of the N-'particle' system (N=$\mid \Phi(\xi) \mid^2$). The term $\Sigma(p,\varpi)$ is called self-energy. It sums all the interactions the particle undergoes with its surroundings and is most likely related to the Berry phase. It has a real part and an imaginary part that we denote by Re $\Sigma(p,\varpi)$ and Im $\Sigma(p,\varpi)$.The real part brings about an energy shift to the bare particle energy ε_p but the Imaginary part , reflects the dynamic loss or absorption of energy as the coherent state Green's function moves on its trajectory . While the real part with the bare-particle energy gives us the pole of the Green's function, the imaginary part, branch cut of the Green's function gives us the spectral weight of the dynamic process: it tells us the spectrum of energy that the system (coherent state) will lose or gain as it travels . Explained in a simpler way, these are the mental processes or excitations involved that we want to characterize or measure , if we can. We display these terms, real part of energy and the spectral function $\mathscr{A}(\mathrm{p},\varpi)$ as

$$\varepsilon_p = \epsilon_p - \mu_p + Re\Sigma(p,\varpi) \tag{51}$$

$$\mathscr{A}(\mathrm{p},\varpi) = \frac{-2\mathrm{Im}\Sigma(\mathrm{p},\varpi)}{[\varpi - \epsilon_\mathrm{p} - \mu_\mathrm{p} + \mathrm{Re}\Sigma(\mathrm{p},\varpi)]^2 + [\mathrm{Im}\Sigma(\mathrm{p},\varpi)]^2} \tag{52}$$

The spectral weight is everything we need to understand a typical N.M.R type of experiment specially the fmri in-vivo expt one performs. It has two key parts: the segment where Im $\Sigma \rightarrow 0$. This is where Green's function reveals its pole or Eigen energy at

$$\varpi = \epsilon - \mu + Re\Sigma(p,\varpi) \tag{53}$$

We note that to find the pole , one needs this equation which has to be solves self-consistently, so that there can be whole region of $\varpi$where there is not going to be any solution . This the region of energy gap and where there is no spectral weight. The spectral weight at the pole is given by

$$Im\Sigma(p,\varpi) = 0$$

$$\mathscr{A}(\mathrm{p},\varpi) = 2\pi\delta\left\{\varpi - \epsilon - \mu + \mathrm{Re}\Sigma(\mathrm{p},\varpi)\right\} \tag{54}$$

It is important to realise that at the pole position , the spectral weight is a sharp delta-function *with no imaginary part* . Strictly speaking it will not show up in an energy loss measuring experiment. On the other hand for all other energy spectrum (excluding the gap region where there is nodensity of states), the spectral weight is spread out in energy giving us an idea the energy absorption and loss (see fig) . This is the inelastic part of the experiment which will take spectral weight away from the elastic or pole part and spread it out in the loss part giving the pole part a renormalized weight R(p)$\leq$ 1

$$A(\mathrm{p},\varpi) = 2\pi R(p)\delta\left\{\varpi - \epsilon - \mu + Re\Sigma(p,\varpi)\right\}$$

This allows us to write the sum rule

$$\frac{1}{2\pi}\int_0^\infty \mathscr{A}(\mathrm{p},\varpi)\mathrm{d}\varpi = 1 \tag{55}$$

Now we are in a position to understand the functional resonance spectroscopy experiment FMRI which gives a sort of image of our brain ar rest or in activity, also called BOLD contrast imaging experiment . BOLD signifies Blood Oxygen Level Determination . What this in-vivo N.M.R experiment does is to give us an approximate *local* ratio of oxygenated blood or hemoglobin to non-oxygenated hemoglobin depending on whether brain is at rest or in activity . The experiment is based on the fact that our neurons do not have internal reserve of energy in the form of sugar or oxygen.When they are active and start firing they need more energy to be brought in quickly and locally. This is the process called hemodynamic response where blood becomes locally oxygen enriched and the ration of oxyhemoglobin to deoxyhaemoglobin increases. Since oxygenated hemoglobin is diamagnetic and deoxygenated hemoglobin is paramagnetic with more free magnetic spins, the deoxygenated region looses quicker its spin resonance peak while the oxygenated region where neurons are more active appears as bright resonance patches. Essentially what the N.M.R probe is giving is a spectrum of energy loss processes. Region of brain where more activity and energy absorption is going on is essentially the more dissipative part. The broad energy spectrum of the spectral region of $\mathscr{A}(\mathrm{p},\varpi)$ of figure () is drawn with one delta-function peak at energy $\varpi = 0$, which because of Im $\Sigma(p,\varpi) = 0$, is completely non-dissipative. But in the rest of the figure , beyond the energy gap region we see the dissipative part extending over a spectrum of width $\sim \triangle\varpi_b$, with a small peak structure at $\varpi = \varpi_C$ with an assumed band width $\triangle\varpi_c \sim \triangle_c$. We assume that this whole dissipative region is where lot of brain or neuronal activity, as seen by bright FMRI patches, is going on . But recently the neurologists ([19]) discovered a very surprising phenomenon. They found that even when the *brain is at rest*, the broad dissipative part remains as intense as if we were at full mental neuronal activity. This was very disturbing. When we are at rest, brain literally is doing nothing, sitting motionless or making our mind a blank or day dreaming without any concentration of any kind , brain seems to be chattering away . Raichle and his group called it *brain's default mode* . They associate the small peak structure with conscious brain activity while its broad background with everything that is going on behind the screen of which we are not conscious of . Consciousness part seems to consume twenty to fifty times less energy than the default mode. If we want to give a time aspect to these two processes, conscious and unconsciousness states , we see that a conscious process is taking about $\tau_c \simeq \frac{\hbar}{\triangle}$ seconds while the background process takes a much shorter time period $\tau_b \sim \frac{\hbar}{\triangle\varpi_b}$. Since $\triangle\varpi_b \gg \triangle_c$, we shall have $\tau_b \ll \tau_c$. We can associate τ_b time with decision making time behind a conscious act, a very short time when lot of high frequency neuronal firing goes on, that we are not aware of or conscious of. Actual consciousness has a much longer lifetime and only then we are aware of what is going on. Consciousness is part of dissipative channel in the brain . Both conscious and non-conscious processes are occurring where Im $\Sigma(p,\varpi) \neq 0$. Hence both are loss processes, only a very small part we are conscious of . Rest is default mode. When we go back to the pole at energy $\varpi =\epsilon - \mu_p + Re\Sigma(p,\varpi) = 0$ by construction , because it is by a

Im $\Sigma(p, \varpi) = 0$ mental 'spot', we cannot be conscious of anything when we are there. Its zero band width confers on it infinite life time but that too we cannot be conscious of ! The coherent state will remain at that energy level indefinitely like an electron in its orbit unless we act on the state by changing Fermi level or move in the parameter space. This 'pole position ' in the $\mathbb{CP}^2$ mental space can be named the *state of Sleep* . BOLD signal will be inexistent or very weak in this state, brain is least oxygenated and energy expenditure rate is at its minimum. We have drawn an energy gap $\triangle$infigure (A) in order to indicate that going from the Sleep state to subconscious default mode or Conscious state and this can only be a first order thermodynamic process initiated in the neuron when it overcomes some threshold potential or at some critical current. It has been conjectured-([20]) that only when our cortical brain is globally activated that one develops a subjective experience.

4.1 Specific Issues

4.1.1 REM Sleep : Oniric dreams

We had mentioned in section 2.1 the two different scenarios of symmetry breaking, A & B. In the last few sections we brought out that scenario B, which is $\frac{SU(3)}{SU(2)\times U(1)}$ gives us the ground state $\mid \phi_s \succ$ which is the state of sleep . The second route of symmetry breaking was shown to be $\frac{SU(3)}{U(1)\times U(1)}$ which will give a new state $\mid \phi_d \succ$ which we shall dream state, characteristic of REM (Rapid eye movement sleep) , a state in which about 50% of our sleeping time is consecrated. It is higher in energy than the deep sleep slate but unlike that state, sleep is more agitated with accompanying eye movements as if the sleeper is acting out visually the vivid scenes he is acting out in his dreams . There are a number of very important cognitive functions of REM sleep but there is no overall consciousness . In this state, the symmetry breaking gives us two wave functions that can be written as a column

$$\phi_d = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \tag{56}$$

The unitary symmetry that operates is

$$\phi'_d = U_d \phi_d$$

Here U_d is a 2×2 matrix that can be written as

$$U_d = U(1) \times U(1) = \begin{pmatrix} exp - iq_1\vartheta & 0 \\ 0 & exp - iq_2\vartheta \end{pmatrix}$$

This is a two dimensional representation of U(1) . As the angle ϑ varies, the matrix generally does not repeat itself and the range of values of the matrix is unbounded.This is because in the dream state the charge $\frac{q_1}{q_2}$ will be an irrational number. This is a very strange electromagnetic gauge space, that is *unbounded* and is of infinite volume and non-compact. U(1)$\times U(1)$ is a torus subgroup. It winds around the torus without ever meeting itself again, just as a dream does never exactly repeating itself. There is no monopole [21] . We call this gauge space, the oniric space , the enchanting magic world of Alice. To end this section we may mention the greek legend of dream state embodied by two divinities that one can write as a column vector

$$Dream = \begin{pmatrix} Eros \\ Hypnos \end{pmatrix}$$

These two states in mythology are completely entangled with each other and Freud seemed to have simply appropriated the whole concept . In our analysis we just discovered one plausible basis of the same ideas about how we dream.

4.1.2 Conscious Mental Order Parameters

The ground state $\mid \phi_s \succ$ is at a energy $\triangle_C$ below the vacuum state energy taken as zero of our energy . The upper four gauge bosons in the consciousness band start its continuum at the consciousness band. Like any photon of the electromagnetic gauge field, where each field tensor can be either an electric field tensor or magnetic field our mixed state gauge photons will also live each in two modes of polarization, in two orthogonal tensor fields . Our four gauge potentials or their tensor fields have altogether eight field tensors . Hence they signify eight modes of mind polarizations, eight different mental functions that we can write down as Conscious mind-order parameter. These are:

$$\chi(x) = \begin{bmatrix} Attention \\ Executive\, function \\ Somato - sensory \\ Motor \\ Memory \\ Emotional\, regulation \\ Visual \\ Auditive \end{bmatrix}$$

The four different gauge fields with their equivalent mental attributes are seen here as pairs of orthogonal polarisation modes (attention & executive function, somatosensory paired with motor, memory & emotion, visual linked with auditive).The eight of the mind's polarisation modes have given rise to *eight order parameters* of brain . Neurologists tell us where some of these order parameters are located .

These order parameters can also be thought as eight centers of a variety of brain-functions which are deeply associated with our whole mentation process. These we see clearly in the attached figure delineated by Brodman known as Brodman's areas ([24])). Brodman's areas show subdivisions all of which carry distinctive mental attributes that we illustrate by the different colorings.The full symmetry of the massless gauge photons of gauge group H is evident here.

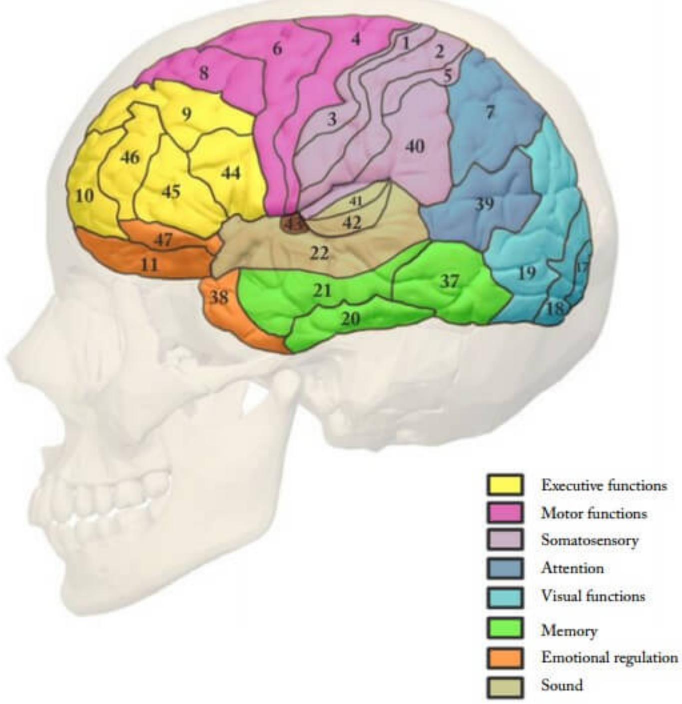


Figure 5: Order parameters of cognitive brain

4.1.3 Memory as a classical soliton

The gap $\triangle_c$ has a whole set of localized states, which can be called *memory states*. This comes about when the wave functions in the consciousness band ϕ_c interact with the four massless gauge photons as can be seen in the accompanying Figure below:

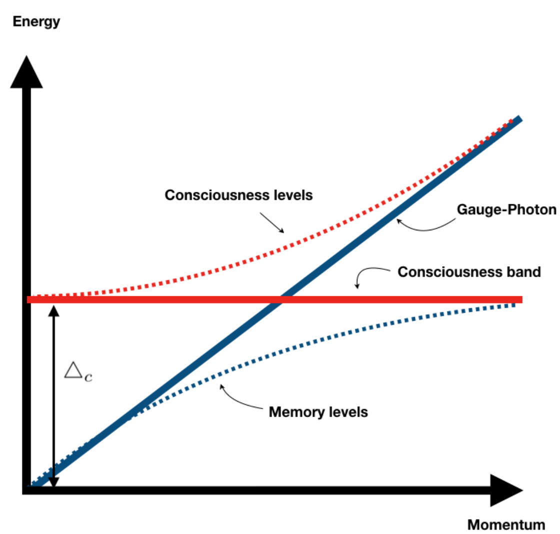


Figure 6: Memory levels

There are now two sets of levels, the lower set is in the energy gap and these are mixed localized states of information field clothed with gauge photons which have developed mass . As the figure shows, the memory states will be spread out all throughout the energy gap. These states are our seats of memory.The upper set is in the continuum and these will carry consciousness. The energy gap will protect the memory states from decaying into the continuum as long as $\triangle_c \geq kT$.

These states are finite positive energy states localized only in certain space-time region, when and where incidents occurred to become memory. Memory by definition is an energy field localized in a slice of three dimensional space and one dimensional Minkwoski time, $\boldsymbol{t}$. In the figure A, the energy is spread over range $\triangle_c$ and momentum range $\triangle k_c$ so that the energy is localized within a temporal range $\triangle t \sim \frac{\hbar}{\triangle_c}$ and a spatial range$\triangle x \sim \frac{1}{\triangle k_c}$. Here x is a three dimensional coordinate $\{x, y, z\}$. The total energy δE that is stocked in this region should be of the order of the field energy of the gauge photons . These photons are now localized within a specific -space-time region size $\sim \triangle x \triangle t$. Such an object can be called a Yang-Mills soliton. These solitons are our memory states. If our time t be taken as the Euclidean time τ (going from -∞ *to* $+\infty$), then we have gotten a pure *four dimensional space* where like space, the time dimension τ does not "flow" and has become an additional space dimension. This buttresses the observation that in our memory the images do not *age* and retain the pristine quality of freshness. They do not live in Minkwoski time.

Let us look a little more critically the problem. Memory may be thought as a correlation function in time between image $\mathbb{I}(0)$ at time t= 0 with that at time t, $\mathbb{I}(\mathrm{t})$ written as

$$\mathcal{M}(\mathrm{t}) = \prec \mathbb{I}(\mathrm{t})\mathbb{I}(0) \succ \tag{57}$$

We can equally express it in the Fourier space of frequency ω through the transformation

$$\mathcal{M}(\omega) = \frac{1}{2\pi}\varrho \int \mathrm{dt}\mathcal{M}(\mathrm{t})\mathrm{expi}\omega\mathrm{t} \tag{58}$$

$$= \varrho\mathcal{M}(\infty)\delta(\omega) + \varrho \int \mathrm{dt}\left[\mathcal{M}(\mathrm{t}) - \mathcal{M}(\infty)\right]\mathrm{expi}\omega\mathrm{t}$$

$$= \mathbb{I}_0\delta(\omega = 0) + \rho \int \mathrm{dt}\,[\mathcal{M}(\mathrm{t}) - \mathcal{M}(\infty)]expi\omega t \tag{59}$$

The first term of the correlation function , the $\omega = 0$ term is very interesting . It gives us the permanent memory , our autobiographical and learned-skills memory part . It is the *static part* and can be thought to be as a Bose-condensation in temporal space instead of conventional physical space.

The finite energy of these states in a localized region of space-time allows to qualify these long-time memories as a $static\ Yang - Mills\ solitons$. Because the theorem of Deser([22]) forbids static Yang-Mills soliton in all spatial dimension n except for n=4, we conclude that the five fold dimension of our mental space , the coset space $\frac{G}{H}$ must be written as the Minkwoski space x,y,z,τ& $\boldsymbol{t}$ where τ is the Euclidean time serving as the fourth axis of space, where the Minkwoski time remains t. The metric of the mental space is then (1,-1,-1,-1,-1). Thus we have to conclude that our long term memory resides in Euclidean time, τ. The short time memory on the other hand decays in Minkwoski real time $\boldsymbol{t}$. Because the spatial dimension of mind space is four $\mathbb{E}^4$ ($\mathbb{S}^3$) and that of our

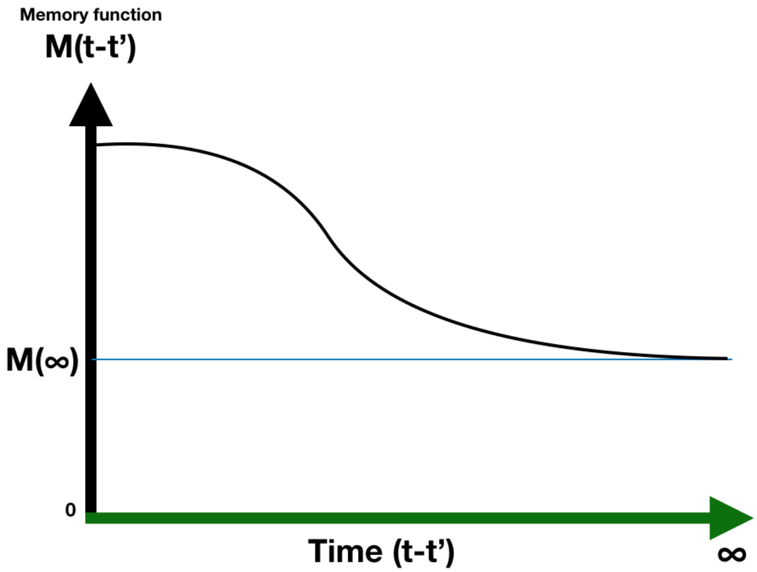


Figure 7: Memory short and long term

ordinary daily physical space is three $\mathbb{E}^3$ ($\mathbb{S}^2$) , mental space cannot be mapped or photographed into physical space: $\Pi_2(\mathbb{S}^3) = 0$.

Conclusion

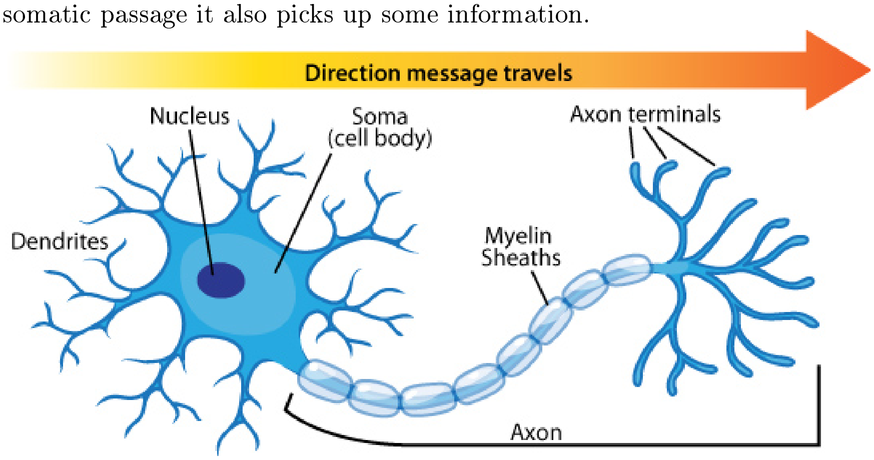
- We have shown that our brain can be described as a Hilbert space in $\mathbb{C}^3$ of three complex functions that exemplify a three level system carrying a SU(3) internal symmetry.
- A symmetry breaking into neuron states can occur either to the deep-sleep state $\frac{SU(3)}{SU(2)\times U(1)}$ or to the dream state $\frac{SU(3)}{U(1)\times U(1)}$.
- Our *mental space* was defined as the projective Hilbert space having a $\mathbb{CP}^2$ manifold, basically the image space. Projection from the brain space to mind-space is a many to one imaging .
- Mind space was shown to be a five dimensional Minkwoski space (space x,y,z,τ&time t) has four *spatial dimensions* which is the space of our permanent memory. As a direct result of the extra spatial dimensionality of mind space over ordinary physical space, images in our mind cannot be projected onto the ordinary space: an image in the mind cannot be photographed!

- Projective Hilbert space was also shown to be the space where consciousness emerges as a coherent state vector.

Appendix **A**

Symmetry Space of a single neuron

A neuron is a single cell creature. Charged excitations dribble into it from outside world (or our body) via the dendrites . But there is no way such an excitation will reach into the axon of neuron without first getting cleared by the genetic soma (fig 5) . The basic architecture of the neurons is clear: that what enters the axon of the neuron is more than the electrical excitation . Through its somatic passage it also picks up some information.



We shall propose a reasonable model that catches the essence of the transformation process from dendritic excitation to axons where genetic molecules will play the key role ([23]).

Genetic space is the space of complicated organic molecules which form the basis of all genetic functions. What one calls genetic alphabet is comprised of just four molecules. These four magic genetic molecules (called nucleotide *bases*) of our genomic alphabet $\mathscr{A}$ are:

$$\mathscr{A} = \begin{pmatrix} Adenine \\ Guanine \\ Cytosine \\ Thymine \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} \tag{60}$$

In the last column we have abbreviated the nomenclature to keep it readable, and think of a, b, c, & d four real numbers enclosing a real space $\mathbb{R}^4$. Although in the elaborate construction of our genetic code, these molecules live as codon units of composites of triplets of such molecules, strung along the double helix, what we really have is repeated use of these four letters that serve as the crucible into which the excitations from outside world plunge into and come out of. Our genome is a vast space of special information carrying molecules on inter-twinning helices , of about 3-billion of such molecules that are arranged in sequences called genes. But there are also genetic material which do not constitute genes. In point of fact only only about 1.5% of the genomic molecules are used as genes, the rest is either inter-genetic and intra-genetic (introns) which do not function neither as genetic code nor as protein producing amino-acids. This vast 98% is simply there as dark matter, like some intergalactic space. We can imagine our genes as a stretch of archipelago strewn and scattered like some beads on a necklace surrounded by a vast ocean of non coding genetic molecules. These other molecules empty of codons will be our center of attention. One does not know what roles these 'empty' molecules play. There is a whole developing science of epigenetic that postulates that these molecules are very susceptible to external environment and through that may influence the actual genes.

As soon as an outside dendritic excitation *enters* the genetic $\mathbb{R}^4$ region, the excitation will take the imprint of four genetic molecules and fragment into a primal information field $\mathscr{I}(\mathrm{r})$ with four real parameters at space point r, which we write as

$$\mathscr{I}(\mathrm{r}) = \begin{pmatrix} f_\alpha(r) \\ f_\beta(r) \\ f_\gamma(r) \\ f_\delta(r) \end{pmatrix} \tag{61}$$

These four f-fields have a representation $\mathbb{S}^3$, a 3-sphere for four real number space $\mathbb{R}^4$. Let us now write the four parameters as two complex information fields or two complex column vectors components of the wave function $\psi(r)$,

$$\xi(r) = \begin{pmatrix} \mid 1 \succ \\ \mid 2 \succ \end{pmatrix} \equiv \begin{pmatrix} z_1(r) \\ z_2(r) \end{pmatrix} = \begin{pmatrix} f_\alpha(r) + i f_\beta(r) \\ f_\gamma(r) + i f_\delta(r) \end{pmatrix}$$

We can write

$$\xi'(r) = U(2)\xi(r)$$

Here the internal symmetry space enjoining the two complex functions is a complex 2×2 matrix denoted bu the general unitary matrix, is called U(2).These transformations of signal space $\mathscr{I}(\mathrm{r})$ in $\mathbb{R}^4$ to two dimensional complex space $\mathbb{C}^2$ is the Hopf map $\mathbb{S}^3 \to \mathbb{S}^2$ [3].

Appendix ***B***

SU(3) generators and Root vectors ([9])

Let $\mathrm{T}_a = \frac{\lambda_a}{2}$ be the generators of the SU(3) group . There are two diagonal generators, namely T_3 & T_8 . They are said to form a Cartan subalgebra . The two diagonal generators form a two dimensional vector H

$$H = (T_3, T_8)$$

The remaining six non-diagonal generators can be grouped together as follows:

$$E_{\pm 1} = \frac{1}{\sqrt{2}} (T_1 \pm iT_2)$$

$$E_{\pm 2} = \frac{1}{\sqrt{2}} (T_4 \pm iT_5)$$

$$E_{\pm 3} = \frac{1}{\sqrt{2}} (T_6 \pm iT_7)$$

The commutators of H with the non-diagonal generators can be written as

$$[H, E_{\pm a}] = \pm w_a E_{\pm a},\ a = 1, 2, 3$$

The vectors w_a are called the root vectors . Their components are

$$w_1 = (1, 0)$$

$$w_2 = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

$$w = \left(-\frac{1}{2}, +\frac{\sqrt{3}}{2}\right)$$

The root vectors have unit length. They are neither orthogonal nor independent and form an over-complete set

$$\sum w_a^i w_a^j = \frac{3}{2}\delta_{ij}$$

References

[1] **Quantum Mechanics** : **Fundamentals**, Kurt Gottfried, p59 (Addison - Wesley, N.Y 1966),

[2] **Topologically Non − trivial phases in Condensed matter physics**, F. Duncan Haldane , 2012 Dirac Medal ceremony, ICTP Trieste, Italy , July 2013

[3] Applications ofTopology to the study of Ordered Systems, R.Shankar, Jl de Physique, Tome 38, p 1405,1977,

[4] M.Gell-Mann, Phys Lettr, Vol 8, p 214, 1963

[5] G.'tHooft, Nucl.Phys B190,p 455 (1981)

[6] **QuantumFieldTheory**, Claude Itzykson & Jean-Bernard Zuber (McGraw-Hill Book, N.Y 1985 Student Edition) p509

[7] Quarks, Leptons & Gauge Fields, Kerson Huang (World Scientific, Singapore 1992) p61

[8] **QuantumFieldTheory**, Lewis H.Ryder (Cambridge University Press, second Edition , 1996, Cambridge)p282

[9] **Dual Superconductor Models of Color Confinement**, Georges Ripka, (Springer-Verlag, Berlin 2004) p77

[10] **Geometry of Quantum States**, Ingemar Bengtsson & Karol Zyckowski (Cambridge University Press paperback Edition, Cambridge 2008), p75

[11] M.V.Berry, Proc . Royal Soc, **A392**, p45 (1984); 'Geometrical Phases in Physics', Alfred Shapere & Frank Wilczek (World Scientific, Singapore 1989)

[12] Y. Aharonov & J.Anandan, Phys. Rev. Lettr, vol 58, p1593(1987)

[13] E. Majorana, Nuovo Cimento, vol9, p 43 (1932)

[14] J.M. Radcliffe, Jl. Phys **A, p313**,1971; Generalized Coherent States', A.M.Perlemov (Springer -Verlag, Berlin 1986)

[15] **Shadows of the Mind**, by Roger Penrose, p279 (Oxford University Press, Oxford 1994)

[16] Wojciech Ganczarek, Marek Kus &Karol Zyckowski, Phys. Rev **A** 85, 032314, 2012

[17] Wei-Min Zhang 'Quantum Field Theory' (Ed: by A.N.Mitra, 1997) p297: ArXiv.hep-th/9908117

[18] G.D.Mahan 'Many Particle Physics' (Plenum Press, N.Y 1981)p.155

[19] Marcus E. Raichle, Scientific American 2010 ,p 28

[20] S.Dehaene & J.P. Changeux , Neuron 2011, Vol 70, p 200

[21] G. Khanna, S. Mukhopadhy, R.Simon & N. Mukunda, Annals of Physics, 1997, vol 253, p55

[22] S. Deser, Phys. Lettr ,vol 64,B463 (1976), see also Huang, ibid, P87

[23] **The Gene**, Siddhartha Mukherjee, (Bodley Head,London 2016) p133

[24] Neuroscience, M. F. Bear, B.W. Connors, M. A. Paradiso, Lippincott Williams & Williams, N.Y., 2001

Acknowledgement

The author wants to thank many discussions with Timothy Ziman of the Theory group of Institut Laue-Langevin, Grenoble, Mario Cuoco of University of Salerno, Italy and Indranil Paul, of University of Paris, Diderot Campus.