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A generic ageing model for prognosis - Application to Permanent Magnet Synchronous Machines

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ABSTRACT

In the context of more electrical aircrafts, Permanent Magnet Synchronous Machines are used in a more and more aggressive environment. It becomes necessary to supervise their health state and to predict their future evolution and remaining useful life in order to anticipate any requested maintenance operation. Model-based prognosis is a solution to this issue. A generic modeling framework is proposed in this paper in order to implement such a prognosis method which relies on knowledge about the system ageing. A review of existing ageing laws is presented, and motivates the choice to develop an ageing model that could incorporates every kind of ageing laws. A generic ageing model is then defined, that allows representing the ageing of any equipment and the impact of this ageing on its environment. It includes the possible retroaction of the system health state to itself through stress increase in case of damage. The proposed ageing model is then illustrated with Permanent Magnet Synchronous Machines. A fictive but realistic scenario of stator ageing is built. It comprises apparition and progression of an inter-turns short-circuit and its impact on stator temperature, which value has an impact on the ageing speed. A prognosis method based on the generic ageing model is proposed, and applied successfully to this scenario.

1. INTRODUCTION

In the context of the more electrical aircrafts, electrical motors such as permanent magnet synchronous machines (PMSM) are more and more used for critical functions in the actuators, such as landing gear extension/retraction, braking systems, or flight control. They are often used in very aggressive environments. The transition from 270V to 540V of supply voltages, and the increase in switching frequencies, also applies a lot of additional stress on the motors. In this aggressive context, PMSMs risk to be subject more and more degradation and faults. In order to ensure the operational availability of critical functions, one option is to implement a Health-Monitoring module. This Health-Monitoring module consists in a detection and diagnosis module, that allows assessing the current health state of equipments, and a prognosis module, that allows predicting the future health state of equipments, and their remaining useful life (RUL). With prognosis, the maintenance action can be anticipated in advance. The goal is to optimize maintenance planification and avoid any operational interruption or flight delays due to equipment faults.

Predicting the future health-state of equipments requires a knowledge about their ageing. This knowledge can take several forms, it can be based on experience, on degradation and ageing data obtained in service or in tests, or on ageing physical models. Different ageing knowledge representations and prognosis methods used in the literature are presented in Section 2. The conclusion is that the ageing knowledge can always be put into the form of an ageing model. A generic ageing model is then presented in Section 3. It allows representing the behavior and ageing of any kind of equipment, that may be heterogeneous and complex. An application is proposed on Permanent Magnet Synchronous Machines, with the modeling of two critical progressive degradation: inter-turns short-circuits and rotor demagnetization. The modeling framework contains all the information that is needed to perform diagnosis and prognosis. A generic prognosis method based on the model is then proposed on Section 4. The prognosis algorithm is presented, and applied to a short-circuit virtual scenario simulated thanks to a complete PMSM virtual prototype. Finally Section 5 proposes some conclusions and perspectives.
2. Ageing models for prognosis

In order to predict the system remaining useful life, prognosis requires knowledge about the system degradation or ageing that is contained in a model. This model describes the evolution of the system ageing state, it is a priori known and used on-line for predictions. In the literature, several prognosis methods already exist which rely on different models. Three classes of methods that rely on three types of available knowledge can be distinguished:

- experience-based prognosis,
- data-driven prognosis,
- and model-based prognosis.

The choice of one of these methods depends on the level of knowledge contained in ageing model and is mainly characterized by the availability of sensors that allow obtaining on-line data of the system state. Every approach has pros and cons, and it is often useful to combine them.

2.1. Experience-based prognosis

Experience-based approaches, like case-based reasoning or reliability analyses, are the only alternative when no sensors nor physical knowledge of the system ageing is available. This form of prognostic model is the simplest and only requires failure history to determine the probability of failure within a future time (Gebraeel, Elwany, & Pan, 2009). Reliability techniques are used to fit a statistical distribution to the failure data.

The Weibull law is often used due to its flexibility in reliability analyses for mechanical or electrical components. It can represent a time-dependent failure rate by describing the different phases of a component life with three parameters. (Bufferne, 2009) represents the impact of corrosion, fatigue, or wear on components with these three parameters. (van Noortwijk & Klatter, 2002) models the cost of structure replacement with Weibull distributions by applying the maximum likelihood estimation method on life data obtained from broken structures. The cost of structure replacement is then computed thanks to their current age and uncertainties related to the predicted replacement date. The main drawback of the Weibull law is the difficulty of estimating these three parameters. The exponential law is simpler as it depends on only one parameter, the failure rate, which is constant. It can represent a component ageing without wear, i.e. the abrupt failures. It is used a lot for life duration of electronic devices. For progressive failure, the Gamma law seems to be well suited. It can represent a failure rate increasing in time and is used to model progressive failures like crack evolution in (Lawless, 2004) or erosion in (van Noortwijk, Kallen, & Pandey, 2005). It is also possible to use several laws simultaneously like in (Huynh, Castro, Barros, & Berenguer, 2012) which combines a Gamma law with a Poisson process to model progressive degradation and abrupt failures.

Models used by experience-based approaches are the less complex ones, and use available data without dedicated effort, except if there are no enough experience feedback: equipment that are new, and/or very reliable, and never fail before a preventive maintenance action, as in aeronautics. This approach brings only few information, and does not take into account the way the equipment is used, or its past. This might be useful for the manufacturer, but not for the user that is interested in one particular component.

2.2. Data-driven prognosis

Evolutionary and trend monitoring methods are used when on-line observed data are available. Prognostic models then simply rely on degradation estimators or indicators (Kalman filters or various other tracking filters for the state estimation). The prognostic method then uses on-line estimators to evaluate the system current degradation state relying on the on-line observations. To get the estimators, failure history is required (identification of fault patterns). Such estimators may be obtained by learning techniques (neural networks or Bayesian networks) or by identifying parameters of classical estimators like for Kalman filters (Hu, 2011).

Neural networks allow building a grey/black box ageing model to estimate and predict the current and future trend of the system degradation from specific indicators. The prediction relies on the network learning from experience that extracts essential characteristics from noisy data (Goh, Tjahjono, Baines, & Subramaniam, 2006). Neural networks are used in (Das, Hall, Herzog, Harrison, & Bodkin, 2011) to perform prognosis on systems of high-speed milling. (Adeline, Gouriveau, & Zerhouni, 2008) tests and compares different methods based on neural networks in terms of prediction precision, computation cost and requirements related to the implementation. (Vachtsevanos & Wang, 2001) combines neural networks with wavelets to predict the RUL of rolling bearings. Fuzzy neural networks combines neural networks and fuzzy logic to deal with ambiguous, inaccurate, noisy or incomplete data (Goh et al., 2006; El-Koujok, Gouriveau, & Zerhouni, 2010). Fuzzy systems use knowledge as expert rules. They are recommended in case where no qualitative information about the system degradation is available but only causal rules describe fault propagation within the system (El-Koujok et al., 2010). They can be automatically adjusted and do not require physics-based knowledge.

Ageing models can be represented by Bayesian networks that are acyclic graphs defined by a set of nodes and relations with conditional probabilities. Each node may represent a potential degradation mode of the system and transition probabilities from a current mode to possible future modes result from a learning phase. A priori probabilities in Bayesian networks have to be introduced by the expert. Theory of
Bayesian networks is well explained in (Bouaziz, Zamai, & Duuvier, 2013) which shows its relevant application in the semi-conductor industry. (Weber, P.Munteanu, & Jouffe, 2004) uses dynamical Bayesian networks and Markov chains to model the ageing of a system composed of a pump and a valve. (Camci & Chinnam, 2010) models the progressive deterioration of drills with hierarchical hidden Markov models that are estimated from tests. For a clear and simple representation, they can also be described by a Bayesian network where a node stands for a health state. The RUL is then predicted from transition probabilities of the network. (?, ?) combines Bayesian networks with an event-based approach to monitor degradation of an automatic mechanical system of laminating. A priori knowledge is based on experience and trend monitoring is performed on line thanks to data. Physics-based knowledge allows determining causal relations of component degradations.

(Greitzer & Pawlowski, 2002) proposes a prognostic method based on the trend monitoring of a health state indicator that results from a composition of observations and is evaluated on-line from a failure threshold. A parametric model of the vibration waveform for different faults (particularly for bearing faults) is estimated to perform prognosis on a diesel motor. (Byington & Stoelting, 2004) performs diagnosis and prognosis on an EMA of a flight control system with a model whose parameters are estimated from on-line data. Diagnosis estimates the current health state of the system with classification tools. Prognosis computes the rate of change of state at current time and anticipates it in the future. In this study, prognosis is a simple temporal prediction of the indicator evolution that does not take into account the equipment environment. (Lacaille, Goubey, & Piot, 2013) studies the wear of turbojets and proposes a simple algorithm to build a degradation indicator from successive measurements of exhaust gas temperature after each flight according to the operating time. The strength of the data-driven method is that it transforms a huge amount of noisy data into a few relevant data for prognosis. It does not require knowledge about failure mechanisms. The main drawback is that the method efficiency highly depends on the quantity and quality of data. Moreover, results are valid in a similar situation but for different configurations, generalization and extrapolation is controversial since the indicators have no physical meaning. In aeronautics, equipment are generally very reliable, and maintenance is preventive and realized before the failure occurrence, so there are very few degradation data. Tests can be done to obtain data, but they are costly, time consuming, and destructive.

2.3. Model-based prognosis

Model-based prognosis is based on deep knowledge of the equipment ageing and relies on a continuous physics-based model of the component degradation. The ageing model is represented as a set of equations which involve physical quantities corresponding to environmental constraints (Onori, Rizzoni, & Cordo-Banares, 2012; Roychoudhury & Daigle, 2011; Bregnon, Daigle, & Roychoudhury, 2012). The model provides more information by extrapolating on-line data by physics-based reasoning. It can be an analytical model based on physical laws or a simulation model identified from tests results. In (Gucik-Derigny, Outbib, & Ouladsine, 2011), the ageing model is represented as a set of non-linear differential equations with multiple time scales (short for the system behavior dynamic and large for its degradation). Three observers with unknown inputs are compared for a linear example with sliding mode. The illustrative example is an electromechanical oscillator whose dynamical and ageing models are known. The fast dynamic state is estimated thanks to the observer and the parameters of the slow dynamic are determined. In (Khorasgani, Kulkarni, Biswas, Celaya, & Goebel, 2013), the ageing of electrolytic capacitors with temperature is represented by a complex nonlinear physics-based model. Particle filtering is then used to estimate the parameters of the degradation model.

Physics-based ageing models can be divided into three types depending on their output format. They can directly compute the remaining useful life or progressive evolution of degradation by evaluating the damage or a failure rate to anticipate the future behavior of the equipment. The Arrhenius law is used to represent the impact of temperature on the lifetime of an electronic device or a component whose degradation process is chemical. (Venet, 2007) uses it to model the ageing of liquid electrolyte capacitors but it can also be applied for dielectric components, semiconductors, batteries, lubricant or plastic filament incandescent lamps. The inverse power law also describes the impact of damaging factors on the component lifetime like voltage on electronic components for example. It is applied to dielectric components, ball bearings, optoelectronic or mechanical components subjected to fatigue. A specific case of the inverse power law is the Coffin Manson law that gives the number of cycles leading to the rupture when components are subjected to temperature variations or thermal shocks. The generalized Eyring model allows taking into account any type of damaging factor like temperature, voltage, humidity, etc. It is used to model the ageing of electronic components, aluminum conductors, mechanical components subjected to rupture.

The Paris law calculates the damage associated to a component. It is used in numerous works like in (Pommier, 2009-2010) where it represents the crack propagation according to the number of cycles. The Miner’s law models the accumulation of linear damages due to fatigue. It can be used for metals only until yield strength. The Wiher curve gives the number of cycle leading to damage thanks to a characteristic parameter like maximal constraint for example.
The American military norm MIL-HDBK-217 gives the failure rates for some components such as transistors, resistors, etc. For example the law Belvoir Research Development & Engineering evaluates the failure rate of a solder joint. The Cox model is mainly used in the medicine and maintenance fields to study the impact of different variables involved in the degradation process of components. The mathematical expression is based on a failure risk function (Letot & Dehombreux, 2009).

A physics-based ageing model can be determined from physical analytical laws or from tests performed in controlled conditions to identify characteristic parameters of the system degradation. In this second case, the damage evolution is assumed to be measured from tests. Moreover simulation is interesting as no component destruction nor deterioration is needed to study the system degradation. All data are assumed to be observable which allows choosing the suited sensors to implement. The main difficulty consists in elaborating and validating the ageing simulation model, since equipment are complex and faults are multiple and difficult to be understood as a whole (Bansal, Evans, & Jones, 2005).

In some cases, it can be useful to combine different types of information in a common ageing model. By combining failure history and physical laws, a statistical physics-based model can be obtained. In such a model, physical stress is represented through a parameter of the statistical law. The statistical law can then be adapted to the operational environment of the component. The difficulty is to assign a physics-based law to one or several parameters of the statistical law like in (Brissaud, Lanternier, Charpentier, & Lyonnet, 2007), (Nima, Lin, Murthy, Prasad, & Yong, 2009), (Gebraeel et al., 2009) or (Byington, Roemer, & Galie, 2002). (Ray, 1999) builds a stochastic model for the crack propagation in a metallic material (in structure or oil pan for example). The physics-based equation is validated from test data. The non-stationary probability density function depends on the instant of crack initiation and its actual size (in order to deduce the speed of the crack propagation). (Hall & Strutt, 2003) proposes a statistical model of physics of failure. It results from Monte-Carlo simulations performed with different parameters of the physics-based degradation model to obtain the failure dates. These values are then represented with the Weibull distribution whose parameters are well chosen to fit data.

2.4. Synthesis

The choice of a prognostic method depends on available knowledge, the presence of sensors or physics-based models that allow monitoring and analyzing the real condition of the system. This ageing knowledge can be represented as an experience, a known qualitative or quantitative model or an estimated model obtained by learning and classification methods. The prognostic model may vary from a very poor model (that cannot handle on-line observations for example) to a very rich one (that can handle on-line observations and can extrapolate these observations in terms of physical reasons for the component to fail in the future). In an industrial context such as aeronautics, a lot of equipment is similar but no identical. The objective is to build a generic model-based prognosis method that relies on a generic representation of component ageing. So in this paper, the challenge consists in defining a generic ageing model whatever the available knowledge about the system degradation.

3. A GENERIC AGEING MODEL AND ITS APPLICATION TO PERMANENT MAGNET SYNCHRONOUS MACHINES

3.1. The generic ageing model

In (Vinson et al., 2013) a structural and functional model is presented. A system Σ is a set of n components Cᵢ. Parameters p represent physical quantities in a component. There are three kinds of parameters. Input parameters ip values depend on the environment, private parameters pp belong to only one component, and output parameters op are a combination of input and private parameters through functional relationships ar. The values of parameters at time t are p(t). The rank r of a parameter p is the set of possible values, such as νt ∈ r(p). Components are connected through the structure st via their input and output parameters to form the system. Two parameters structurally connected are such as ipᵢ→j = st(opᵏᵢ→l) ⇒ ∀t, ipᵢ→j(t) = opᵏᵢ→l(t). This structural and functional model is represented on the first layer of the modeling framework on Figure 1. The ageing model developed hereby enriches the functional model.

3.1.1. Damage and ageing laws

3.1.2. Damaging factors

During operational life an equipment ages, it is damaged. Ageing is due to stresses, that can be thermal, electrical, mechanical or chemical. Stresses are modeled with damaging factors. The set of damaging factors of one component Cᵢ is Dᵢ = {dfᵢ}. The set of damaging factors of the system Σ is D = ⋃ᵢ Dᵢ. The value of a damaging factor at time t is df(t). Ranks are defined for damaging factors, they are noted r(df) and they are such as ∀t, v(dfᵢ, t) ∈ r(dfᵢ).

3.1.3. The damage

The equipment ageing is characterized by its damage. Damage is irreversible. It is null at the beginning of the equipment life and increases with the ageing.

Since they do not vary for functional purposes and they are intrinsic to one component, we decide to use private parameters and their values to represent the system and component health state. A private parameter modification represents therefore a damage. The damage eᵢ→j at time t is modeled as the distance
between $pp^{i,j}(t)$ and the initial value $pp_{0}^{i,j}$:

$$e^{i,j}(t) = d(pp_{0}^{i,j}, v(pp^{i,j}, t))$$

(1)

with $pp_{0}^{i,j} = pp^{i,j}(t_0)$ and $e^{i,j}(t_0) = 0$.

There is one damage per private parameter, but every component may have several damages represented by different private parameters.

The damage depends on stresses. The ageing law $ag$ allows the calculation of damage $e$ as a function of the damaging factor values $df_{1},...df_{n}$:

$$\{ ag : \mathbb{C} \times T \rightarrow \mathbb{C} \\
(\{ df_{1},...df_{n}, t \} ) \rightarrow e^{i,j}(t) = ag(df_{1},...,df_{n}, t) \}$$

(2)

It is possible to define a global damaging factor as a combination of damaging factors, in order to have a unique parameter for the ageing law, and to include known ageing laws (described in Section 2) in this approach.

3.1.4. The retroaction law

The stress that undergoes an equipment depends on its environment but also sometimes on its own damage. Indeed a damaged component often has a more negative impact on its environment and on itself. For instance the wear of a component will increase the level of pollution in a mechanical system, and pollution is certainly a stress for the component and its environment.

This is modeled by the fact that damaging factors values depend on the system health state. The function $f_{df}$ assesses a damaging factor rank. The rank may depend only on the system environment. Otherwise, if the rank of a damaging factor depends on the system health state, the function $f_{df}$ is defined as follows:

$$\{ f_{df} : D^\Sigma \times Supp(df_{i}) \rightarrow I_R \\
\quad \quad \quad \quad \quad df_{i} \rightarrow r(df_{i}) = f_{df}(\{ e^{x,y}(t) \}) \}$$

(3)

We highlight that the damage depends on damaging factors through ageing laws and that damaging factors depend on the damage through the retroaction laws. Figure 1 presents both the functional and structural model on the first layer and the ageing model on the second layer. The two models communicate through the private parameters, that is to say through the health state: the ageing model affects the functional model.

All kind of knowledge can be represented with this generic modeling framework, as will be shown on our industrial application.

3.2. Application: the ageing model of PMSMs

3.2.1. The functional model of PMSMs

The functional and structural model of PMSMs is shown on Figure 2. The PMSM has two components, the stator and the rotor that are combined to perform the PMSM function: to transform supply voltage $U_{ab}, U_{bc}, U_{ca}$ into a given mechanical speed $\Omega$, independantly of the torque $C$ applied by the environment on the shaft of the PMSM. The stator transforms the voltages into phase currents, $I_a, I_b, I_c$, dependently of the induced voltages $E_a, E_b, E_c$ produced by the rotor. The stator private parameters are the phase resistances $R_a, R_b, R_c$ and inductances $L_a, L_b, L_c$. The rotor transforms the phase currents into a mechanical speed. Its private parameters are the magnets electromagnetic remanent field $B$, the rotor inertia $J$ and the friction coefficient $K_f$. The relationships between parameters are explained in details in (Vinson, Combacau, & Prado, 2012).

Figure 2. Modeling of the PMSM.

After a Failure Modes Effects Analysis and Criticity two faults were selected as candidates for model-based prognosis, corresponding with the two components of the PMSM: inter-turns short circuits in the stator and demagnetization of a part of the rotor.
3.2.2. The stator ageing : inter-turns short-circuits progression

A common and critical degradation of PMSMs are short-circuits, and especially inter-turns short-circuits, that come from the stator insulation ageing and degradation. A short-circuit model is proposed in (Vinson, Combacau, & Prado, 2012) and (Vinson, Combacau, Prado, & Ribot, 2012). There is the creation of a short-circuit loop in one of the three phases, phase A for instance. Two fault parameters, \( R_f \) and \( S_a \), represent the intensity of the short-circuit. \( R_f \) is the resistance of the insulation at the short-circuit point and progressively decreases until 0\( \Omega \) in case of direct short-circuit. \( S_a \) is the percentage of short-circuited turns and varies between 0 and 100%.

The private parameter that represents the damage of the stator is chosen to be the short-circuited phase resistance, \( R_a(t) \), for the three following reasons. It varies with short-circuit, it depends on the two fault parameters, \( R_f \) and \( S_a \), and unlike them it can actually be measured on a real PMSM. \( R_a \), the equivalent resistance of phase \( A \) with the short-circuit loop of resistance \( R_f \), is expressed as:

\[
Ra(t) = R_a(t) = R_a(t)(1 - S_a(t)) + \frac{R_a(0)S_a(t)R_f(t)}{R_a(0)S_a(t) + R_f(t)} (4)
\]

The stator damage \( e^s \) is then:

\[
e^s(t) = |Ra_0 - Ra(t)|, (5)
\]

During the stator ageing the damage \( e^s \) progressively increases. Two thresholds are defined to estimate the gravity of the short-circuit: the degradation threshold \( e^*_d \) and the fault threshold \( e^*_p \). According to the comparison between the damage value and these thresholds, the stator is considered nominal when \( e^s(t) < e^*_d \), degraded when \( e^*_d < e^s(t) < e^*_p \), or faulty when \( e^s(t) > e^*_p \).

**Ageing law** The insulation degradation is due to thermal and electrical stresses. The damaging factors are the magnitude \( V \) and frequency \( f \) of the supply voltage, and the statoric temperature \( T_s \): \( DF^3 = \{ V, f, T_s \} \).

Since no real ageing data are available to estimate the stator ageing law, a law obtained in (Lahoud, Faucher, Malec, & Maussion, 2011) is used for illustrative purpose. This law was obtained with tests on insulation boards. We consider that the shape of the law is correct for the stator, and the parameters \( K_1, K_2, K_3 \) and \( b \) values are adjusted to fit with realistic life duration known from experience. \( L \) is the stator life duration and depends on the statoric temperature \( T_s \):

\[
L(t) = K_1 + K_2 \times exp(-b \times T_s(t)) (6)
\]

The proposed ageing law \( a^p \) is then:

\[
e^p(t) = a^p(T_s, t) = \frac{K_3}{L(t)} (7)
\]

\( V \) and \( f \) are constant so we consider that the ageing law only depends on \( T_s \). There is a correlation between \( L \) and \( e^s \) that is known from experience.

**Retroaction law** Short-circuits increase the temperature \( T_s \) because of the high currents that circulate in the phases and in the short-circuit loop. The following retroaction law is proposed:

\[
T_s(t) = f^*_a(e^s, t) = \begin{cases} 70^\circ C & \text{if } e^s(t) < e^*_d \\ 80^\circ C & \text{if } e^*_d < e^s(t) < e^*_p \\ 90^\circ C & \text{if } e^*_p < e^s(t) \end{cases} (8)
\]

This is the only retroaction function of the stator ageing model since we consider that there is no influence of the short-circuit on \( f \) and \( V \).

3.2.3. The rotor ageing : demagnetization progression

Another degradation that may occur on PMSMs is rotor demagnetization, which means that the remanent electromagnetic field \( B \) of one or several magnets decreases. This can be due to two kinds of degradation. Cracks or breaks of the magnets induce air gaps, which consequence at the electromagnetic level is the diminution of \( B \). High currents or high temperature variations can modify the physical composition of magnets which also leads to a diminution of their remanent electromagnetic field \( B \).

An analytical demagnetization model is proposed in (Vinson, Combacau, Prado, & Ribot, 2012). The fault parameter is the percentage of demagnetization of one magnet, which is proportional with the loss of \( B \) of this magnet. The private parameter that represents the damage of the rotor is \( B \). The rotor damage \( e^r \) is then:

\[
e^r(t) = |B_0 - B(t)| (9)
\]

At every effort cycle the fatigue of the magnet is accumulated because it is sized to resist to the effort. There is a macroscopically elastic deformation. The maximal number of cycles that the magnet can bear being reached, it breaks up. From this state, every part of the magnet undertakes a similar ageing process than the first one until it breaks again.

During this evolution the brutal rupture of a magnet is expressed with the Wohler curve described on Figure 3. It represents the limit of endurance \( \sigma \) of a material as a function of a number of fatigue cycles. When the limit is reached the material breaks.
We assume that the more the magnet is broken the more it becomes fragile. Calling $N_i$ the date of the $i^{th}$ rupture, we suppose that $\forall i, N_i - N_{i-1} > N_{i+1} - N_i$, because the duration between two breaks is shorter and shorter.

If the number of cycles between breaks $i$ and $i+1$ is divided by a factor $k > 1$ compared with the number of cycles between breaks $i-1$ and $i$, the number of breaks increases more and more rapidly. We define $T_x = \frac{N_t + 1}{N_t}$ as the acceleration factor of the degradation. The number $n$ of ruptures at time $t$ is defined as:

$$n(t) = \frac{\log(T_x) - \log(T_x + t \times (1 - T_x))}{\log(T_x)} \tag{10}$$

Every break divides the remanent induction of a factor $K > 1$, due to the air gap. We obtain a law giving the remanent induction as a function of the number of use cycles. The proposed rotor ageing law $ag^r$, is then:

$$e^r(t) = ag^r(t) = B_0(1 - K^{n(t)}) \tag{11}$$

In this ageing law, the only considered damaging factor is the time (i.e. the number of fatigue cycles). As a perspective, if sufficient data are available, it would be possible to add other damaging factors, such as short-circuit currents $I_{cc}$ or statoric temperature $T_s$, that may accelerate the rotor degradation.

4. THE PROGNOSIS

4.1. The generic prognosis method

A Health-Monitoring module is proposed in (Vinson et al., 2013). It is based on the generic model of the system and comprises a fault detection and diagnosis module. The prognosis algorithm is developed here. Its input is the result of diagnosis $\Delta \Sigma$, which allows estimating all the parameter values, even if they are not observable, at current time $t$.

The prognosis module predicts the future values of damaging factors thanks to retroaction laws (Equation 3). It then predicts the future values of private parameters thanks to ageing laws (Equation 7), and the input and output parameters values thanks to the knowledge of the future external solicitation of the system, and to the analytical laws between parameters. The future values of damages are estimated (Equation 1) and the time of degradation or fault can be predicted. The principle of the prognosis operation is presented on Figure 4.

The prognosis operation is similar to a diagnosis operation, but realized in the future. The main difference is that parameters values are predicted instead of being observed. The parameters or damaging factors are observable if their value at current time is known, for instance they are measured with sensors. The parameters or damaging factors are predictable if their future value can be estimated thanks to the ageing model or the functional model. The sets of predictable parameters and damaging factors are $P_{pred} \subset P$ and $DF_{pred} \subset DF$.

The prognosis is a sequence of diagnoses realized at future degradation time $t_i$, until the fault time $t_f$:

$$\Pi^\Sigma(t) = \{\Delta^\Sigma(t), \Delta^\Sigma(t_1), \ldots, \Delta^\Sigma(t_f)\} \tag{12}$$

The prognosis algorithm uses the generic formalism developed in this paper, as shown in Algorithm 1. It is developed on Matlab and needs to be validated on degradation and fault data. Since no real data are available, a virtual prototype is built on Matlab Simulink.
4.2. Development of a virtual prototype

The virtual prototype is a very precise and complete functional and ageing model of the PMSMs. It is used only for simulation purposes in order to obtain a realistic set of data to validate the prognosis algorithm, built with a simple functional and ageing model of PMSMs. In the virtual prototype the equation of dissipation of thermal power allows predicting the stator temperature $T_s$. Phase resistances are computed thanks to an ageing law that depends on $T_s$, $V$ and $f$, and thanks to the equation of copper resistivity that depends on $T_s$. This coupled phenomena are represented on Figure 5.

To model the virtual prototype we consider the following hypothesis:

- the ambient temperature is constant (the ventilation is working well);
- the motor shell acts as a constant thermal resistance $R_{th2}$, and a uniform temperature;
- the insulator acts as a constant thermal resistance $R_{th1}$;
- the winding temperature is uniform;
- only the steady state is considered since the transient state is short.

**Variation of the short-circuit resistance** The ageing law allows deducing the short-circuit resistance value $R_f$. The health points $PV$ are used to correlate the life duration $L$ with $R_f$.

The initial number of health points $PV_0$ corresponds with the initial life duration $L_0$. Between $t$ and $t + dt$ the proportion of consumed health points is $PV(t) - PV(t + dt) = \frac{dt}{L(t)}$, so

$$PV(t) = \int_0^t \frac{1}{L(z)} \, dz$$ (13)

The integration of the ageing law can be done by approximation with a piecewise continuous function having the value $L(T(t_{k+1}))$ between times $t_k$ and $t_{k+1}$:

$$\left\{ \begin{array}{l} PV(0) = 0 \\ PV(t_{k+1}) = PV(t_k) + \frac{(t_{k+1} - t_k)}{L(T(t_{k+1}))} \end{array} \right.$$ (14)

To the best of our knowledge the law that gives the short-circuit evolution as a function of health points does not exist. We choose an exponential shape because we assume that the degradation accelerates with time:

$$R_f(t) = R_{f0}(1 - \exp(-k \frac{PV(t) - PV_0}{PV_0})))$$ (15)

**Variation of phases resistivity** At temperature $T$ the resistance $R$ of a coil is $R(T) = (\rho(T) \times L)/s$, where $l$ is the length of the cable and $s$ is its section. $T_0$ is the nominal temperature, $R_0 = R(T_0)$. Besides the short-circuited phase resistance modification due to the short-circuit loop with resistance $R_f$, the three phase resistances $R_a$, $R_b$ and $R_c$ respect the following equation:

$$R(T) = R(T_0) + \frac{l}{s} \times (\rho(T) - \rho(T_0))$$ (16)

where the copper resistivity is $\rho(T) = 17.24 \times (1 + 4.2 \times 10^{-3} \times (T - 20)) \times 10^{-6}$. 

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**Algorithm 1 Prognosis**

Input: $\Sigma, t, \Delta \Sigma$

Output: $\Pi_{\Sigma}(t)$

Initialization: $k \leftarrow 1$

while $R\Sigma L \neq 0$ do

$t \leftarrow t + \Delta t$

for all $pp^{k,i} \in P \Sigma P$ do

$r(pp^{k,i}) = r_{\Sigma}(pp^{k,i}) \%$ values known from diagnosis

end for

for all $df^i \in DF$ do

$r(df^i) = f_{df}(\{r(pp^{k,i})\})$

end for

for all $pp^{i,j} \in P_\Sigma P_{pred}$ do

$r(pp^{i,j}) = ag^{i,j}(\{df^i\})$

end for

for all $ip^{i,j} = st(op^{k,j}, t) \in P_{\Sigma}P_{pred}$ do

$ip^{i,j}(t) = op^{k,j}(t)$

end for

for all $op^{i,j} \in OP_{P_{pred}}$ do

$op^{i,j}(t) = ar(\{p^{i,j}\})$

end for

for all $pp^{i,j} \in P_\Sigma P_{pred}$ do

if $e^{i,j} \geq e_x^i j$ then

$t_k \leftarrow t$

go out of loop

end if

end for

Diagnose the system at time $t_k$

$\Pi_{\Sigma}(t) \leftarrow \Delta \Sigma(t_k)$

$k \leftarrow k + 1$

end while

Return $\Pi_{\Sigma}$

---

**Figure 5. Virtual prototype: relationships between stator temperature and phase resistance**
Thermal power dissipation

\[ T_s = (R_{th1} + R_{th2}) \times P_d + T_a \quad (17) \]

The stator temperature is obtained from the dissipated statoric thermal power \( P_d \), that depends on phase resistance \( R_a \), \( R_b \) and \( R_c \), on the short-circuit intensity through \( S_a \) and \( R_f \), and on phase and short-circuit currents. The equation can be found on (Vinson et al., 2013).

4.3. Application: Permanent Magnet Synchronous Machine prognosis

A short-circuit scenario is simulated on the virtual prototype. The resulting fault resistance and statoric temperature can be seen on Figures 6 and 7. The short-circuit resistance decreases progressively with the short-circuit, until 0Ω when the short-circuit is direct. Meanwhile, the statoric temperature progressively increases with the degradation.

During the degradation progression, phase currents are observed on the virtual prototype. This allows the diagnosis of the stator and the PMSM thanks to the diagnosis algorithm developed in (Vinson, Combacau, Prado, & Ribot, 2012) which uses a short-circuit indicator based on the phase currents. The damage \( e^s \) is estimated thanks to the diagnosis algorithm, as shown on the top left of Figure 8. The diagnosis module assesses the health-state of the stator according to the damage value: it is first nominal, the degraded, and then faulty (top-right on Figure 8). The prognosis module is run every time when a threshold is passed by the statoric damage. It can predict the future values of the statoric temperature \( T_s \) thanks to the retroaction law described by 8 (bottom-left on Figure 8). It can then predict the life duration \( L \) of the stator thanks to the ageing law represented by Equation 7 (bottom-right on Figure 8. Two predictions are realized with two different values of the parameter \( b \) (Equation 7), in order to represent uncertainties on the ageing law. The real life duration can be compared with the two predicted life duration.

5. CONCLUSION

In this paper a study about related work on existing ageing models and prognosis methods was first proposed. It motivated the idea of designing a generic ageing modeling framework in order to represent every kind of known ageing law, whatever the nature of available knowledge. Indeed it can be experience-based, data-driven, or physics-based ageing models.

The proposed generic modeling framework contains all information to perform diagnosis and prognosis. Besides a diagnosis algorithm presented in details in a previous paper (Vinson et al., 2013), a prognosis algorithm is developed based on this generic ageing model. It uses predictable parameters and damaging factors to estimate the future degradation and faults occurrences.

An application is shown on Permanent Magnet Synchronous Machines, which ageing is successfully modeled by the proposed method. A virtual short-circuit scenario is predicted by the prognosis algorithm. When implemented in service, this prognosis will allow anticipating any maintenance operation for PMSMs.

The developed modeling framework and prognosis algorithm are intended to be applied to other critical equipment in aeronautics, such as hydraulic pumps or electromechanical actuators. In order to adjust the proposed ageing model with ageing and retroaction laws, it seems essential to perform some degradation tests. The generic ageing model we pro-
posed is a common representation of ageing of any equipment type. But the level of knowledge contained in the model is directly characterized by the availability of sensors, experience or physics-based models and may vary from one component to another. The higher the level of knowledge about ageing is, the more accurate the prognosis results. It becomes interesting to define and implement performance metrics for prognosis based on the level of knowledge contained in out generic aging model in order to compare the results obtained for the components and qualify the prognosis result at the system level.

REFERENCES


