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An Adaptive Control Strategy of Urban Signalized Intersection Using Petri Nets

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Abstract—The dynamic behavior of traffic signals controlling an isolated intersections can be seen as a discrete event system that can be modeled by Petri nets. The approach used in this paper proposes an adaptive control strategy of urban signalized intersection in real time using T-timed Petri Nets (T-TPNs). This strategy allows to manage the green light time autonomously according to the queue size for each flow in each lane of the intersection. A decentralized approach is proposed for the decision-making, where every phase in the intersections can be considered as a sub-system to be modelled and controlled (by traffic lights) separately, then a global controller allows to manage the conflicts between several phases.

Keywords—Timed Petri net, discrete event systems, decentralized approach, traffic signal control

I. INTRODUCTION

The problem of regulation within road traffic constitutes a major challenge for operators and developers of transport. Traffic management systems inscribe the purpose of reducing congestion, vehicles delay time, fuel consumption and pollution. The most popular technique to regulate and manage urban traffic is traffic signal control. Many control strategies of traffic lights were developed. The currently available traffic control strategies can be splitted into two main categories as explained in [1], [2]: fixed timing strategies and traffic-responsive strategies. In the first group fixed timing plans are determined by the control system using an offline optimization method implemented by computer programs. In this case, the traffic management is far from an optimal exploitation of the resources because it is not possible to pursue the request fluctuations [3]. The second category is traffic-responsive strategies which employ actuated signal timing plans and offer an online optimization and synchronization of traffic signals. These adaptive strategies regulate the traffic lights according to the number of presented vehicles on the intersection: sensors detect traffic on the intersection phases and furnish information to the real-time controller [4].

As a matter of fact, a network of traffic lights can be observed as a complex discrete event system. The events in the road traffic are translated by the arrivals and departures of vehicles at the intersection, and beginning or completion of the various phases in the signal timing plans of the traffic lights controlling junctions [5]. Thanks to the well-known strength of Petri Nets (PNs) to achieve concurrency and synchronization phenomena, PN based models may be suitably derived for urban traffic systems. More precisely, PNs can be applied

for describing traffic signals controlling and related concurrent activities. Moreover, PNs models suggest easy graphical representations to understand, study and analyze the behaviour of DES [6], [7].

Several research works developed approaches using PNs with the purpose of avoiding the problem of combinatorial explosions such as the modular approach [16] and the decentralized approach [17], [18]. For the traffic management in the intersections, every phase can be considered as a sub-system to be modelled and controlled separately. This shows the shows the decentralized nature of the system. Considering a such decentralized configuration, the traffic control approach require the use of a global coordination structure [19]. The coordinator in such structure represents the controller which allows managing the conflicts in the intersection.

The paper is organized as follows. Section 2 presents a literature review about the problematic proposed in this paper. Section 3 recalls the basics of T-TPNs and some requirements for traffic signals control. Moreover, section 4 presents the control model of isolated intersection using Petri Nets and decentralized approach. Performed simulations and discussions about obtained results are reported in section 5. Finally, a conclusion and some perspectives are given in section 6.

II. LITERATURE REVIEW

Several methods and approaches have been developed in the literature for modeling traffic lights in urban intersections. Among them: Dioid algebra [22], cellular automata [23]. Further methods were based on Multi-agent systems to control traffic at signalized intersections in a distributed way [24]. Moreover, in [25] an approach enabling to combine a set of traditional learning methods was proposed for self-adaptive decision making. Petri nets have been introduced for traffic modelling and control in [8] then many other works have been proposed for modelling and controlling traffic into road network intersections [9], [10]. More recently the authors of [11] have discussed the use of PNs for modelling traffic signal control and perform a structural analysis of the developed control PN model. In addition, [12] presents a traffic model using timed PN formalism, where tokens represent vehicles and places represent parts of lanes and intersection. There are other works where time is introduced into the model to represent phase duration are given in [13], [14], [15]. The durations of traffic signals in these works are based on fixed predetermined intervals represented in timed PNs. In the literature, there are some research works dealing with

the supervision of decentralized systems. A supervision study of decentralized discrete event systems using automata model was presented in [20]. Moreover, a decentralized supervisory control theory using PN's were introduced in [21].

The purpose of this paper is to propose a management strategy for the traffic lights at a road intersection using variable and adaptive times for green light. In addition to firing times associated with model transitions, we introduce inhibitor arcs in the model with the aim to well-manage the traffic light cycles. Our modeling approach is further developed in the rest of this paper.

III. PRELIMINARIES

A. T-timed Petri Nets

A Petri net is a bipartite directed graph composed of three types of components. Places, transitions, and directed arcs connecting transitions to places and vice versa [7]. In a Timed Petri Nets (TPN), a temporization is associated with each transition and represents firing delay of the transition, in this case we talk about T-TPN. In the second class of TPN, we associate temporizations with places. This class is called P-TPN. In this paper, we focus on the use of T-TPN with two types of transitions: immediate ones without firing delays, and deterministic ones with non null firing delays. Formally, a TPN is given as follows [7]:

$TPN = (P, T, I, O, H, M_0)$, where:

- $P = \{P_1, P_2, \dots, P_m\}$ a finite set of places.
- $T = \{T_1, T_2, \dots, T_n\}$ a finite set of transitions.
- $I : P \times T \rightarrow N$ is the input function that defines directed arcs from places to transitions. N is the set of relative integers.
- $O : T \times P \rightarrow N$ is the output function that defines directed arcs from transitions to places.
- $H \subseteq T \times P$ a set of inhibitor arcs from P to T .
- $M_0 : P \rightarrow N$ is an initial marking.

B. Arcs with variable weights within PN model

To be able to express the functioning of the systems with a variable value, the concept of "arcs with variable weight" is necessary. This concept is already introduced for the modelling and management of stocks in supply chains [26]. In this article, we shall see that this concept is very useful in our modelling approach. More precisely, it will be used for traffic regulation issue. So, the weights associated with some arcs of the model can be variable according to the current marking of the network. These arcs allow taking a group of tokens simultaneously, which is useful in our approach.

C. Requirements for traffic signals control in a network

For the proper functioning of traffic signals, we underline some important requirements for traffic signals in a given intersection. As given in [9]:

- The green state in the traffic signal must not allow to two conflicting road sections simultaneously.

- A defined sequence of active color lights must be followed for each traffic signal, normally from green to amber and red, and then back to green.
- The green light must be given successively to all phases (all vehicles have the right to use an intersection).

Responsive controllers are equipped to receive information of traffic flow from various measuring detectors at present time intervals. The controller sends the signalling scheme to traffic signals, which controls traffic flow as illustrated in Fig.1.

Fig. 1. traffic scheme [27].

IV. THE CONTROL MODEL OF ISOLATED INTERSECTION USING PETRI NET

The traffic area reported in Fig.2 is considered, composed of an intersection with four directions: west, south, east and north. This crossroad includes sensors to detect arrival of vehicles into the intersection. Traffic flow information in each lane could be obtained via detectors located at a given distance from the stop line. This intersection features a typical two-phase traffic signal control plan, as shown in Fig.3. The green light can be given simultaneously to the opposite directions east-west or north-south without loss safety.

Fig. 2. Intersection with traffic lights.

In this paper, the sequence of active color is from green to red, and then back to green. We suppose that the time of amber light is included in the red time. The solution proposed in the next section is to control the crossroads using variable temporization which determines the duration of the

Fig. 3. Example of phases in a given intersection .

green light, according to the number of vehicles waiting in the intersection. Using a decentralized approach and separating the local controllers for each phase, a global controller will be developed to manage the conflicts between the considered two phases of the intersection as shown in Fig.3.

A. Petri net based control model

In this subsection, we build the traffic flow and the signal control models for each intersection phase using T-Timed PNs. The following fundamental constraints are considered to assure the local control: 1) the vehicle is authorized to cross the intersection only if the traffic light is green; 2) the time of green light depends on the number of existing vehicles at the in the intersection during a cycle (after the evacuation of all existing vehicles on opposite lanes of a phase, the traffic light becomes red). For more clarification, we are going to describe each part of the global model separately.

1) *The local controller*: is modeled by the TPN of Fig.4 and represents **lanes and traffic flow**. In this intersection, the structures of the phases are similar. So, only the Petri model of the first phase east-west are depicted in Fig.4. The transitions T_{11} and T_{15} are used to generate tokens (arrival of vehicles on the intersection) when the detectors, located in a given distance from the stop light, detect vehicles coming respectively from the two opposite directions east and west. Tokens in places P_{11} and P_{14} represent vehicles waiting to enter the intersection. The timed transitions T_{12} and T_{16} can be fired every t_1 time unit, their firing add tokens respectively into the two places T_{12} and T_{16} . Time t_1 is a delay which allows to fixe number of vehilces during making-decision. The role of this delay is to save the same compared number of vehicles to be evacuated from the intersection if the traffic light is green. After the firing of T_{12} and T_{16} , all tokens will be respectively added to places P_{12} and P_{15} . The role of these places is to give the traffic flow information, in the first phase, to the global controller (Fig.6). Token in place P_{37} (see global controller) allows the firing of transitions T_{13} and T_{17} (bidirectional arrows are used here to represent reading of place P_{37}). With these firing, all tokens of places P_{12} and P_{15} are added respectively to places P_{13} and P_{16} . The transitions T_{14} , T_{18} , and T_{19} represent the stop line and the place P_{17} represents the green light for the first phase. So in order to respect the first constraint, the firing of these transitions is controlled by presence of a token in the place P_{17} (reading of P_{17} by bidirectional arrows). The time t_2 associated to these timed transitions represent the necessary time for each vehicle to leave the intersection. While firing the transition T_{19} (if there are tokens in P_{13} , P_{16} , and P_{17}) the number of tokens in P_{13} and P_{16} will be reduced by one every t_2 time unit. When P_{13} (or P_{16}) becomes empty the transition T_{18} (or T_{14}) fires every t_2 until P_{16} (or P_{13}) becomes empty.

Fig. 4. Traffic flow model.

Fig.5 shows the T-timed Petri net model for **traffic light** control. As aforementioned, we suppose in this work that the amber light is included within red time, so just two lights are considered in this study: the green and the red. The green light can be given to the same phase for two successive cycles. This is modelled by the two places P_{17} and P_{18} representing the green light (P_{18} allows to save green time within making decision by the global controller). If we have token in P_{37} the decision is given to the first phase, a conflict situation occurs for the two transitions T_{31} and T_{32} . The tokens in P_{18} and P_{37} means that the decision is given to the first phase for the second time and the green light will be saved after firing of transition T_{32} . Token in P_{19} and token in P_{37} mean that the light is red and will be changed to the green light after firing of T_{31} . If the decision is given to the other phase (token in P_{38}) the lights change from green to red by firing transition T_{34} . In order to respect the second constraint about the green light time (token in P_{17}) a controller policy is proposed using inhibitor arcs. When P_{13} and P_{16} become empty that means all authorized vehicles leave the intersection and the green light stays on during making decision, by firing of transition T_{33} .

Fig. 5. Traffic light model.

2) *The global controller:* The global coordination is represented by the controller of Fig.6. We rename N_1 and N_2 the number of vehicles detected respectively in the phases east-west and north-south:

$$N_1 = M(P_{12}) + M(P_{15}); N_2 = M(P_{22}) + M(P_{25})$$

The places P_{31} and P_{32} represent the comparator used to compare the number of vehicles detected in each phase of the intersection. The transition T_{41} becomes enabled once both P_{31} and P_{32} contain a token. Its firing is immediate. After each firing of T_{41} , a token is removed from P_{31} (N_1) and P_{32} (N_2), the number of firing equals to $\min(N_1, N_2)$ because of the synchronization phenomenon that this transition presents. If $N_1 > N_2$ after the last firing of T_{41} , P_{32} becomes empty from tokens, while P_{31} contains yet some tokens. If $N_2 > N_1$ after the last firing of T_{41} , P_{31} becomes empty from tokens, while P_{32} contains yet some tokens. If $N_1 = N_2$ the priority will be given to the first phase then we find P_{31} , P_{32} empty after the last firing of T_{41} . If $N_1 = N_2$ or $N_1 > N_2$ the transition T_{42} or T_{43} fire respectively, so the places P_{3i} and P_{4i} (for $i = 1, 2$) become empty and a token is added into P_{37} to represent that the priority is given to the first phase (E-W). If $N_2 > N_1$ the transition T_{44} can be fired, so the places P_{3i} and P_{4i} (for $i = 1, 2$) become empty and a token is added into the place P_{38} representing that the priority is given to the second phase (N-S).

Making decision is authorized just if the green light is allocated to the first phase twice (token in P_{18}) or to the second phase twice (token in P_{28}). To make a decision T_{51} and T_{54} (or T_{52} , T_{53}) fire simultaneously, if there is a token in P_{18} (P_{28}), after this firing, the model marking evolves and we will have $M(P_{31}) = M(P_{12}) + M(P_{15})$ for the phase E-W and $M(P_{32}) = M(P_{22}) + M(P_{25})$ and then the comparison process can start. Token in P_{41} (resp. P_{42}) prevents the firing of T_{51} and T_{52} (resp. T_{53} and T_{54}), via inhibitor arcs, after the first firing (because tokens still in P_{12} and P_{15} (resp. P_{22} and P_{25})).

If one of the phases had the green light all time the other have not possibility to use the intersection. In order to avoid this situation and manage the vehicles' crossing with a balanced way, the number of successive green light is limited to twice for each phase. In Fig.7, P_{45} and P_{46} are added to the model in order to ensure this alternative functioning. Initially, we consider that the first phase had two successive green light. For every firing of T_{31} or T_{32} , a token is added to P_{46} . When P_{46} contains two tokens, and P_{28} contains one token, a conflict situation will be generated between transitions T_{51} , T_{54} and T_{57} . To avoid this conflict the time t_4 is added to transitions T_{52} and T_{53} to delay their firings. After a firing of T_{57} , the place P_{46} becomes empty from token and token is added to P_{38} , so the decision to move to the second phase is taken without comparison process. The time t_3 is added to transitions T_{35} and T_{36} to assure that T_{27} and T_{23} fire before T_{36} . The time t_3 is less than t_4 in order to clear P_{28} and prevent the firing of T_{5i} (for $i = 1, 2$), because there is no need to use the comparator, the decision is already taken by firing T_{57} .

V. MODEL ANALYSIS AND SIMULATION RESULTS

By gathering the developed submodels of Fig. 4, 5 and 6, the global T-TPN model of the studied intersection is given in Fig.7.

Fig. 6. Controller model.

A. Verification and validation of model properties

The background of PNs will be used in this section to verify and validate the proper functioning of the studied system. In what follows we address some relevant qualitative properties that we will check in order to ensure the efficiency of the proposed model.

1) *Accessibility:* In a Petri net model, for the initial marking M_0 , we say that the marking M_f is accessible if we can all-time find a sequence of firing S which allows passing from an initial marking M_0 to a final marking M_f , where W is the incidence matrix of PN model:

$$M_f = M_0 + WS$$

Related to intersections management and traffic light, the notion of accessibility is bound to the evacuation of vehicles according to the state of traffic lights. It means that the network allows to change the traffic lights in both phases and evacuate the number of the authorized vehicles, so the system can always pass from an initial to a final state if the conditions are combined (sequence of firing).

2) *Boundedness:* The places which represent the state of traffic lights are limited because they cannot contain more than token. For the places which represent the number of vehicles, they are limited because they depend on the capacity of limited section of the lane (bound to the location of the sensor in the way).

3) *Vivacity and deadlock-free:* A marked PN is alive for an initial marking M_0 if all its transitions are alive for this initial marking. A transition is alive for an initial marking M_0 if for any accessible marking M_k , there is a sequence of firing S from M_k containing T_j :

$$\forall M_K \in *M_0, \exists S \ M_K \mid S > \text{et } S = \dots T_j \dots$$

A blocking situation is a marking for which no transition is validated. A marked PN is said deadlock-free for an initial marking if no accessible marking is a blocking. According to the obtained results in (Fig.9) the system allows the switching of the green lights between both phases, what assures that all the transitions of the network are alive and without blocking.

Fig. 7. Global controlled model.

4) *Reversibility*: A PN is said reversible for an initial marking M_0 , if for any marking M accessible from M_0 , there is a sequence of firing S which returns M_0 . The reversibility in our case is ensured for the state of traffic lights.

B. Simulation results

For simulation issue, random values of traffic flow are generated for each phase and each direction of the intersection: east, west, north, and south. The Petri net model is implemented and simulated by Matlab. Among the obtained results, those presented in Fig.8 and Fig.9.

Fig. 8. Evolution of traffic light state over time.

The evolution of traffic lights for the two phases over the time are shown in Fig.8, this simulation proves that this solution prevents the conflict situations into the intersection. The proposed model shows that the traffic light is managed with an alternative way between the two phases while taking into account the number of vehicles within each phase. As shown in Fig.8, when the light is green for one phase the other will be red, and vice versa. Fig.9 shows the evacuation of vehicles for each phase over the time. For the simulation issue, the necessary time to cross the intersection (firing of the transitions T_{i4} , T_{i8} or T_{i9} for $i \in \{1, 2\}$), for each vehicle, is fixed on one second. The evacuation of vehicles (for the

Fig. 9. Evacuation of vehicles according to the state of the traffic light over time.

phase with green light) in Fig.9 depends on the number of vehicles in each direction: evacuation two by two (one from each direction of the phase) if there are vehicles in the two sides of the phase and evacuation one by one when all vehicles of one side of the phase are evacuated and there are still vehicles on the other side of the same phase. For example, in the interval of time $[60s, 85s]$ (Fig.9) the green light is given twice successively to the first phase according to the high number of presented vehicles in this phase. In this case, the model suggests a decision without comparator by assigning

the green light to the first phase for the next cycle [85s, 100s].

This performed simulation allows to verify the properties of The Petri net model proposed in Fig.7, and the proper functioning of the intersection by respecting the system specifications.

In this paper, making a decision depends on the sum of vehicles in each phase (with its two opposite directions). In this solution, only the evacuation of the maximal number of vehicles in the minimal time criteria is respected. Deadlock situation at the intersection is not considered in this work.

VI. CONCLUSION

This article presents a novel timed Petri net model, in order to control and manage the traffic lights at a given road intersection. A decentralized approach is used to control each phase separately respecting some system constraints. It is based on a coordinator which represents the global controller avoiding the conflict situations at the intersection. The properties of the proposed TPNs model (representing the functioning properties of the studied system) model are proved by simulation. The advantage of the proposed approach is that the time of the traffic lights can be identified in real time and according to the number of presented vehicles into the intersection. This article has demonstrated how to use TPNs to model traffic control system. In this work, we focused only on two phases of the intersection (E-W and N-S), as a perspective all possible scenarios (phases) will be considered and the global controller can be developed using other controlling methods. The following dilemma will be taken into account: between waiting delay and the evacuation of maximum number of vehicles in a minimum time. As shown in global PN model (Fig.7), there is symmetry of the model components, so we can use colored PN to reduce the model representation and to consider other functioning constraints.

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