On Motivations for Designing Analog Filters Under LFT Framework

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On Motivations for Designing Analog Filters Under LFT Framework ⋆

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Abstract: The objective of this paper is to motivate the design of Radio Frequency (RF) analog filters under the LFT framework. Current increase in filter complexity makes traditional design methods outdated. New methods need then to be developed to tackle this issue. The Linear Fractional Transformation (LFT) is a general tool enabling to represent, analyse and synthesize interconnected systems. Coupling with the so-called KYP lemma, and its extensions, it allows to transpose complex analytical optimization problem into a finite dimensional convex problem under Linear Matrix Inequality (LMI) constraints. That is to say a problem which is optimally and efficiently solvable. All through this paper, illustrations are provided on how using the LFT framework for LC filter design. Finally, potential benefits of such an approach are enumerated, which are mainly based on the generic property of the LFT.

Keywords: Robust Linear Matrix Inequalities (LMI); Analog Filter Design; Linear Fractional Transformation (LFT); KYP Lemma; Radio Frequency (RF) Applications;

1. INTRODUCTION

Analog filters design is one of the oldest subjects of interest of System Theory (with its branches : Control and Circuit Theory) and Signal Processing but is still a topical issue. In fact, new challenges are still arising. In Radio Frequency (RF) electronics application, analog filters are especially appreciated as they do not need external energy supply and tend to be more robust to uncertainties than digital filters. However, due to the important growth in frequency bands and standards in the near future, a big increase in filter complexity is expected (Hashimoto et al., 2015). As a result, new filter design methods are required to replace outdated methods in order to tackle this challenge.

First filters design methods were developed for reactive filters, based on inductances L and capacitances C, also known as *LC-filters* (Belevitch, 1962). Due to poor properties of inductances for the integration on chip, they were progressively replaced by new components (such as SAW/BAW resonators) with better characteristics for RF applications. But no theoretical design methods were developed, and researches have been focused on getting better performance of components, while very simple design methods are still used in practice. As future tunable filters may be based on resonators and capacitors (Hashimoto et al., 2011), new appropriate design methods are required.

In this paper, it is proposed to formulate the usual LC filter design problem using recently developed tools of Control Theory. With the development of efficient methods for solving convex optimization problems under Linear Matrix Inequality (LMI) constraints (Boyd et al., 1994a), new ways of representing, analysing and synthesising systems have emerged (Doyle et al., 1991). The aim is to illustrate how this problem can be formulated with this approach, and how new filter design problems can then be addressed by using the generalizing properties of these new tools.

This paper is organised as follow. The LC filter design problem is first reviewed. The usual design method, based on two stages, is briefly explained. Then, a new tool of post-modern control, the Linear Fractional Transformation (LFT), is presented. It especially allows to represent graphically and mathematically a system in a generic way. The Kalman-Yakubovitch-Popov (KYP) lemma is introduced to make the link between frequency constraints of LFT systems and LMI optimization problems. Finally, potential benefits of formulating the usual LC filter design problem with these tools are enumerated.

The following notations are used. The variable s stands for the complex Laplace variable $s \in \mathbb{C}$. 0 and I_d respectively represent the zero matrix and the identity matrix of appropriate dimensions. For a matrix M, its transpose and complex conjugate transpose are denoted by M^T and M^*. For a Hermitian matrix $M = M^*$, inequalities $M > (\geq) 0$ and $M < (\leq) 0$ denote positive (semi)definiteness and negative (semi)definiteness, respectively. Finally, $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ means that the matrix is partitioned.

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2. REACTIVE FILTER SYNTHESIS

2.1 RF Reactive Filter

A RF filter is a 2-port network, composed of lumped elements, inserted between a generator and a resistive load (Fig. 1). Specifically, a *reactive* RF filter is a RF filter made of inductances L and capacitances C.

A 2-port may be represented by an *impedance* matrix $\mathcal{Z}$, linking the characteristic electrical quantities $U = [U_1\ U_2]^T$ and $I = [I_1\ I_2]^T$:

$$U = \mathcal{Z}I \qquad \text{with} \qquad \mathcal{Z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \tag{1}$$

The *input impedance* z_{in} allows then to represent the behaviour of the 2-port terminated on a resistive load R_l :

$$z_{in} = \frac{u_1}{i_1} = z_{11} - z_{12}(R_l + z_{22})^{-1} z_{21} \tag{2}$$

In order to characterize the input-output power transfer of a loaded 2-port, the *scattering* matrix $\mathcal{S}$ is introduced :

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \mathcal{S} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \qquad \text{with} \qquad \mathcal{S} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \tag{3}$$

where a_1, a_2 refer to the incident *waves* and b_1, b_2 to the reflected *waves* and are defined as :

$$2\sqrt{R_g} b_1 = U_1 - R_g I_1 \qquad 2\sqrt{R_g} a_1 = U_1 + R_g I_1$$
$$2\sqrt{R_l} b_2 = U_2 - R_l I_2 \qquad 2\sqrt{R_l} a_2 = U_2 + R_l I_2$$

Using convention of Fig. 1, one can note that $U_1 + R_g I_1 = E_g$ and $U_2 + R_l i_2 = 0$. Little calculation gives then :

$$s_{21}(j\omega) = \frac{U_2(j\omega)/\sqrt{R_l}}{E_g(j\omega)/2\sqrt{R_g}} \tag{4}$$

Typical filtering constraints are set on the *power gain* G, defined by the ratio of P_l, the average power delivered to the load R_l, to P_g the available generator power (Youla, 1971) :

$$G(\omega^2) = \frac{P_l(\omega^2)}{P_g(\omega^2)} \tag{5}$$

where

$$P_l(\omega^2) = |U_2(j\omega)|^2/R_l \qquad P_g(\omega^2) = |E_g(j\omega)|^2/4R_g \tag{6}$$

Comparing (4) and (5), one gets :

$$G(\omega^2) = |s_{21}(j\omega)|^2 \tag{7}$$

Then, power frequency-domain constraints are expressed on the square magnitude of the scattering parameter s_{21}, which should also satisfies stability and causality conditions.

The filter synthesis problem is usually solved in a two-stage fashion. First, one computes a scattering matrix with the scattering parameter s_{21} satisfying stability, causality and frequency-domain filtering constraints. This is called the approximation step. Then one computes a circuit synthesizing this scattering matrix. This is called the realisation step. This process requires to design first a prototype low-pass filter, and then other versions are obtained using usual frequency and component transformations (Baher, 1984).

Remark 1. For RF applications, the generator internal resistance and resistive load are considered equalled to the same value R :

$$R_g = R_l = R$$

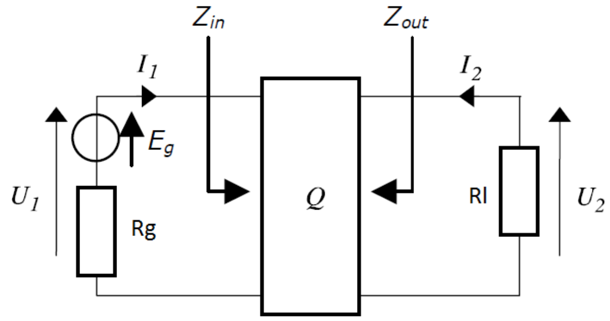

Fig. 1. Radio Frequency Filter

The value of this resistance can be set to $R = 1\ \Omega$ and impedance scaling may be done subsequently. In this case, the *reduced* voltage and currents are respectively defined by $u = U/\sqrt{R}$ and $i = \sqrt{R}I$ while the *normalised impedance* matrix Z is then given by

$$Z = \mathcal{Z}/R \tag{8}$$

The *normalised* scattering matrix S is defined as :

$$(u - i) = S(u + i) \tag{9}$$

Alternatively, the normalised scattering matrix S can be linked to the normalised impedance matrix Z. Assuming $(Z + I_d)^{-1}$ exists,

$$S = (Z - I_d)(Z + I_d)^{-1} = -I_d + 2Z(Z + I_d)^{-1} \tag{10}$$

2.2 Approximation Stage

This stage consists in approximating, along the imaginary axis, a desired non-negative, real, frequency-domain function by a rational function, which should be the squared magnitude frequency response of a complex rational function. This approximation is achieved by minimizing an error measure between the desired function and the rational function. The transfer function s_{21} is extracted from the squared magnitude using spectral factorisation.

The function to be approximated is a normalized *ideal* low-pass filter (see left-hand side of Fig. 2) which can provide other versions, such as ideal bandpass filter (right-hand side of Fig. 2), using standard frequency transformations. The four usual approximations (Butterworth, Chebyshev I, Chebyshev II, Elliptic) can be generated using two criteria. A Taylor series approximation can be achieved at $\omega = 0$ and $\omega = \infty$, which provides flat responses. The Chebyshev approximation results in minimising the maximum error over pass-band or stop-band, and produces equi-ripple filters (Parks and Burrus, 1987).

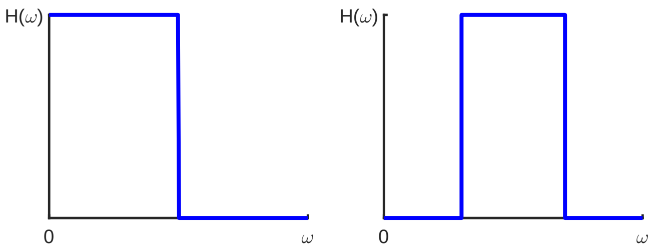

Fig. 2. Ideal Low-Pass and Band-Pass Filters

This approach suffers from two main drawbacks in practice. First, frequency transformations only allow to design bandpass filters with *identical* transition bandwidths. In RF applications, a transition bandwidth may often be significantly shorter than the other (e.g. duplexers). Resulting filters order may then be unnecessary high. Second, none of the usual design criteria takes explicitly into account the order of the rational function in the minimization process. This is of special interest as the order is related with the number of components.

2.3 Realisation Stage

The realisation stage consists in computing a circuit which synthesizes the desired scattering parameter s_{21}. A circuit can be determined by the values of its components and the configuration they are set in, namely the *topology.* In practice, a design criteria of interest is the total number of reactive components used for the synthesis, which should be as small as possible. A circuit with the least number of reactive components is called *minimal.*

In order to ensure that a realisation exists or is minimal, to characterize the types of components used or the topology of the circuit, several conditions on the representative matrices (S, Z) should be verified. For the synthesis of a reactive filter in a *ladder* form, most of these constraints are satisfied when using one of the usual approximation methods (Youla, 1971).

Remark 2. For a *lossless reciprocal* network, such as a reactive filter in a ladder form, these constraints imply that finding the scattering parameter s_{21} determines the whole scattering matrix S.

Example 1. The impedance matrix Z of a circuit made of positive inductances and capacitances ($L > 0$, $C > 0$) is a *positive real* rational matrix.

Positive realness is an important property in *passive* filter synthesis.

Definition 1. (Anderson and Vongpanitlerd, 1973) A matrix $\widetilde{Z}$ of rational functions is positive real if :

(1) All elements of Z are analytic in $Re\ [s] > 0$.
(2) $\widetilde{Z}(s)$ is real for real s.
(3) $\widetilde{Z}(s) + \widetilde{Z}^*(s) \geq 0$ for $Re\ [s] > 0$.

Algebraic Realisability Conditions Realisability conditions, such as positive realness, are typically expressed in an analytic way. In practice, given a representative matrix, it may be truly complicated to check if it satisfies or not these conditions. For instance, one has to test an infinite number of points in the condition (2) of definition 1.

Alternatively, using state-space representation of the matrix, equivalent algebraic conditions exist for most of these conditions (Anderson and Vongpanitlerd, 1973; Willems, 1976). This allows to solve a *finite*-dimensional *convex* optimization problem.

Theorem 1. **Positive Real Lemma** (Anderson and Vongpanitlerd, 1973) . Let $\widetilde{Z}$ be a matrix of real rational functions of a complex variable s, with $\widetilde{Z}(\infty) < \infty$. Let A, B, C, D be a minimal state-space representation of $\widetilde{Z}$. Then $\widetilde{Z}$ is positive real if and only if there exist real matrices $P = P^T > 0$, L, W such that :

$$\begin{aligned} PA + A^T P &= -LL^T \\ PB &= C^T - LW \\ W^T W &= D + D^T \end{aligned} \tag{11}$$

Remark 3. If it exists, a solution of (11) may be obtained by computing the stabilizing solution $Q = -P < 0$ of the Algebraic Ricatti Equation (Anderson and Vongpanitlerd, 1973) :

$$A^T Q + QA - (QB + C^T)(D + D^T)^{-1}(B^T Q + C) = 0 \tag{12}$$

Unlike the analytical case, these solutions can be easily computed, with the aid of a computer (Zhou et al., 1996).

3. ON THE LFT FORMULATION

3.1 The Linear Fractional Transformation

Definition 2. (Doyle et al., 1991) Suppose $M = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \in \mathbb{C}^{(p_1+p_2)\times(q_1+q_2)}$ is a complex partitioned matrix. Let $\mathcal{D}_1 \subset \mathbb{C}^{p_1\times q_1}$ and $\mathcal{D}_2 \subset \mathbb{C}^{p_2\times q_2}$. Then the *upper* and *lower* Linear Fractional Transformations (LFTs) are the maps :

$$\mathcal{F}_l(M, \cdot) : \mathcal{D}_2 \mapsto \mathbb{C}^{p_1\times q_1} \qquad \mathcal{F}_u(M, \cdot) : \mathcal{D}_1 \mapsto \mathbb{C}^{p_2\times q_2}$$

with

$$\mathcal{F}_l(M, \Delta_l) = A + B\Delta_l(I_d - D\Delta_l)^{-1}C \tag{13}$$

$$\mathcal{F}_u(M, \Delta_u) = D + C\Delta_u(I_d - A\Delta_u)^{-1}B \tag{14}$$

assuming that $(I_d - D\Delta_l)^{-1}$ and $(I_d - A\Delta_u)^{-1}$ exist.

The LFTs allow to *mathematically* and *graphically* represent in a natural way feedback interconnection (Fig. 3).

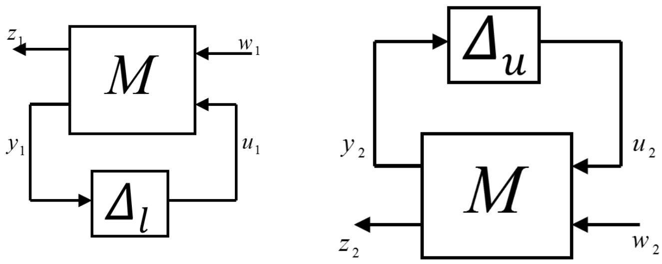

Fig. 3. Lower and Upper LFTs

For instance, the diagram on the left of Fig. 3 represents the following system :

$$\begin{bmatrix} z_1 \\ y_1 \end{bmatrix} = M \begin{bmatrix} w_1 \\ u_1 \end{bmatrix}$$
$$u_1 = \Delta_l y_1$$

The resulting closed-loop transfer function from w_1 to z_1 is then $\mathcal{F}_l(M, \Delta_l)$.

The LFT framework actually enables to represent other system interconnections (serial, parallel, inverse, ...).

Example 2. Let consider $G_1 = \mathcal{F}_l(\left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array}\right], \Delta_1)$ and $G_2 = \mathcal{F}_l(\left[\begin{array}{c|c} A_2 & B_2 \\ \hline C_2 & D_2 \end{array}\right], \Delta_2)$, both LFT systems . Then, their

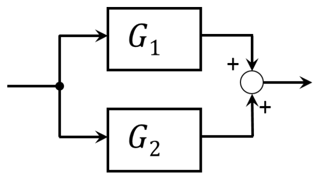

Fig. 4. Parallel Connection of LFT systems

parallel connection (see Fig. 4) is the new LFT system G defined by

$$G = G_1 + G_2 = \mathcal{F}_l\left(\left[\begin{array}{c|cc} A_1 + A_2 & B_1 & B_2 \\ \hline C_1 & D_1 & 0 \\ C_2 & 0 & D_2 \end{array}\right], \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix}\right)$$

Another valuable property of LFTs is that they allow to easily represent rational functions.

Example 3. The input impedance z_{in} of (2) (with $R = 1\Omega$) can be viewed as the normalised impedance matrix Z with a negative feedback : $z_{in} = \mathcal{F}_l(Z, -1)$.

Example 4. The scattering matrix S of (10) may also be expressed as an LFT of the normalised impedance matrix Z : $S = \mathcal{F}_l\left(\begin{bmatrix} -I_d & \sqrt{2}I_d \\ \sqrt{2}I_d & -I_d \end{bmatrix}, Z\right)$.

3.2 Linking LFT and LMI : the KYP Lemma

A Linear Matrix Inequality (LMI) is a matrix inequality of the form :

$$F(x) = F_0 + \sum_{i=1}^{m} x_i F_i \ > 0 \ (\geq 0) \tag{15}$$

where $x \in \mathbb{R}^m$ is the variable, and $F_i = F_i^T$, $i = 1, ..., m$ are given symmetric matrices.

LMIs play an important role in many Control Theory problems (Boyd et al., 1994b).

Example 5. It can be shown (Willems, 1971) that solving the ARE of (12) with symmetric $Q = Q^T$ is equivalent to minimize a linear objective function under the LMI constraints :

$$\begin{bmatrix} A^T Q + QA & QB + C^T \\ B^T Q + C & D + D^T \end{bmatrix} \geq 0 \tag{16}$$

Optimization problem involving only LMI constraints are computationally attractive as they are convex and can be solved efficiently, by interior-point methods.

The KYP Lemma The Kalman-Yakubovitch-Popov (KYP) lemma, also known as the extension of the positive real lemma and the bounded real lemma, is an important result for dynamical systems analysis. It allows to equivalently transpose the problem of solving an infinite number of frequency domain inequalities into a finite dimensional LMIs feasible problem. A modern version of the KYP Lemma (Rantzer, 1996) can be formulated as follows.

Theorem 2. **KYP Lemma** Consider $\mathcal{M} = \mathcal{M}^T$ a real symmetric matrix and $T(j\omega) = \mathcal{F}_u(\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right], \frac{1}{j\omega} I_d)$, with A, B, C, D real matrices of appropriate dimensions, such that $det(\frac{1}{j\omega} I_d - A) \neq 0$ for $\omega \in \mathbb{R}$ and (A, B) controllable. The following two statements are equivalent :

(1) $\forall \omega \in \mathbb{R}$, $T(j\omega)^* \mathcal{M} T(j\omega) \leq 0$
(2) There exists a real symmetric matrix $P = P^T$ such that
$$\begin{bmatrix} A^T P + PA & PB \\ B^T P & 0 \end{bmatrix} + \begin{bmatrix} C^T \\ D^T \end{bmatrix} \mathcal{M} \begin{bmatrix} C \\ D \end{bmatrix} \leq 0$$
holds.

This lemma has several extensions to consider other types of variables and constraints (for example see Iwasaki and Hara (2005)).

4. AIM

Our aim is to formulate the reactive filter synthesis under the LFT framework, and to solve it using extended versions of the KYP lemma. This would bring substantial benefits detailed at the end of this section. To achieve this, two approaches might be developed.

4.1 2-stage Approach

First, the approximation problem of the usual synthesis method can be solved with the LFT and the KYP lemma tools. In (Rossignol et al., 2001), the problem of synthesizing a transfer function of *minimum order*, satisfying *polynomial* frequency constraints, is solved using an adapted version of the KYP lemma. In practice, constraints are expressed as frequency bounds and no curve fitting is required. This makes this approach more valuable and allows to ensure to compute the *minimum* order.

Realisability conditions, which are inherently satisfied in the usual approximation stage, need to be added to this method. This may then enable to achieve the realisation stage as in the usual approach. These realisability conditions have to be formulated as LMI constraints, using for instance extended versions of the KYP lemma.

In (Anderson and Vongpanitlerd, 1973), most of these conditions have been expressed in terms of the state-space representation of the representative matrices to be synthesized. This paper is an attempt to illustrate the relation between constraints on the state-space representation and equivalent LMI constraints. Once this is achieved, the usual realisation method can be applied.

4.2 Unified Approach

Alternatively, one can consider a unified approach using the LFT framework. This is based on the following observation. The RF electronics filter design problem consists in synthesising an electronics circuit, of which a representative function satisfies frequency constraints. The LFT tool enables to represent *graphically* and *mathematically* a system, its subsystem interconnection, and its input-output behaviour.

Therefore, the LFT representation may be viewed as a generic way of representing systems, and electronics filter circuits are specific systems. In one hand, one has to include practical constraints which are implicit in circuit representations. In the other hand, the generic property of the LFT representation may allow to create new interconnections of practical interest, which are too complex to be synthesized with current circuit representations.

4.3 Potential Benefits

Several benefits are expected to be obtained by formulating the reactive filter synthesis problem with new Control tools (LFT, LMI, KYP).

First, the synthesis problem may be optimally solved, in the sense that the resulting filter will have the minimal number of reactive components. As already mentioned, usual methods for designing the function s_{21} do not guarantee minimality of its order, which is linked to the number of reactive components used.

Second, the generic property of the LFT representation may enable to extend the method to other types of filters than LC filters. In practice, inductors have been tried to be avoided due to their poor integration characteristics for the benefits of *resonators* (namely SAW/BAW resonators). In RF electronics, tunable filters made of capacitors and resonators may have a crucial role in the near future but raise formidable design problems (Hashimoto et al., 2011). Usual simulation methods for designing resonator-based filters are inadequate for tackling such challenge. A theoretical method allowing to design these tunable filters is therefore required. Additionally, the synthesis of MIMO (Multiple Input Multiple Output) filters may also be considered.

Third, the LFT framework enables to make the so-called μ-analysis, widely developed in Control Theory (Zhou et al., 1996). This may then allow to make a precise robustness analysis of the synthesized circuit. This is of practical interest as sources of uncertainties are numerous (values of the components, lossy components, simplified model, ...).

Finally, this approach is not restricted to analog filters design and may be extended to digital filters. For instance, in (Iwasaki and Hara, 2005), the finite impulse response filters design problem is solved using an extended version of the KYP lemma. Infinite impulse response filters may be obtained by using the well-known bilinear transformation. It can also be noticed that the transfer function of an infinite impulse response filter is a rational function of the discrete variable z, and can therefore be represented using the LFT framework.

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