Analysis of some basic creep tests on concrete and their implications for modelling

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INTRODUCTION – BASIC CREEP OF CONCRETE – experimental observations

The ability to accurately predict the delayed deformation of prestressed concrete is important for the correct design of prestressing. In modern codes, delayed strains are broken down into four components: autogenous shrinkage, drying shrinkage, basic creep and drying creep - see for instance MC2010, the most recent fib model code [1]. In this paper only basic creep will be considered.

Basic creep is conventionally obtained by measuring the deformation of a concrete specimen protected from desiccation and loaded under a constant stress σ. In the laboratory, this is achieved by protecting the concrete from drying. To obtain the basic creep, simultaneous measurements are made of autogenous shrinkage, so that basic creep can be deduced from raw creep measurements on the specimen. The compliance J is defined such that the mechanical deformation ε (which is the deformation due to the applied load, that is the total strain minus the shrinkage) is equal to the product of J and the applied stress:

$$\varepsilon = J\,\sigma \qquad \text{(Eq. 1)}$$

Acker and Ulm [2], analyzing Le Roy's tests [3], examined the derivative of the compliance dJ/dt. Considering different loading ages, they showed that this derivative tends to $1/Ct$ when t is large, with the same value of C for a given concrete regardless of the age of loading t_0 (see Figure 1). Their conclusions were that two mechanisms are involved in basic creep: a short-term mechanism corresponding to the stress-induced movement of water towards the largest diameter pores and a long-term mechanism due to irreversible viscous behavior, and related to viscous flow in the hydrates (slippage between layers of C-S-H).

The microprestress theory offers also an explanation for the long term creep: assuming that aging is due to a variation of the viscosity at a microscopic level, a variation of the stresses at the same level is induced [4, 5]. In the case of basic creep, Bažant has shown that the assumption of a flow growing as a logarithmic function of time is in accordance with this explanation of the long term creep and is introduced in the B3 and B4 models [6, 7].

If the derivative of the compliance of the experimental results presented before (figure 1) is expressed as a function of the age since loading (t-t_0), an almost identical behavior is observed (Figure 2).

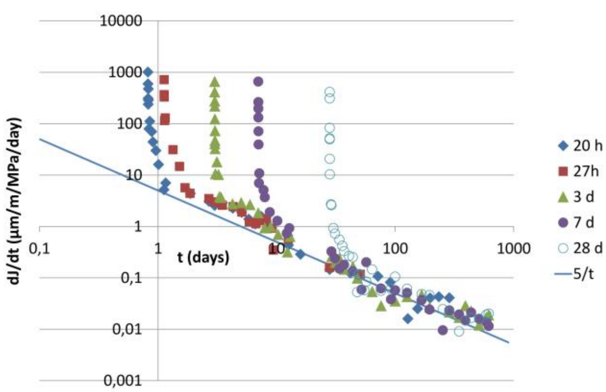

Figure 1. Derivative of the compliance of basic creep: tests performed by Le Roy [3] with respect to time, as proposed by Acker et Ulm [2].

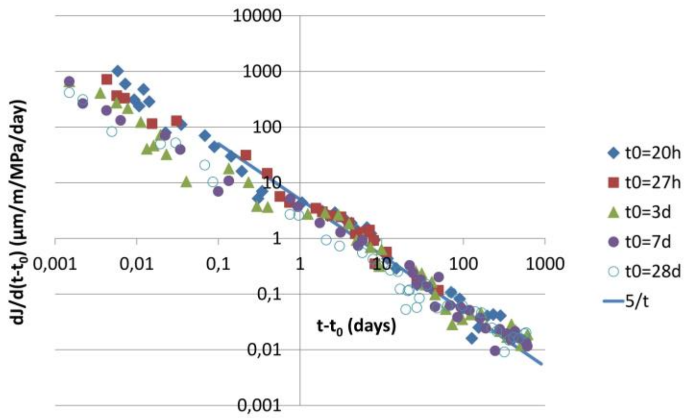

Figure 2. Derivative of the compliance of basic creep: tests performed by Le Roy [3] with respect to age since loading

By integrating the derivative of the compliance we are able to obtain an expression for basic creep. The relationship obtained is similar to that proposed in MC2010 [1]: the compliance may be expressed as the sum of an elastic component and a delayed component where the term *C* is homogeneous to a stiffness and independent of t_0, and the characteristic time $\tau(t_0)$ which depends on the age of loading (Equation 2).

$$J(t_0, t - t_0) = \frac{1}{E(t_0)} + \frac{1}{\beta_1 C}\, log\left(1 + \frac{t - t_0}{\beta_2 \tau(t_0)}\right) \text{ [Eq.2]}$$

with when $t \to \infty$ $dJ/dt \to \frac{1}{\beta_1 C t}$

The parameters C and $\tau(t_0)$ may be estimated from the relevant expressions in MC2010:

$$\frac{1}{C} = \frac{1{,}8}{E_{28}\, f_{cm}^{0,7}} \qquad \text{[Eq. 3]}$$

$$\frac{1}{\tau(t_0)} = (0.035 + \frac{30}{t_{0,adj}})^2 \qquad \text{[Eq.4]}$$

$$\text{with } t_{0,adj} = t_0\left(1 + \frac{9}{2+t_0^{1,2}}\right)^{\alpha} \qquad \text{[Eq.5]}$$

where $\alpha = -1$ for a CEM32.5N (or SL) cement, 0 for a 32.5R or a 42.5N (or N) cement and +1 for 42.5R, 52.5N, 52.5R (or R) cements, E_{28} is the Young modulus after 28 days and f_{cm} is the mean strength after 28 days.

β_1 and β_2 are parameters that may be adjusted to match experimental results when these are available. These parameters are equal to 1 when Equations 3 and 4 give values of C and $\tau(t_0)$ that fit the experimental results.

This behavior resembles that which has been observed on small specimens subjected to nanoindentation [8], but also on real structures [9, 10] although in the case of structures other phenomena should also be considered (drying shrinkage, drying creep and the relaxation of prestressing for example). This type of function may also a have a closed form when a continuous retardation spectrum is used for an evaluation of coefficients in Dirichlet series that approximate the compliance function [11].

Using values obtained from Equations 3 to 5, Figure 3 shows the influence of the term $\tau(t_0)$ on the basic creep strain for three different loading ages (3, 7 and 28 days). For basic creep, the effect of ageing is included in the parameter $\tau(t_0)$. This explains why in the expression for basic creep in MC2010, the stiffness C is independent of t_0 (and does not depend, for example, on Young's modulus at the age of loading t_0). In Figure 4, for the same sets of values for $\tau(t_0)$, when the basic creep strain is expressed as a function of $(t\text{-}t_0)$, a single plot is obtained very rapidly, explaining experimental observations (Figure 2). The only differences are at the beginning of the curve, i.e. during the first days after loading.

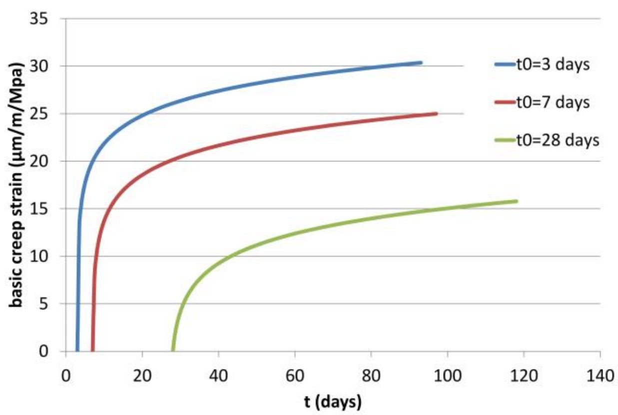

Figure 3: Basic creep strains as a function of time; influence of the parameter $\tau(t0)$ on the basic creep strains. The different values of $\tau(t0)$ were obtained from Equation 4 assuming $\alpha = 0$ (Equation 5): $\tau(t0 = 3 \text{ days})=0.01$ day, $\tau(7)=0.05$ day and $\tau(28)=0.8$ day.

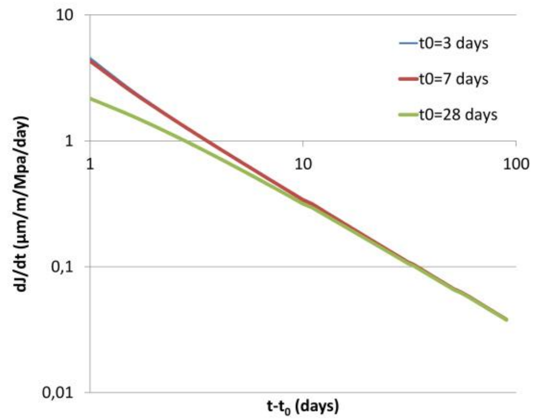

Figure 4. Derivative of the compliance function of the time since loading and for different loading ages; the values of τ(t0) are identical to the values used in Figure 3.

ANALYSIS OF BASIC CREEP TESTS FROM THE LITERATURE

Equation 2 for basic creep has already been compared with Le Roy's basic creep tests performed on different concretes with varying loading ages, proportions of water, and volumes of silica fume and paste [12]. Here, we shall compare this equation with several results from basic creep tests with different loading ages, obtained from the NU database (http://www.civil.northwestern.edu/people/bazant/CreepShrinkData_131127.xlsx).

Below we shall consider basic creep tests in which several loading ages were tested. Table 1 shows the characteristics of the concrete used and the corresponding values of parameter C for each concrete according to MC2010 relation (equation 3). Note that to assess the value of Young modulus when it is not indicated in the NU database, the instantaneous deformation of the creep test at 28 days is used. Figures 5 to 10 show the comparison between experimental results, direct application of MC2010 equations (equations 3, 4 and 5, i.e. with β_1 and β_2 equal to 1) and equation 2 with adjusted parameters.

It could be seen from the comparison between experimental results and the prediction with MC2010 that the accuracy of the long term prediction of basic creep with the MC2010 relations without adjustment on tests is limited. This scatter could not be completely avoided: the error between the calculated long term values of the creep function and the observed long term creep function extrapolated from experimental results from a data set used to establish MC2010 relations is around 25% despite the fact that these relations are the result of a regression analysis on this very data set [13]. Note also that the relations of the MC2010 were mostly fitted using experimental results of tests on concretes using European cements and are neither fitted on Japanese nor North American concretes.

Figures 5 to 10 show also that applying a fitted constant coefficient $\beta_1 C$ for a given concrete and by varying the parameter

$\beta_2\tau(t_0)$ for each loading age enables the proposed relationship to represent the physical phenomena very accurately.

Parameter fitting was performed on each concrete by the least squares method considering a single value for $\beta_1 C$ for all the tests performed by each author with the same concrete. The aim is not there to find the best fit for the relation of the parameters with the strength from tests coming from a large database like the NU database. In this case a more sophisticated approach is mandatory [13, 14]. Table 2 presents the parameters obtained for the concretes in question.

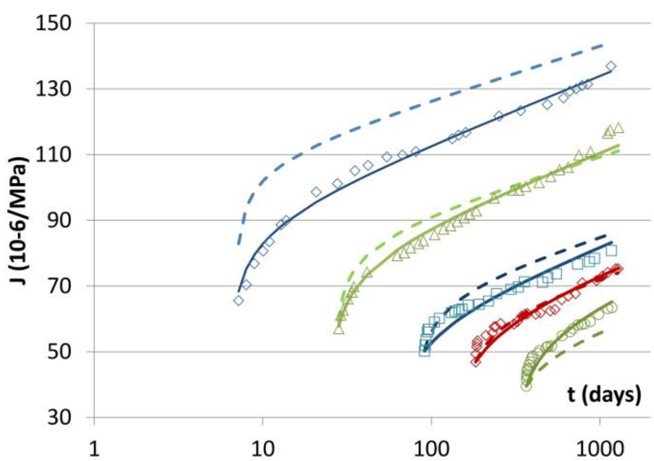

Figure 5. Comparison between experimental results and Equation 2 – Kawasumi's tests [15]. The corresponding files in the NU database are J_018_01 to 05. The solid lines correspond to the use of equation 2 with adjusted parameters, the dashed lines to the application of MC2010 relations (with β_1= β_2= 1) and the dots to experimental results. Loading ages are 7, 28, 91, 183 and 366 days.

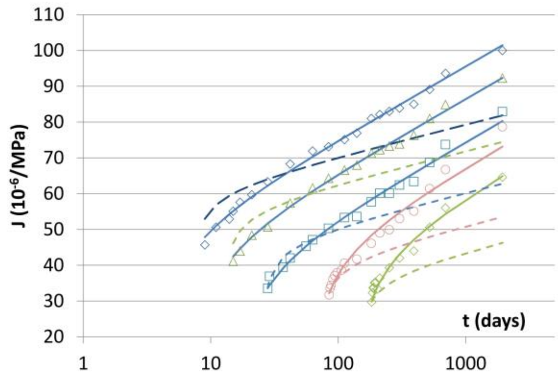

Figure 6. Comparison between experimental results and Equation 2 – Shritharan's tests [16]. The corresponding files in the NU database are C_079_07 to 12. The solid lines correspond to the use of equation 2 with adjusted parameters, the dashed lines to the application of MC2010 relations (with $\beta_1 = \beta_2 = 1$) and the dots to experimental results. Loading ages are 8, 14, 28, 84 and 182 days.

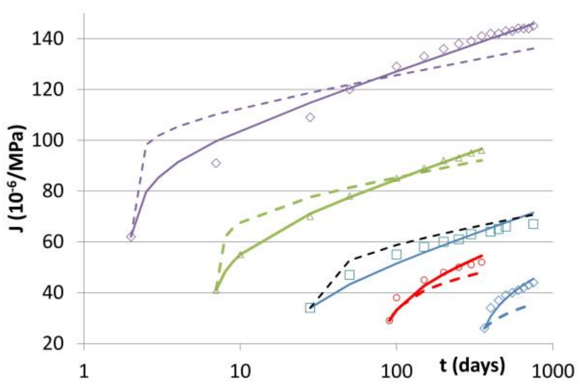

Figure 7. Comparison between experimental results and Equation 2 – Hanson's tests [17]. The corresponding files in the NU database are C_002_02 to 06. The solid lines correspond to the use of equation 2 with adjusted parameters, the dashed lines to the application of MC2010 relations (with $\beta_1 = \beta_2 = 1$) and the dots to experimental results. Loading ages are 2, 7, 28, 90 and 365 days.

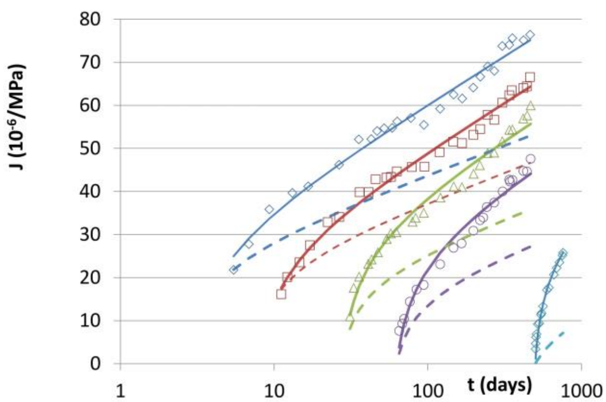

Figure 8. Comparison between experimental results and Equation 2 – Nagamatsu's tests [18]. The corresponding files in the NU database are J_027_01 to 05. The solid lines correspond to the use of equation 2 with adjusted parameters, the dashed lines to the application of MC2010 relations (with $\beta_1 = \beta_2 = 1$) and the dots to experimental results. Loading ages are 2, 7, 28, 90 and 365 days. Note that in this test only the delayed deformations are considered.

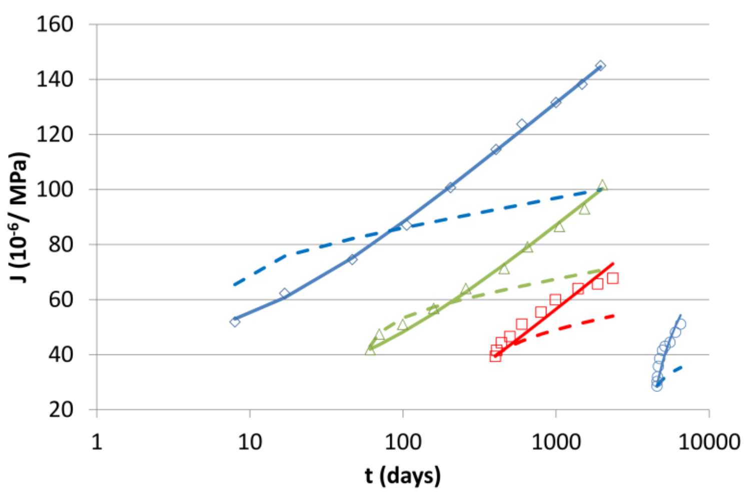

Figure 9. Comparison between experimental results and Equation 2 – Browne's tests [19]. The corresponding files in the NU database are C_110_01 to 04. The solid lines correspond to the use of equation 2 with adjusted parameters, the dashed lines to the application of MC2010 relations (with $\beta_1 = \beta_2 = 1$) and the dots to experimental results. Loading ages are 7, 60, 400 and 4560 days.

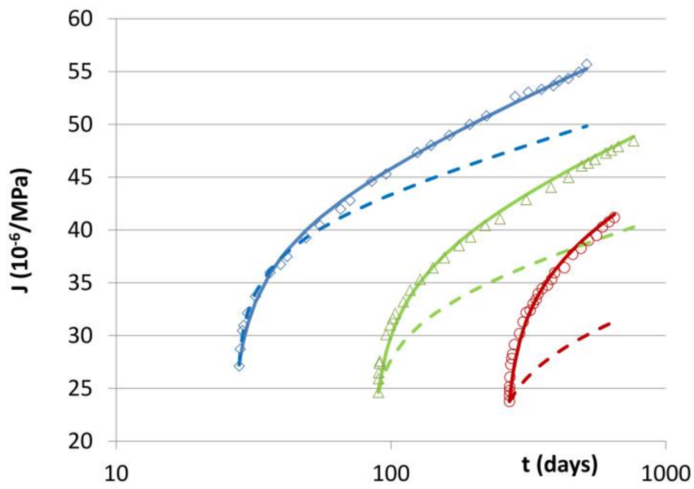

Figure 10. Comparison between experimental results and Equation 2 – Kommendant's tests [20]. The corresponding files in the NU database are C_104_01 to 03. The solid lines correspond to the use of equation 2 with adjusted parameters, the dashed lines to the application of MC2010 relations (with $\beta_1 = \beta_2 = 1$) and the dots to experimental results. Loading ages are 28, 90 and 270 days.

COMPARISON WITH MC2010

We can then compare the values obtained by fitting Equation 2 to the experimental results with those proposed by the Model Code 2010. Table 3 indicates the values of parameters $\beta_1$and β_2. The values of the parameter β_1 are below 1 indicating that eq.3 overestimates the value of the parameter C in the case of the tests used in this study. The values of parameter β_2 exhibit greater variation, especially when loading is applied before 28 days. In this case, the value of $\tau(t_0)$ is very low and a small difference between experimental fitting and MC2010 will result in a high β_2 value. But this difference is very important because this parameter will result in greater creep strains.

This also shows that Equations 2 to 5, which are based on a small number of mechanical parameters, are unable to capture all the sources of variation in creep (for example the nature of the aggregates or the binder).

Figures 11a and 11b present the comparisons for the parameter $\tau(t_0)$ in the case of SL and R cements respectively. Agreement is satisfactory, but some variability in these parameters is observed between the different concretes. For the case of sensitive structures in which the prediction of creep is important these parameters should be calibrated on the basis of prior laboratory experiments.

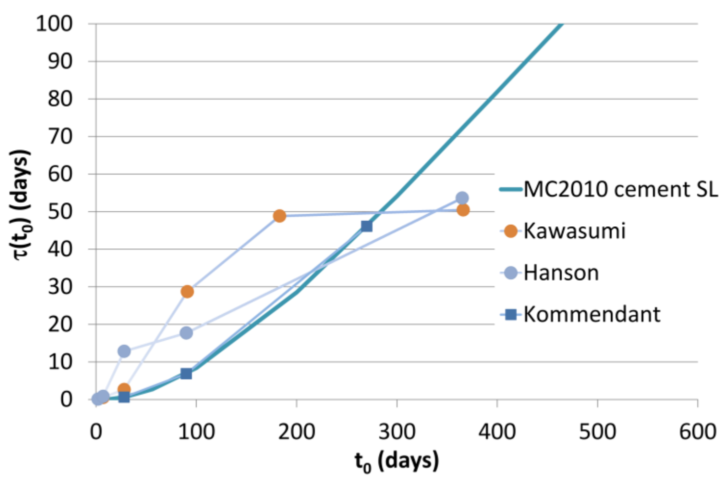

Figure 11.a Changes in τ(t0) predicted by MC2010 and obtained by fitting experimental results – cement type SL

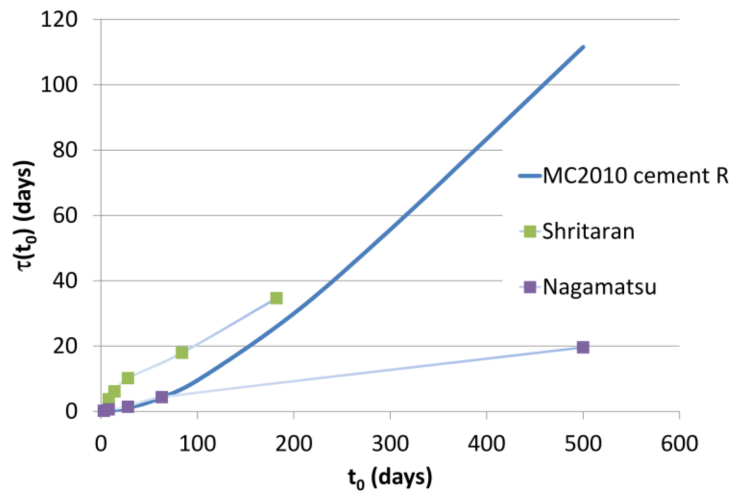

Figure 11.b Changes in τ(t0) predicted by MC2010 and obtained by fitting experimental results – cement type R

CONCLUSION

The analysis of experimental results for basic creep ordinary and high performance concretes shows that when expressed as a function of time since loading, in the long term the derivative of the compliance is linear in log-log space.

This analysis shows also that basic creep compliance may be expressed as a logarithmic function of time, involving two parameters C and $\tau(t_0)$. The comparison of this expression with a number of experiments in the literature, if we fix the parameter C for a given concrete, and $\tau(t_0)$ for each loading age, shows that the long-term temporal behavior of basic creep at different loading ages can be satisfactorily predicted. A comparison of the expression proposed in MC2010 for basic creep with experimental results shows that the accuracy of the model is improved when the two parameters C and $\tau(t_0)$ are adjusted to match creep experiments. This procedure is recommended for structures that are sensitive to creep.

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Table 1. Properties of the concretes considered in this study (characteristics obtained from the NU database).

Authors	W/C	Cement type	f_{cm} [MPa]	E_{28} [MPa]	C [MPa]
Kawasumi [15]	0.47	SL (slow)	33	21950	14100
Shritharan [16]	0.47	R (rapid)	50	29800	25600
Hanson [17]	0.56	SL	34	29400 (estimated)	19300
Nagamatsu [18]	0.55	R	32	26650	16750
Browne [19]	0.42	N (hypothesis)	50	25400 (estimated)	21900
Kommandant [20]	0.38	SL	45	36900 (estimated)	29400

Table 2. Values obtained for the parameters $\beta_1 C$ and $\beta_2\tau(t_0)$ for the tests presented in Figures 5 to 10.

Kawasumi [15]	$\beta_1 C$ [GPa]	t_0 [days]	7	28	91	183	366
	0.11	$\beta_2\tau(t_0)$ [days]	0.5	2.7	28.7	48.8	50.5
Shritaran [16]	$\beta_1 C$ [GPa]	t_0 [days]	8	14	28	84	182
	0.11	$\beta_2\tau(t_0)$ [days]	3.7	6.1	10.2	17.9	34.7
Hanson [17]	$\beta_1 C$ [GPa]	t_0 [days]	2	7	28	90	365
	0.11	$\beta_2\tau(t_0)$ [days]	0.1	0.8	12.8	17.7	53.6
Nagamatsu [18]	$\beta_1 C$ [GPa]	t_0 [days]	3	8	28	63	500
	0.10	$\beta_2\tau(t_0)$ [days]	0.2	0.6	1.4	4.4	19.6
Browne [19]	$\beta_1 C$ [GPa]	t_0 [days]	7	60	400	4560	

	0.05	$\beta_2\tau(t_0)$ [days]	17.7	104	428	716	
Kommendant [20]	$\beta_1 C$ [GPa]	t_0 [days]	28	90	270		
	0.2	$\beta_2\tau(t_0)$ [days]	1.8	5.4	11.3		

Table 3. Values of the parameters $\beta_1$and β_2.

Kawasumi [15]	β_1	t_0 [days]	7	28	91	183	366
	0.78	β_2	29.0	4.3	4.1	2.0	0.7
Shritaran [16]	β_1	t_0 [days]	8	14	28	84	182
	0.44	β_2	20.1	16.2	9.4	2.5	1.3
Hanson [17]	β_1	t_0 [days]	2	7	28	90	365
	0.56	β_2	188	47.1	20.9	2.6	0.7
Nagamatsu [18]	β_1	t_0 [days]	3	8	28	63	500
	0.6	β_2	3.2	3.3	1.3	1.0	0.2
Browne [19]	β_1	t_0 [days]	7	60	400	4560	
	0.23	β_2	326	30	5.2	1.2	
Kommendant [20]	β_1	t_0 [days]	28	90	270		
	0,7	β_2	2.8	0.8	0.2		