Seals Based on Magnetic Fluids for High Precision Spindles of Machine Tools

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Abstract

The research work reported in this paper is focused on the use of magnetic fluids as active elements in seals for improving sealing capacity and minimizing friction torque, with application to the spindles of high precision machine tools. The prototype design was optimized following numerical computation of the magnetic field in the rings of the seal. Two magnetic fluids were analysed for their use in the seals: a ferrofluid and magnetorheological fluid. The sealing capacity of the MRF based seals was higher than 45kPa per ring, but the friction of the seal in the bearing was 8 N·m, too large for the use in precision spindles of machine tools. The ferrofluid seal achieved sealing capacity around 9kPa per ring, good enough to be used in the spindles of machine tools, with a friction 0.25 N·m. The feasibility of using ferrofluids for developing high performance seals for high precision spindles and the validity of the simulation models has been demonstrated experimentally.

Keywords

Magnetic fluids, ferrofluids, magnetorheological fluids, magnetic fluid sealing, machine tool spindle.

1. INTRODUCTION

The introduction is divided in three sections, 1) a brief introduction to classic seals and labyrinths for high precision spindles in machine tools, 2) behavior of magnetic fluids and their technological applications, and 3) seals for spindles and bearing houses based on magnetic fluids.

1.1. Seals and labyrinths for high precision machine tools

High precision spindles of machine tools, as used in grinding machines, usually implement hydrostatic bearings in their main shafts (wheelhead, workhead or tailstock) in order to ensure the highest quality and accuracy of the grinded parts, since hydrostatic lubrication provides best performance in terms of positioning resolution, damping capacity and motion smoothness [1]. Hydrostatic lubrication requires handling pressurized oil within the machine, fed through a manifold into the bearing and back into the tank, and the sealing of the whole system is a critical issue in the reliability of the bearings [2].

The sealing of the bearing housing must be ensured whether the shaft is at rest or rotating at nominal velocity, avoiding the contamination of the lubricant oil with the cutting fluid, chips from machining, abrasive micro grains from grinding, and any other potential contaminant within the machine tool. A wide review of available sealing technologies was given by Müller and Nau in their book about the principles and applications of fluid sealings [3].

One of the most active fields in the study of fluid seals is turbomachinery and rotordynamics, focused on energy applications [4]. CFD studies were carried out to analyse the force perturbation in the system, key issue to ensure fluid stability [5], and those forces were also studied from the tribology point of view, where the forces are reduced with pattern based surface microtexturing [6].

In the particular case of hydrostatic journal bearings lubricated with low viscosity fluids, as in the lubrication of very high speed bearings sealing (i.e. CBN grinding applications), it is very important to avoid leakage in the spindle. An extreme case due to very low viscosity is the water based hydrostatic lubrication, as presented in [7].

Therefore, high sealing capacity and low friction force are the main performance criteria for bearing seals. A solution based on magnetic fluids is proposed in this work.

1.2. Magnetic fluids, behaviour and applications

Magnetic fluids are materials whose physical properties can be tuned by external magnetic fields. Two main groups can be distinguished: ferrofluids and magnetorheological fluids, MRF hereinafter. All magnetic fluids are formed by: carrier fluid, additives to improve fluid stability, and magnetic particles. The size of the magnetic particles, nanometric or micrometric, categorize them as ferrofluids or MRF respectively. Ferrofluids are compose by nanometric (5-12nm diameter) particles, and due to their size and energy balance (gravity < particle’s thermal energy) the colloidal suspension is stable in time. These particles are usually formed by iron or cobalt oxides, covered with surfactants (like oleic acid) to avoid aggregation and final settling of the particles. The most relevant reference in ferrofluids behaviour is a book by Rosenweig (Rosensweig 1985), while a wide overview of the synthesis, characterization and application of ferrofluids is presented in the compilation book by Odenbach [8].

The MRF are similar to ferrofluids but the magnetic particles are in the range of micrometres, within 1 to 10µm, mainly made of carbonyl iron powder (CIP). They are strongly unstable, settling of the particles is inevitable, but to avoid agglomeration and to have a quick restoration of homogeneous MRF, the fluids include several additives, usually kept confidential by commercial manufacturers.

Engineering applications of magnetic fluids range from active dampers, bearings lubrication, hydraulic actuators to sealing of spindles [9]. In this research paper the application to spindle seals will be analysed.

1.3. Face sealing based on magnetic fluids for machine tool spindles

Magnetic fluids based seals were presented in the early 1980's where the basic design parameters were established [10], such as the gap, size and the distance between the rings. Simulations of those seals with non-linear models to consider the real magnetization curve were discussed [11]. Looking for a better response in the ferrofluids based seals, the centrifugal force was analysed in the case of high speed systems [12], computing Reynolds equations together with the magnetic pressure to consider high speed effects in the fluid.

A theoretical study of the sealing capacity of large seals was presented , including centrifugal, gravitational and magnetic effect [13]. The main parameters to be considered in the design were identified: Width of the rings, distance between them and the effect of the gap and magnetic fluid volume. Other papers went further in the theoretical study and analysis of ferrofluids based seals with free surface consideration ([14]–[16]) discussing the self-sealing capacity of ferrofluids for applications such as bearings.

The use of MRFs for sealing applications was considered by Vardarajan [17], where the pressurization and the wear generated in the rings by the magnetic fluids with strong yield stress was studied.

Finally a deep theoretical and experimental study of ferrofluids based seal was presented in [18], discussing the maximum pressure achieved by the rings and the recovery pressure after the fluid seal was broken, once the maximum pressure was overcome.

This research work was focused in the use of magnetic fluid seals for the application of high precision spindles in machine tools. The work was based on the models presented in the literature, two commercial magnetic fluids that were characterized experimentally in the research, and a final construction of a prototype that was used to validate the seal in close to industrial conditions. A section view of a magnetic fluid based seal is described below following the numeration of Figure 1:

1 Semi-shaft: Rotating shaft. Sealing rings (part 2) is assembled in the semi-shaft.

2 Sealing rings: Ferromagnetic part filled with magnetic fluid. Sealing rings are machined in this part, and magnetic fluid seals are formed in the edges. Different ring edge shapes are studied in the research.

3 Magnetic poles: Steel made rings to conduct magnetic field from the magnets (part 4) to the seal rings (part 2).

4 Magnets: Strontium ferrite made permanent magnets

5 Seal body: Main structure of the seal made in aluminium. Air pressure is achieved into the body, air inlet (orifice 7).

6 Assembly cover: Cover ring to assemble the sealing prototype.

7 Air inlet: Compressed air inlet to the main body. Interior volume of *seal body* (5) is pressurized with compressed air, introduced by *air inlet* (7), with sealing capacity given by *sealing rings* (2).

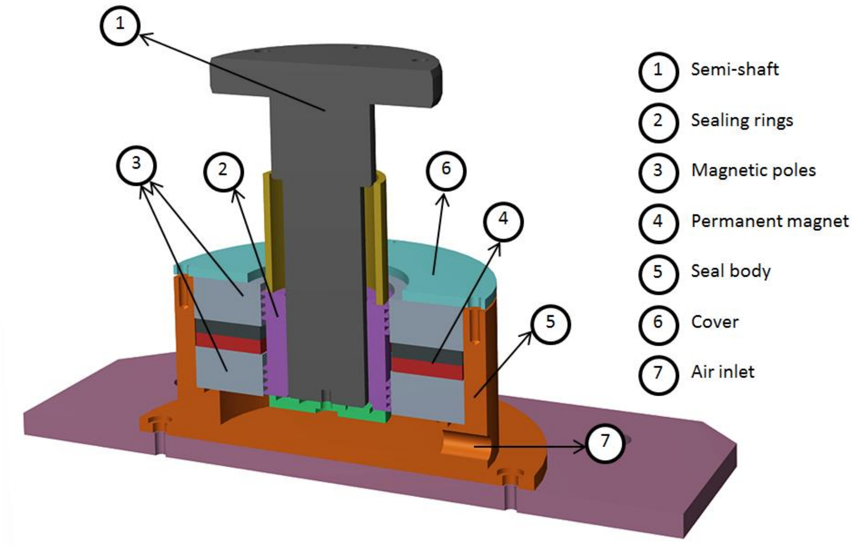

Figure 1 Magnetic fluids sealing, section view with parts description.

In Section 2, the experimental characterization of the magnetic fluids used in the research is performed; Section 3 deals with the modelling of the seals; Section 4 presents the test bench and the prototype for experimental validation; Section 5 summarizes the results and finally in Section 6 the main conclusions are outlined.

2. CHARACTERIZATION OF FERROFLUIDS AND MAGNETORHEOLOGICAL FLUIDS

Most magnetic fluid based seals found in the literature use ferrofluids, but there are also some solutions based on MRF. In order to analyse both alternatives, the following commercial magnetic fluids were used in this research work: a ferrofluid from FERROTEC® APG s10n and a magnetorheological fluid from LORD® Corp. model MRF 122-2ED. Data given by manufacturers are usually approximate values, both for magnetic and rheological behaviour, and therefore a detailed characterization of these fluids was performed, looking at the magnetic response and the rheological properties under the magnetic field and the temperature. The characterization of those fluids was essential to have an accurate computation of fluids magnetization and sealing capacity of the rings, as it is presented in section 3.

The magnetic characterization was performed with a Magnet-Physik Permagraph L magnetometer and the electromagnet EP-3, from which the magnetic field strength, flux density (Figure 2), and magnetic fluid permeability were obtained (Figure 3). The magnetorheological characterization was carried out with a Thermo HAAKE RheoStress RS150, with a magnetic module and a thermally controlled plate.

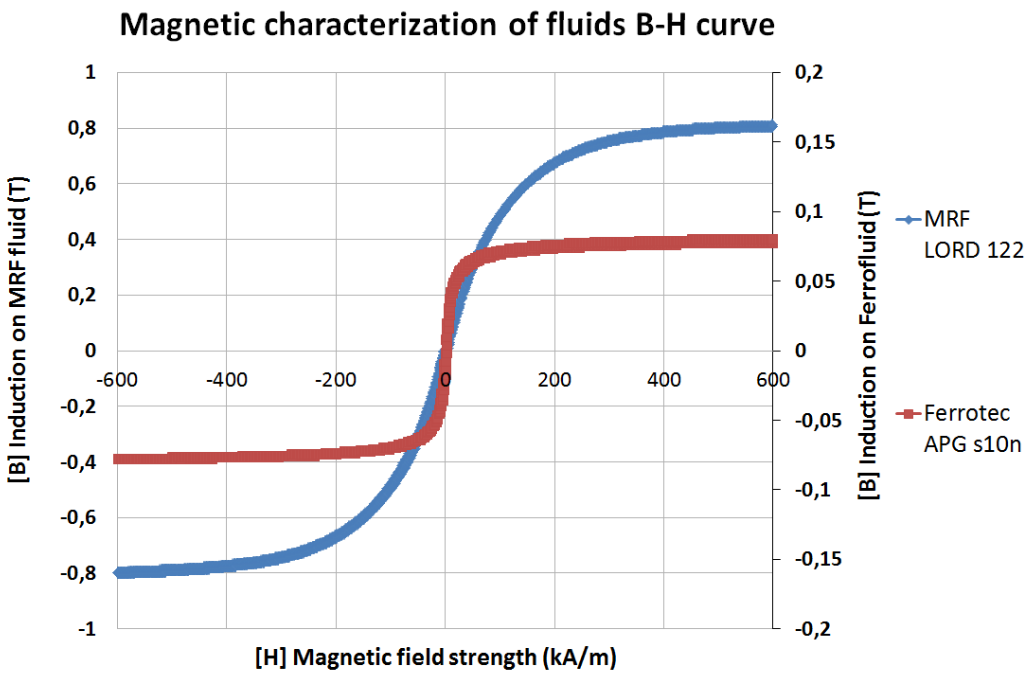

Figure 2 Magnetic characterization of the magnetic fluids: flux density and magnetic field strength.

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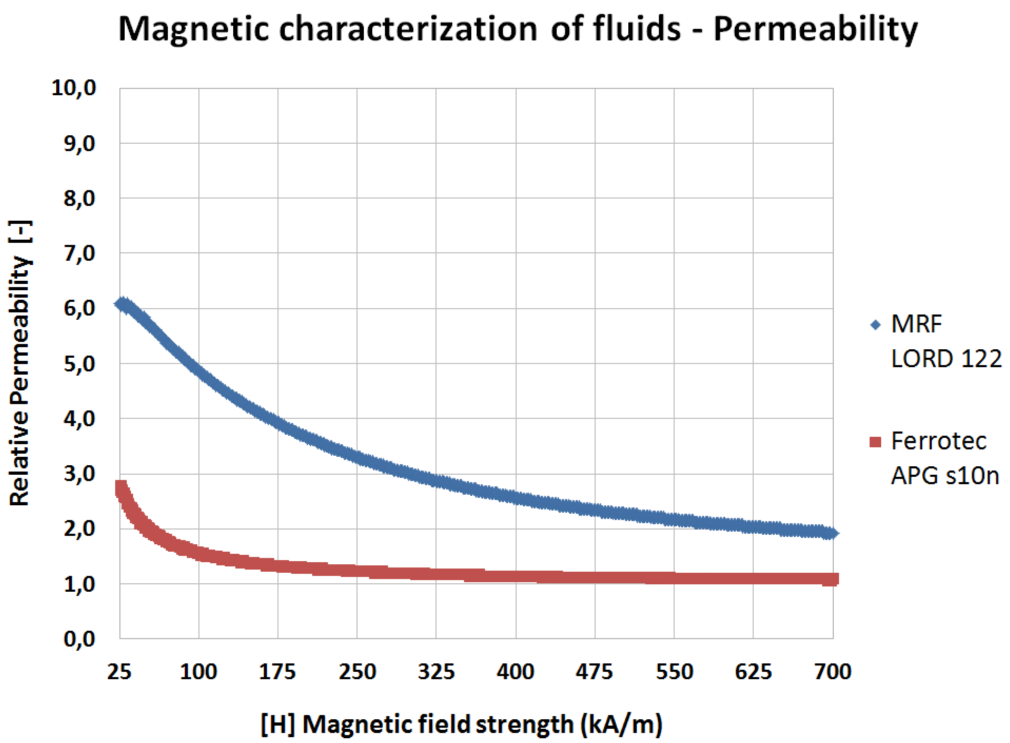


Figure 3 Magnetic characterization of the magnetic fluids: relative permeability.

Rheological analysis of magnetorheological fluid has been carried out following Bingham model, Equation (1). Total shear stress of the fluid (τ) is the sum of yield stress (τ_0), which is in function of magnetic field (H), and the product of base viscosity of the fluid (K) and shear rate ($\dot{\gamma}$). Experimental values for MRF characterization are shown in Figure 4:

$$\tau = \tau_0(H) + K \cdot |\dot{\gamma}| \quad (1)$$

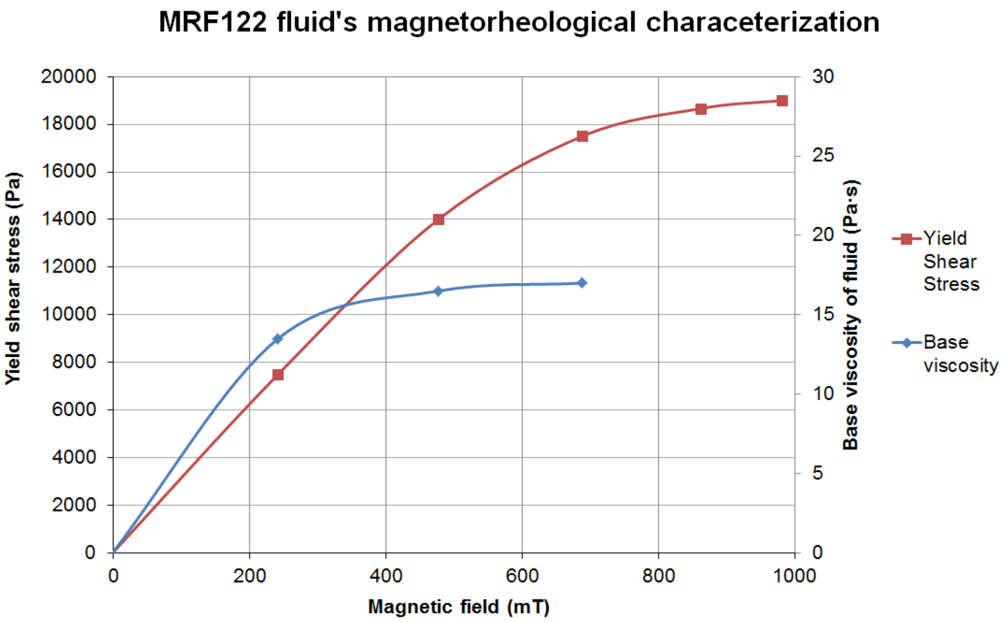


Figure 4 Rheological characterization of MRF: shear stress and base viscosity.

Ferrofluids rheology was characterized following tuneable Newtonian fluid (2), where fluid dynamic viscosity (η) it is in function of temperature (T) and applied magnetic field (H).

$$\eta = \eta_m(H, T) \quad (2)$$

The values for ferrofluids characterization are shown in Figure 5, where magnetic tests were done at room temperature and ferrofluid temperature characterization without magnetic field:

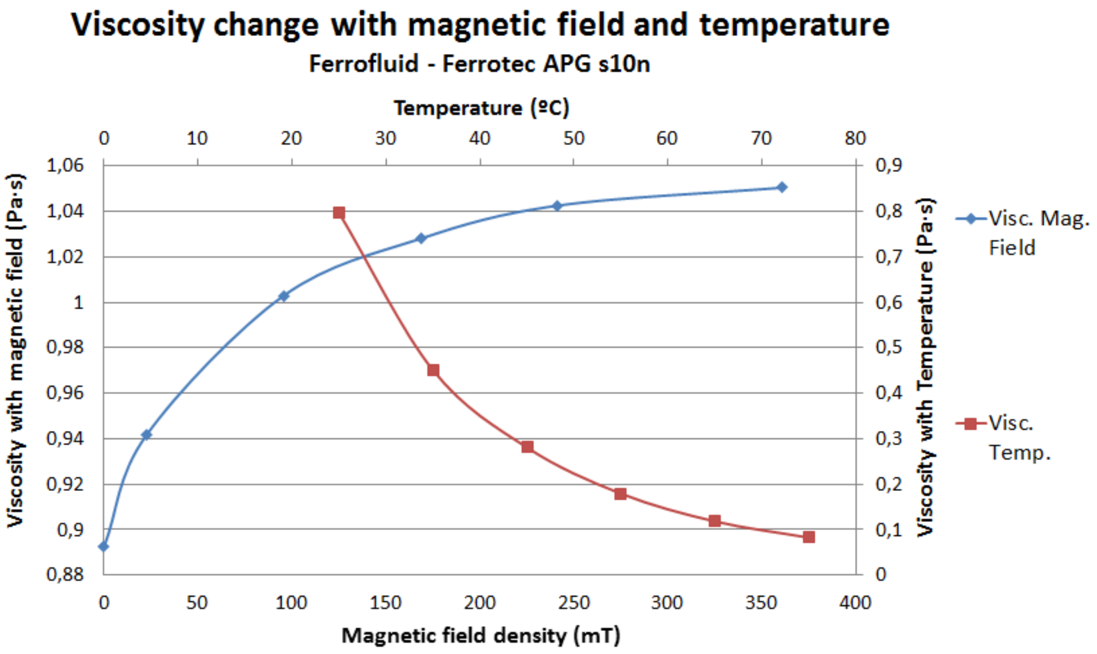


Figure 5 Rheological characterization of ferrofluid: viscosity and temperature.

3. DESIGN AND MODELLING OF SEALING WITH MAGNETIC FLUIDS

A model of the magnetic fluid based seal was developed to support the prototype design process (see Section 4), and they were later validated with the experimental evaluation (see Section 5). The model considers the magnetic field in the seal and the pressure in the fluid.

Magnetic simulations were performed to determine the magnetic field strength and flux density with the FEMM finite element software. Then, fluid pressure was simulated based on the model presented by Park, Kim, Hahn and Lee, [11], solved by integration in in-house software. Those results are based on the experimental values of magnetic fluids characterization, section 2.

3.1. Design of seals with magnetic fluids

The ring of magnetic fluids based seal, as presented in Figure 1 (part 2), is manufactured by different groves and edges. In this section the shape of these ring edges was discussed and designed. Four different designs were studied for the ring, which are presented in Fig.6: a) plain seal, b) square rings, c) saw shape and d) triangular shape. In all those cases the gap, overall geometry and magnet were the same, so only the effect of rings shape was analysed.

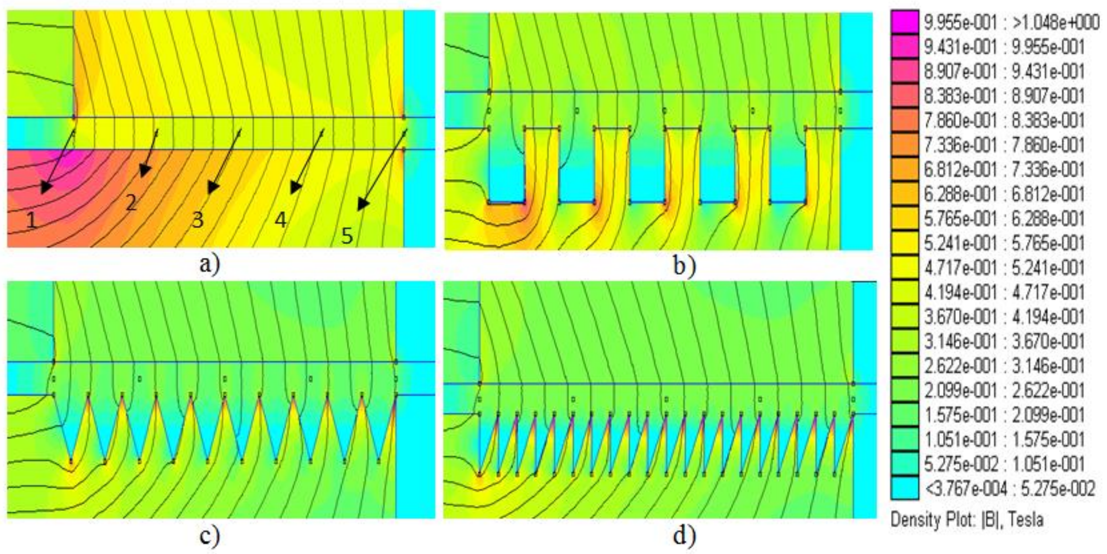


Figure 6 Magnetic field distribution for different rings shape studied in the paper : a) plain seal, b) square rings, c) saw shape and d) triangular rings.

The magnetic flux density was simulated in the fluid for these four different shapes, where the field is generated by a permanent magnet. The simulations were carried out considering axis-symmetry assumption (cylindrical structure like shown in the section view of Figure 1), and magnetic model of the seal is depicted in Figure 7:

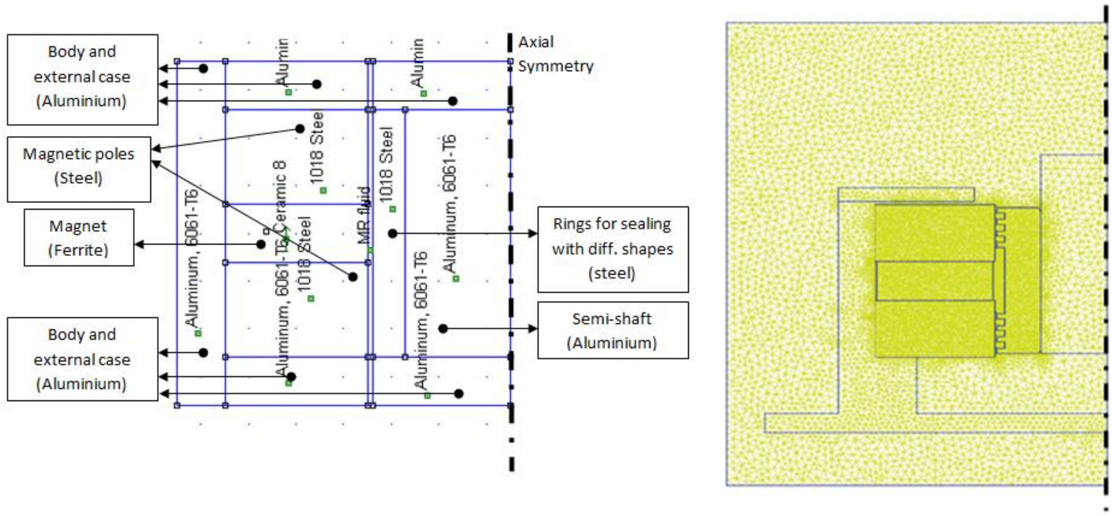


Figure 7 Magnetic fluid seal magnetic model with parts description and detail of mesh.

Magnetic field simulations were carried out with the open source software FEMM (http://www.femm.info). The minimum, maximum and average values of the magnetic field and the magnetization in the fluid for different rings shape (a, b, c and d), and in five control points (shown in Figure 6-a) are summarized in Table 1.

Table 1 Magnetic flux density (B-Tesla) values in the fluid.

	Control points in the model							
Shape	1	2	3	4	5	B_{min}	B_{max}	B_{ave}
Plain	0.4167	0.4719	0.4661	0.4620	0.3443	0.3433	0.4760	0.4167
Square	0.4481	0.3770	0.4641	0.5026	0.3832	0.3591	0.5105	0.4398
Saw	0.4885	0.4206	0.4603	0.4127	0.3534	0.3534	0.4939	0.4236
Triang.	0.4620	0.4508	0.4459	0.4424	0.3486	0.3486	0.4729	0.4107

Considering the higher magnetic flux density achieved (from 0.359T to 0.510T), square rings were selected for the development of a magnetic fluid seal prototype.

3.2. Sealing capacity of magnetic fluid

The final step in the modelling of magnetic fluids seals was to determine the sealing capacity (pressure difference withstood by the rings) that they could achieve. In the case of MRF based seals, the sealing capacity is defined by the yield stress (function of magnetic field), which was experimentally described in Section 2 by Equation (1) and experimental results summarized in Figure 4.

For ferrofluids, the pressure achievable by each ring of the seal was determined following Equation (4),[18].

$$p_m = \max\left[\int_{H_1}^{H_2} \mu_0 M dH\right] \tag{3}$$

$$p_m \approx \max\left[\mu_0 M_S (H_2 - H_1)\right] \tag{4}$$

Where (p_m) is the pressure in the fluid due to the magnetic field, (μ_0) is the permeability in the vacuum, (M_s) the saturation magnetization of the fluid and (H) the magnetic field intensity, H_1 and H_2 are the values of the magnetic field intensity on both free ferrofluid surfaces within one sealing stage.

In Equation (3), M is function of H, as shown in Figure 8. If the variation of magnetization with field is neglected at high fields within the bearing seal, and the magnetization is put equal to saturation magnetization (M_s), than one can use Equation (4). The maximum pressure corresponds to the maximal difference of fields H_1 and H_2 and is calculated imposing a certain volume of the ferrofluid ring and displacing this volume along the shaft direction by small steps.

The model was fed by the magnetic characterization of the ferrofluid (section 2) and the magnetic field simulation carried out in section 3.1. Finally, the sealing capacity is determined calculating the integral in Eq. (3) as presented graphically in Figure 8, and finding its maximum for a given volume of the ferrofluid contained in one sealing stage:

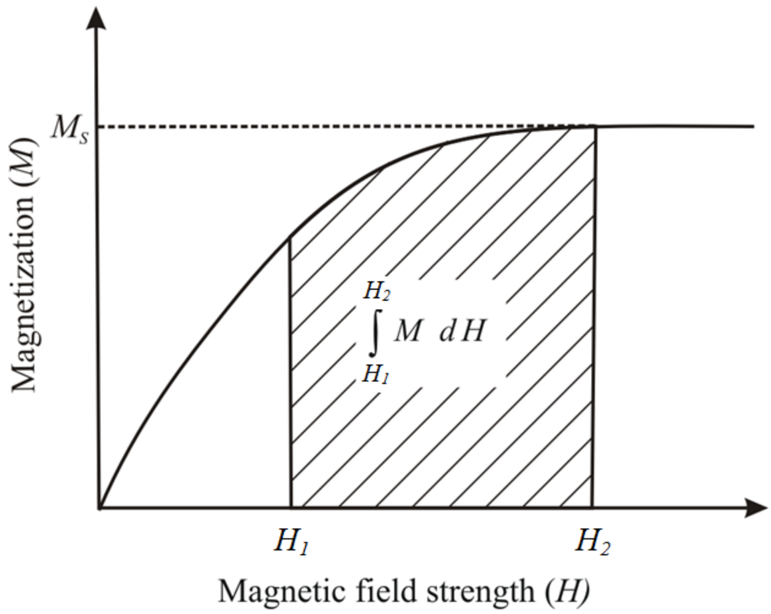


Figure 8 Fluid magnetization and magnetic field strength, pressure computation (Equation (3)).

The nature and therefore the response of MRF are different from ferrofluids so the calculations were based on the yield stress in the rings and the average pressure achievable by the seal.

Once the yield stress was obtained (from magnetic simulations and fluids experimental analysis), the pressure in the ring was achieved with the computation of Equation (5):

$$F_m = 2\pi\left(R_i\int \tau_0(H_i)dz + R_e\int \tau_0(H_e)dz\right) \quad (5)$$

Where R_i and R_e are the shaft radius and the internal radius of the magnetic poles; H_i and H_e – the magnetic field intensity on the shaft and magnetic pole surfaces. If the field H does not change significantly across the gap of the seal ($H_i \approx H_e \approx H$), Equation (6) is applied:

$$F_m = 2\pi(R_i + R_e)\int \tau_0(H)dz \qquad (6)$$

Finally, the fluid magnetic force *(F_m)* was divided by the annular area *(A_r)* of the ring and gapto get the sealing pressure *(p_m)*, or sealing capacity, as presented in Equation (7).

$$p_m = {F_m}/{A_r} \qquad (7)$$

4. DESING OF SEAL PROTOTYPE AND TEST BENCH

The experimental analysis was carried on a test bench (see Figure 9) where the prototype of magnetic fluid seal was analysed under variable operating conditions (rotational speed and pressurized air).

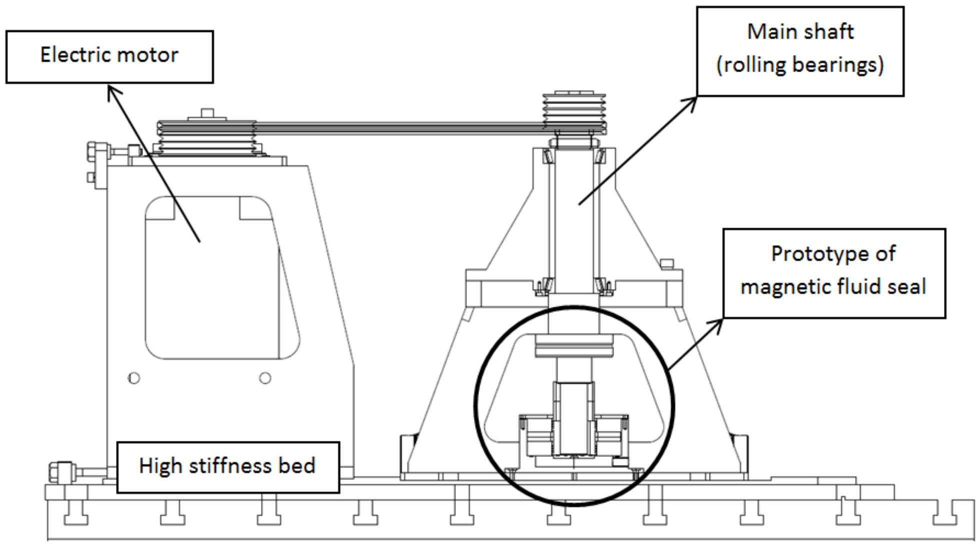


Figure 9 Test bench with the prototype of magnetic fluid based seal.

The magnetic fluid based seal prototype was located in the semi-shaft, which can rotate up to 3000 rpm with a run-out below 5µm. The pressurization of the rings, tested with an air inlet in the body, was evaluated in function of the rotational speed.

Based on the results from magnetic calculations, a prototype of magnetic fluid seal was designed. This prototype is analysed in the test bench presented in Figure 9, assembled in the area marked with a circle. In Figure 10 a section view of the prototype with main dimensions is presented.

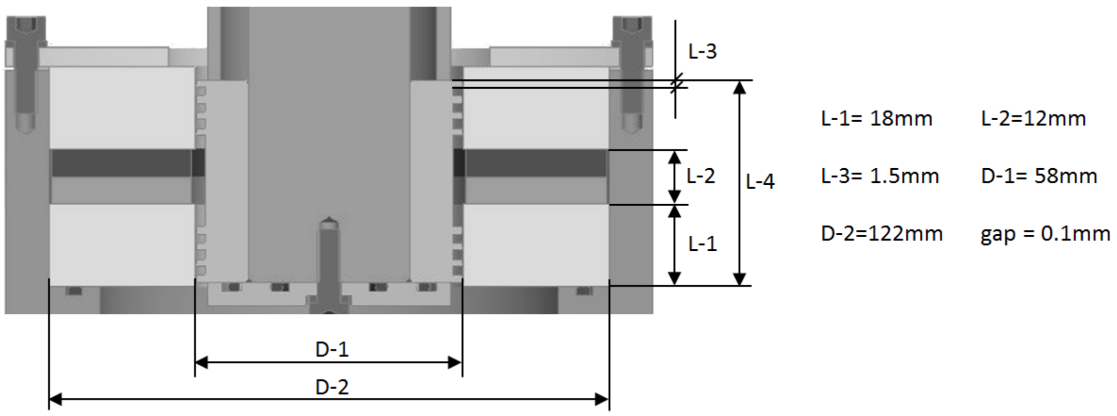
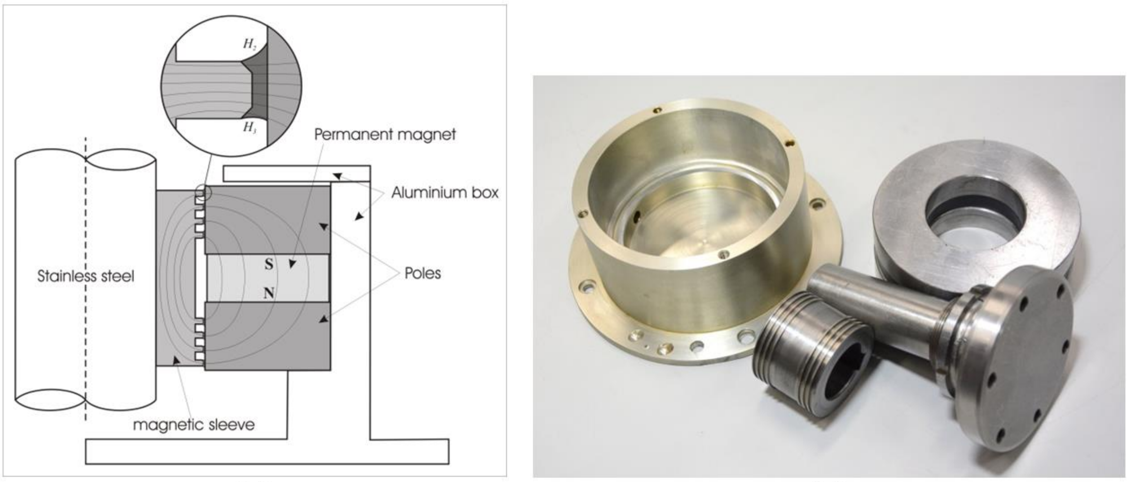


Figure 10 Magnetic fluids sealing, main dimensions

The magnetic field was created with a strontium ferrite permanent magnet located in the sealing body and having an axial polarization (120mm external diameter, 60mm internal diameter and 12mm thickness). The average magnetic field strength of the magnet (given by manufacturer) was around 260kA/m, when flux density achieved is around 0.4T. This magnetic field was guided with two annular poles towards the rings of the sealing, 8 rings in this case as shown in Figure 10. The magnetic field in the rings and a photo of a disassembled seal are shown in Figure 11.



(a) (b)

Figure 11 Magnetic fluids sealing, working principle (a) and manufactured set-up (b)

5. RESULTS AND DISCUSSION

The experimental tests to analyse the behaviour of magnetic fluids based seals are presented in this section. The section is structured in three parts: Analysis of the friction torque, sealing capacity of MRF and sealing capacity of ferrofluids.

5.1. Friction torque analyse

Friction torque was evaluated for two different magnetic fluids, FERROTEC APG s10n and LORD MRF122. In addition, two other magnetic fluids were tested: a dilution of 30% and 50% of MRF 122. Finally, a lubricant oil fluid (viscosity ISO-VG 46) and the torque without any fluid were also tested to determine friction of the kinematic chain. Results are shown in Figure 12 . The friction torque with MRF fluids, even under diluted concentration, is very high, between 2 and 8 N·m at 100rpm, too large to be used as sealing for spindles of machine tools. The torque with ferrofluid in the seal was slightly higher than base friction, around 0.25N·m, at the same rotation speed of the shaft eligible to be used in machine tool applications.

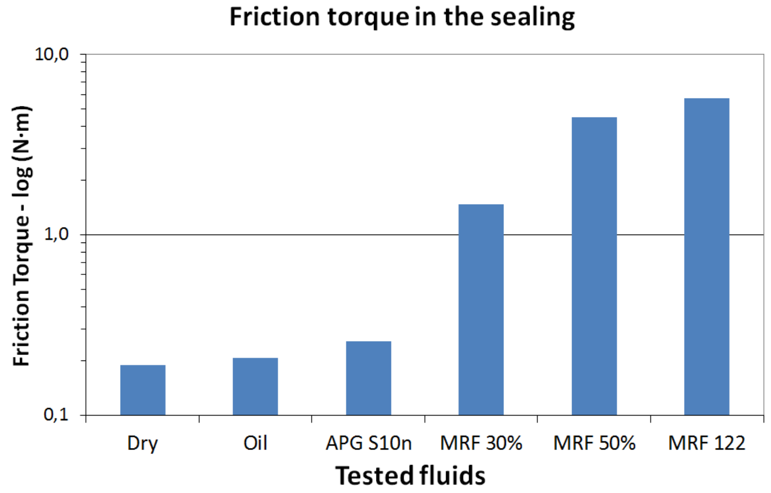


Figure 12 Friction torque in the sealing with different magnetic fluids at 100rpm.

5.2. MRF based sealing results

Tests were carried out to determine the sealing capacity of the system, evaluated as the pressure difference that it can sustain. The experimental procedure was the same for both magnetic fluids, MRF and ferrofluids (section 5.3), and it was the following the scheme shown in the Figure 13: the sealing body was filled gradually with compressed air, controlling the inlet air with a flow valve. The flow rate and pressure were measured with a flowmeter and a pressure gauge, respectively. This set-up was used for the experimental validation of the magnetic seals, and the characterization of sealing capacity of the tested magnetic fluids.

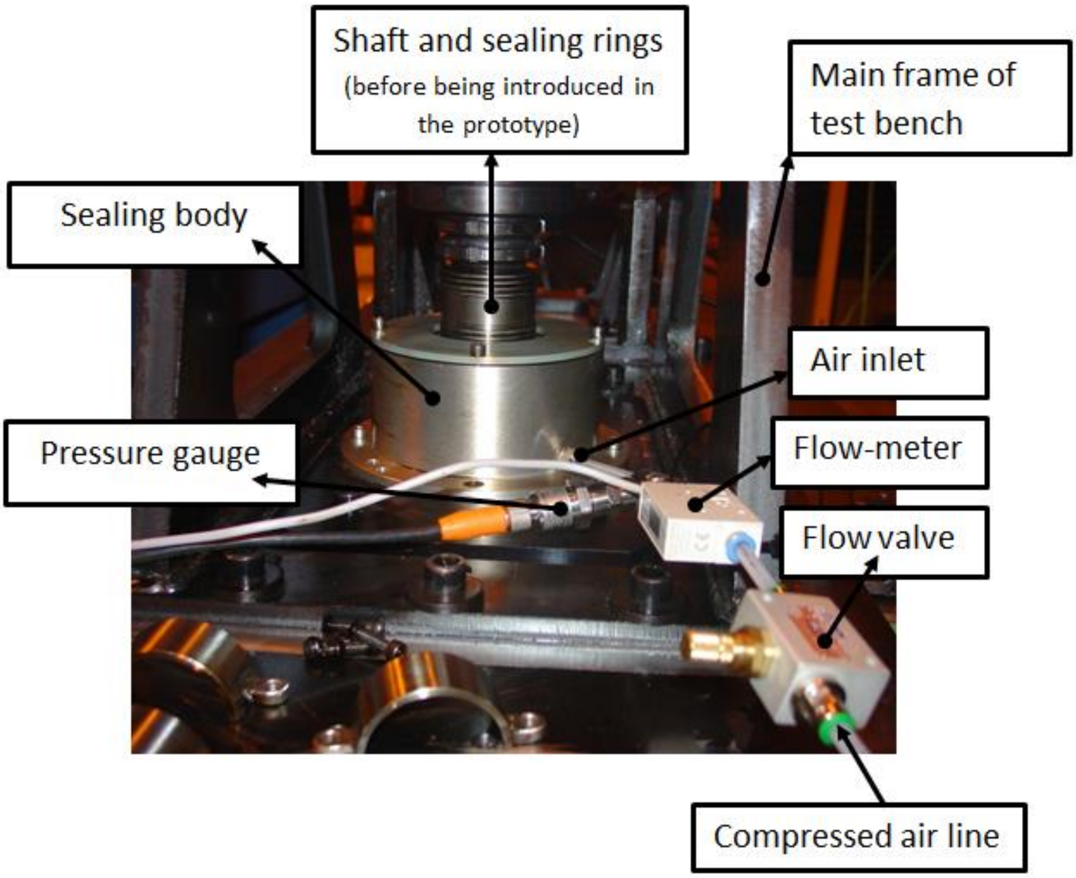


Figure 13 Friction torque in the sealing with different magnetic fluids.

Once the maximum pressure was reached two behaviours were shown: In the case of MRF the pressure stayed constant, and with the ferrofluids the magnetic seals broke (falling down the pressure) and the flowrate increased instantaneously. After a recovery time the seal was formed again and the air pressure increased again till the previous limit (maximum pressure) periodically.

The prototype was composed of eight sealing rings, therefore the sealing capacity of each ring, evaluated with the integration of equation (4) for ferrofluids and Equation **Erreur ! Source du renvoi introuvable.** and **Erreur ! Source du renvoi introuvable.** for MRF, it was multiplied by eight. The results from model and the experimental tests are summarized and compared in Table 2:

Table 2 MRF based seal, sealing capacity (Pa).

Sealing capacity (kPa)		**MRF 122 (dilution 30%)**		**MRF 122**	
		per ring	*Seal*	*per ring*	*Seal*
Calculated sealing capacity	Min	12.36	98.88	41.2	336.8
	Max	15.75	126.0	52.5	420.0
Experimental value in the seal (shaft at rest)		----	130.0	----	320.0

In the case of MRF the pressure for each ring goes from 41.2kPa to 52.5kPa in function of the magnetic field in the fluid, which as shown in Figure 6, is not constant over the ring. For the case of 30% dilution, assuming a proportional reduction of yield stress, and therefore the sealing capacity, the theoretical values go from 12.36kPa to 15.75kPa. Summarizing, the theoretical sealing capacity of MRF for all system with eight rings was: 336.8Pa (3.3bar) to 420kPa (4.2bar), and for 30% diluted MRF: 98.9kPa (0.99bar) and 126kPa (1.26bar).

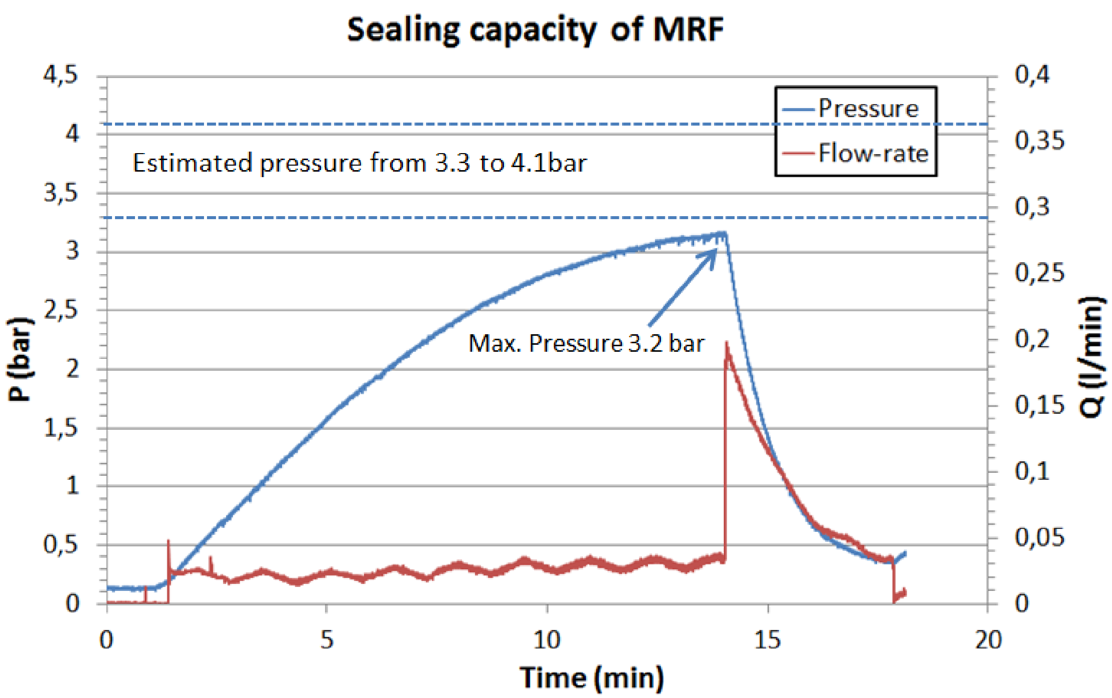


Figure 14 Seal capacity of the system with MRF.

As shown in the Figure 14, the pressure achieved within the tests was around 3.2bar slightly lower than the expected values, which go from 3.3 to 4.1bar following the simulations and calculations described above. Due to the high friction torque of MRF, a diluted fluid at 30% was analysed, where the results are shown in Figure 15. In this case the experimental value matches with expected pressure, from 0.99 to 0.126bar.

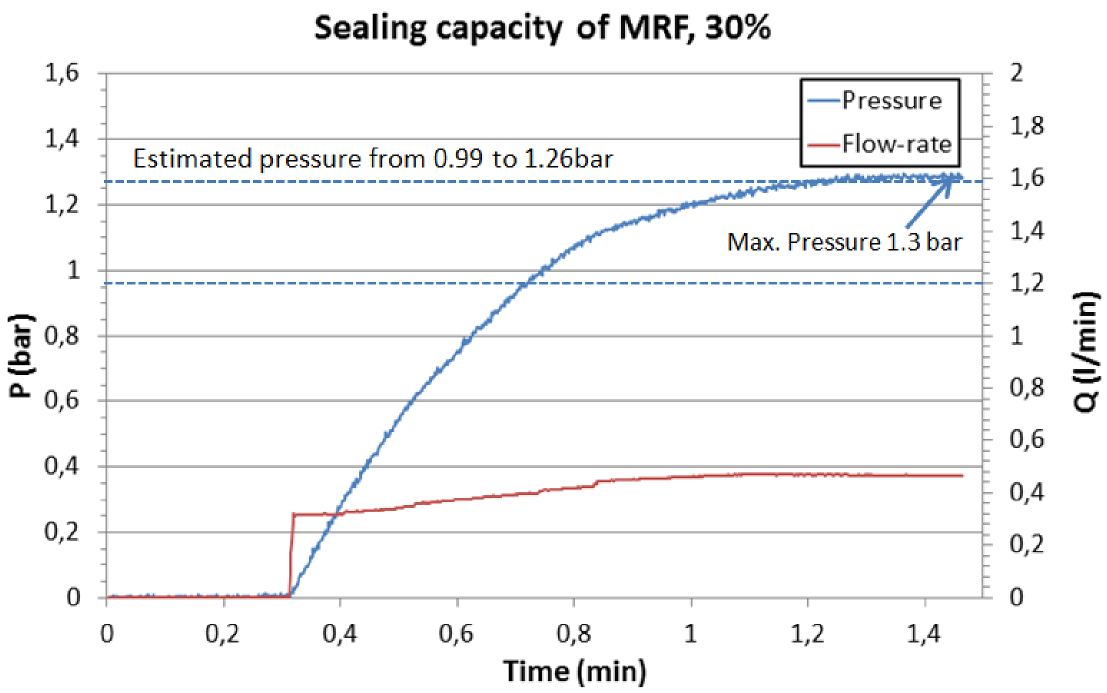


Figure 15 Seal capacity of the rings with MRF at 30% dilution.

Those results were obtained at rest because the pressure in the rings was the same for analysed rotational speed, up to 100rpm. Rupture and recovery pressure were not detected as in ferrofluid based seals, (see section 5.3).

MRF based seals have high sealing capacity, but the friction torque presented in the system is not compatible with machine tool spindle application.

5.3. Ferrofluid based seals results

In Figure 16 the pressure value estimated in each one of the ring filled with ferrofluids is shown, where as calculated the average pressure value was around 9kPa, which means a total pressure capacity (by eight), around 72kPa (0.72bar) in the prototype. The summary of results is presented in Table 3:

Table 3 Ferrofluid based seal, sealing capacity (Pa).

Sealing capacity (kPa)		**Ferrofluid**	
		per ring	*Seal*
Calculated sealing capacity		9	72
Experimental (at rest)	Max	----	70
	Recovery	----	48
Experimental (50r/min)	Max	----	68
	Recovery	----	58
Experimental (100r/min)	Max	----	60
	Recovery	----	58

Recovery pressure, as shown in Figure 17, was the pressure where the magnetic seals (after rupture at maximum pressure) got again pressurization, recovering sealing capacity. These values are a very good pressurization capacity to be used in machine tool's spindles, which together with the low friction torque in the shaft it could be considered as a good candidate for industrial application.

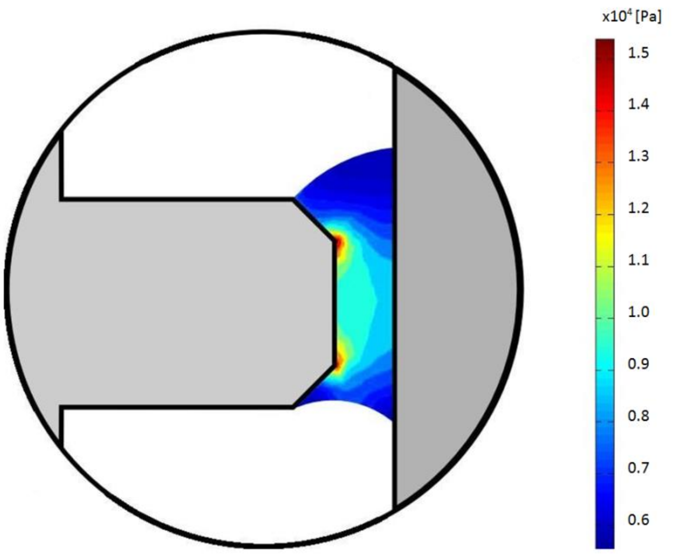


Figure 16 Sealing capacity of each ring with ferrofluid FERROTEC APG s10n.

In Figure 17, Figure 18 and Figure 19 the evolution of air pressure and flow rate for 0 rpm, 50rpm and 100rpm respectively are shown.

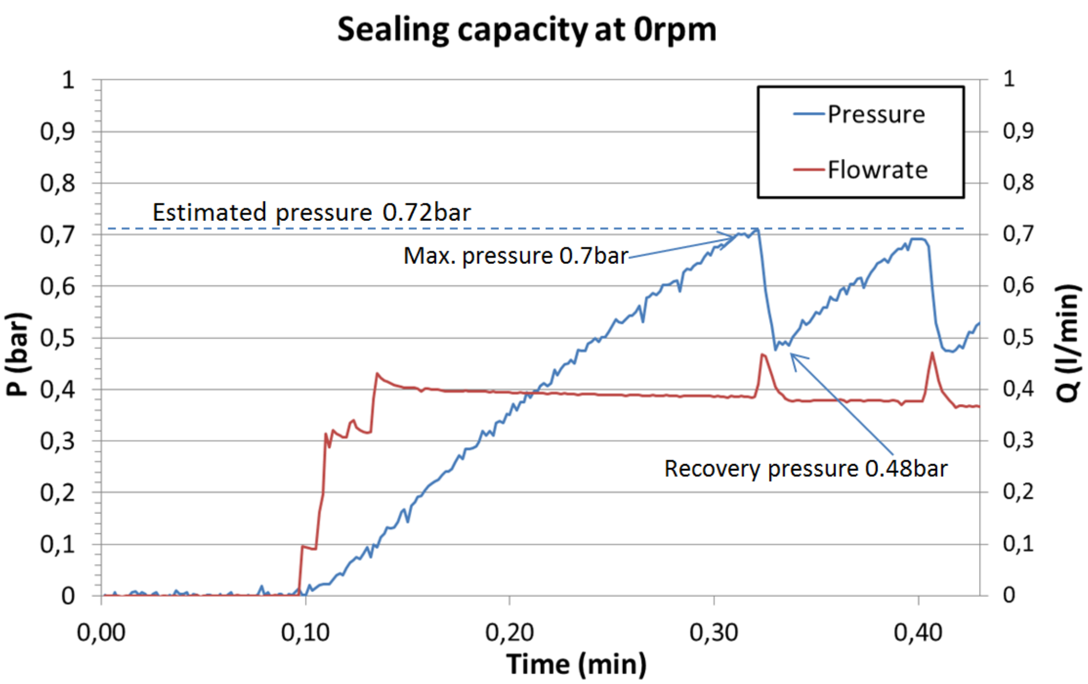


Figure 17 Sealing capacity at rest with FERROTEC APG s10n

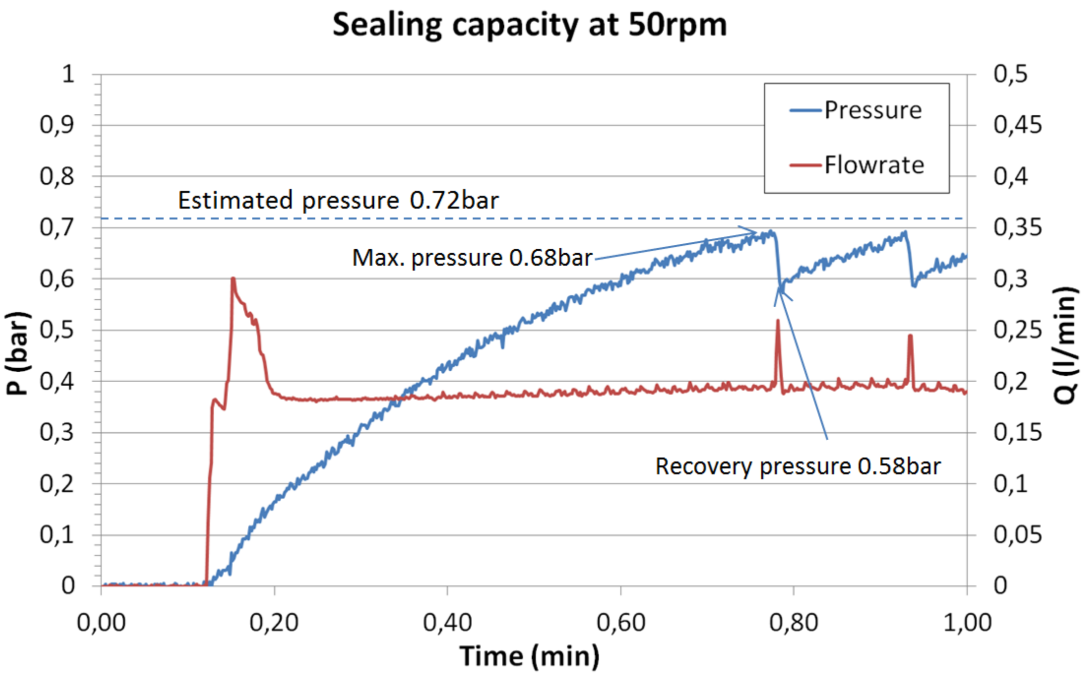


Figure 18 Sealing capacity at rest with FERROTEC APG s10n

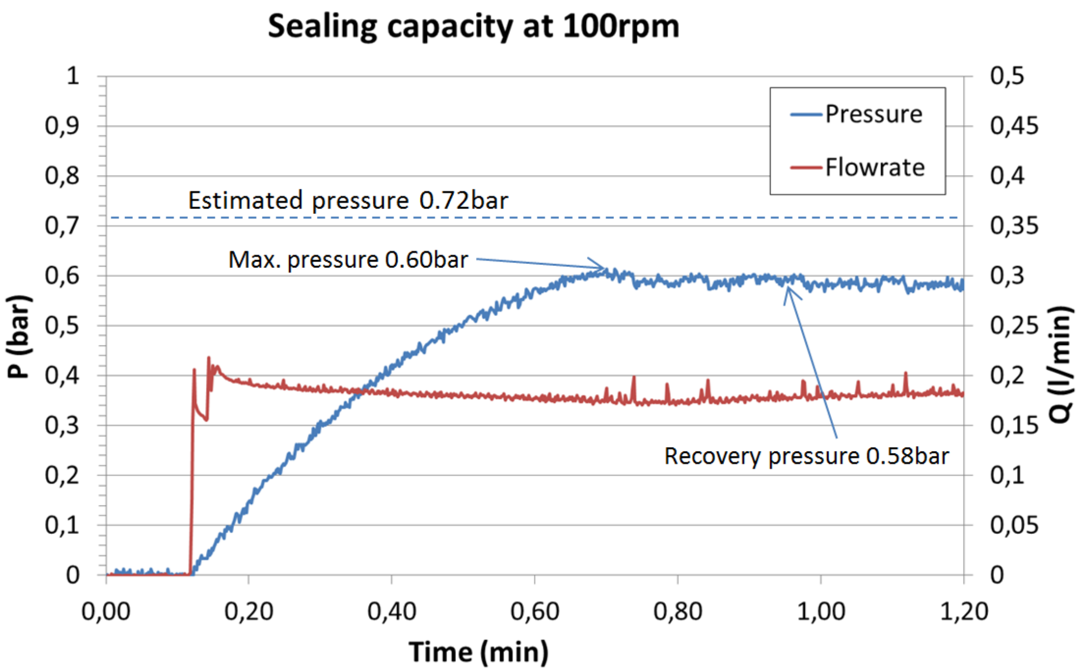


Figure 19 Sealing capacity measured in the test bench with FERROTEC APG s10n.

As shown in Figure 17, the maximum pressure of the sealing matched quite well with the estimated value, around 72000Pa. When the shaft was rotating (see Figure 18), the difference between maximum pressure and recovery pressure was reduced, and at 100r/min (see Figure 19), both pressures were almost equal, converging at a value around 0.6bar. Above this rotational speed it was not noted variations in the results.

6. CONCLUSIONS

The use of magnetic fluids as active elements in seals was analysed in this research work. A close to industrial application prototype has been developed, and the validity of the simulation models and the final performance of the systems were demonstrated experimentally.

- Two magnetic fluids (a ferrofluid and a MRF) were experimentally analysed, obtaining deeper information about their behaviour than that offered by the manufacturers. Such a detailed experimental characterization is highly recommended for any studies in the field of magnetic fluids.
- MRF and ferrofluid based seal models were developed, simulating the magnetic field and the sealing capacity measured in pressure per ring.
- The seal based on MRF showed very good sealing performance (over 3 bar) but with an excessive friction torque (8Nm). Even diluted at 30%, sealing was still good but the torque was too high for machine tool applications.
- The seal based on ferrofluids demonstrated a good enough sealing capacity (0.9bar) and with a friction torque slightly higher than without the seal.
- The seal based on ferrofluids showed thus true potential for industrialization in machine tool spindles.

7. REFERENCES

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