Estimation of Parsimonious Covariance Models for Gaussian Matrix Valued Random Variables for Multi-Dimensional Spectroscopic Data
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Introduction

Satellite remote sensing makes it possible to observe landscapes on large spatial scales. The Sentinel-1 and Sentinel-2 satellites currently provide full coverage of the national territory of France every 5 days. Due to the orbit of the satellites, coupled with the presence of clouds, the sampling of the pixels are temporally irregular. The project aims to develop, study and implement supervised and unsupervised classification methods when the data are of different natures (heterogeneous) and have missing and / or aberrant data. The methods implemented are developed to process satellite and aerial data for ecology and cartography.

Classification

Figure 1. Classification of the Satellite Data

The Model

We model the data as \( \mathbf{Y} \sim N(\mu, \Sigma) \), where \( \mathbf{Y} \) represents the pixel (taking into account the spectra and time sampling) modeled as a Normal distribution with mean \( \mu \), and covariance matrix \( \Sigma \).

The covariance matrix \( \Sigma \) can be estimated as:

- Full model (Naive)
  - Number of parameters is: \( pd + \frac{pd(pd+1)}{2} \), which is huge, where \( p \) is the no. of spectra and \( d \) is the number of time intervals.

- Parsimonious model
  - \( \Sigma = \Sigma_s \otimes \Sigma_T \)
  - Kronecker product of \( \Sigma_s \) and \( \Sigma_T \), where covariance of spectra \( \Sigma_s \) is a \( d \times d \) matrix and covariance of time \( \Sigma_T \) is a \( p \times p \) matrix.

- Number of parameters reduces to: \( pd + \frac{(pd+1)(pd+2)}{2} \).

We can assume:

- Independent model
  - Spectrum independence
  - Time independence

- Kernel model
  - Gaussian
  - Exponential
  - Quadratic
  - Circular
  - Uniform

- Unknown model
  - When distribution of both \( \Sigma_s \) and \( \Sigma_T \) is unknown, we use Flip-Flop Algorithm, using Maximum Likelihood Estimation until convergence of \( \Sigma_s \) and \( \Sigma_T \).

The code is available at: https://github.com/asmitapoddar/BayesSentinel

References