Bayesian Nonparametric Mixtures Why and How?
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Bayesian Nonparametric Mixtures
Why and How?

www.julyanarbel.com, Inria, Mistis

Introduction
Bayesian nonparametric framework
- Massively many parameters
- Inference on curves: pdf, cdf, hazard, link…
- Mixtures, exchangeable data \( X^n = (X_1, \ldots, X_n) \)
  
  \[
  X_1, \ldots, X_n | P \sim \left\{ \begin{array}{ll}
  P & \rightarrow 1 \\
  \int k(\cdot | \theta) P(d\theta) & \rightarrow 2 \end{array} \right. \]
- Natural uncertainty quantification
- Flexibility, avoids over-fitting by regularization (prior)
- Adapt to data complexity
- Underlying clustering
- Justify prior, expert
- Efficient posterior sampling
- Quantify truncation error

What prior for \( P \)?
- Learn about data through posterior dist.
- Discrete random probability measure prior
- Random weights \( (p_i) \) and locations \( (\theta_i) \)
  
  \[
  P = \sum_{i=1}^{\infty} p_i \delta_{\theta_i} 
  \]
- Dirichlet process \( DP(\alpha, G_0) \) (Ferguson, 1973)
  Predictive: Chinese Restaurant Process
  
  \[
  \mathbb{P}(X_{n+1} \in \cdot | X^n) = \frac{\alpha}{\alpha + n} G_0 + \frac{1}{\alpha + n} \sum_{j=1}^{n} p_j \delta_{\theta_j}
  \]
- Or for varying \( \mathbb{P}(X_{n+1} \text{ new } \cdot \cdot \cdot \cdot \cdot) \)

Survival Analysis
Bayesian hazard mixture (Arbel et al., 2016c)
- Data are (remission) times possibly censored
- Prior on hazard rate \( h(t) \) for every time \( t \)
- Induces prior on survival function \( S(t) \)
  - Availability of post. mean, median, mode
  - Smooth estimator \( \hat{V} \) Kaplan–Meier
  - Proper uncertainty quantification

Open Questions
- How to best use underlying clustering? (Wade and Ghahramani, 2015)
- Find consistent estimator of number of clusters: posterior inconsistent (Miller and Harrison, 2014), what about posterior mode?
- Devise efficient posterior sampling, truncation error (Arbel and Prünster, 2016)

Species Modeling
Data can be species, microbes, words, genes…

Discovery probabilities (Arbel et al., 2016a)
- Estimation of \( \ell \)-discovery
  
  \[
  D_{\ell} \equiv \mathbb{P}(X_{n+1} \text{ is a seen \( \ell \) times})
  \]
- Comparison with Good-Turing estimator
  - Closed form posterior and estimators
  - Uncertainty quantifi., unavailable for GT
  - 2nd order (fast) approximations

Diversity in ecology (Arbel et al., 2015, 2016d)
- Assess impact of pollution on microbial community via study of diversity
  
  \[
  \text{Div} = -\sum_i p_i \log p_i
  \]
- Model detects an hormetic effect
- Uncertainty quantification
- Prediction across full range of covariates

Density Estimation
Ecological risk assessment (Arbel et al., 2016b)
- Data are species critical effect concentrations (CEC), possibly censored
- Estimation of species sensitivity distribution (SSD), the density of CEC
- Safe concentration which protects most of the species: 5th percentile of the SSD (HC5)
- Very moderate sample sizes, \( \sim 10 \rightarrow 50 \)
- BNP describes well variability of the data, without being prone to over-fitting
- Species clustering as an outcome

References & Collaborators
Bayesian nonparametric inference for discovery probabilities: credible intervals and large sample asymptotics.
Statistics Sinica.

Bayesian nonparametric modelling of species sampling distributions. In preparation.
Computational Statistics & Data Analysis.


