Bayesian Nonparametric Mixtures Why and How?
Julyan Arbel

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Bayesian Nonparametric Mixtures
Why and How?

www.julyanarbel.com, Inria, Mistis

Introduction

Bayesian nonparametric framework
- Massively many parameters
- Inference on curves: pdf, cdf, hazard, link...
- Mixtures, exchangeable data $X' = (X_1, \ldots, X_n)$

\[ X_1, \ldots, X_n \mid P \sim \left\{ \begin{array}{c} P \sum \delta_{\theta_i} (\theta) \bigg\}\rightarrow (1) \\
\end{array} \right. \]

- Natural uncertainty quantification
- Flexibility, avoids over-fitting by regularization (prior)
- Adapt to data complexity
- Underlying clustering
- Justify prior, expert
- Efficient posterior sampling
- Quantify truncation error

What prior for $P$?
- Learn about data through posterior dist.
- Discrete random probability measure prior
- Random weights $(p_i)$, and locations $(\theta_j)$

\[ P = \sum_{i=1}^{\infty} p_i \delta_{\theta_i} \rightarrow \text{Dirichlet process DP}(\alpha, G_0) \] (Ferguson, 1973)

Predictive: Chinese Restaurant Process

\[ P(X_{n+1} \in \cdot \mid X_n) = \frac{\alpha}{\alpha + n} G_0 + \frac{1}{\alpha + n} \sum_{j=1}^{n} p_j \delta_{\theta_j} \]

- Or for varying $P(X_{n+1} \text{ new } \in \cdot) \rightarrow \text{Bayesian Nonparametric Mixtures (BNP)}$

Survival Analysis

Bayesian hazard mixture (Arbel et al., 2016c)
- Data are (remission) times possibly censored
- Prior on hazard rate $h(t)$ for every time $t$
- Induces prior on survival function $S(t)$
- Availability of post. mean, median, mode
- Smooth estimator VS Kaplan–Meier
- Proper uncertainty quantification

Open Questions
- How to best use underlying clustering? (Wade and Ghahramani, 2015)
- Find consistent estimator of number of clusters: posterior inconsistent (Miller and Harrison, 2014), what about posterior mode?
- Devise efficient posterior sampling, truncation error (Arbel and Prünster, 2016)

Species Modeling

Data can be species, microbes, words, genes...

Discovery probabilities (Arbel et al., 2016a)
- Estimation of $\ell$-discovery
- Comparison with Good-Turing estimator
- Closed form posterior and estimators
- Uncertainty quantification, unavailable for GT
- 2nd order (fast) approximations

Diversity in ecology (Arbel et al., 2015, 2016d)
- Assess impact of pollution on microbial community via study of diversity

\[ \text{Div} = - \sum_i p_i \log p_i \]
- Model detects an hormetic effect
- Uncertainty quantification
- Prediction across full range of covariates

Density Estimation

Ecological risk assessment (Arbel et al., 2016b)
- Data are species critical effect concentrations (CEC), possibly censored
- Estimation of species sensitivity distribution (SSD), the density of CEC
- Safe concentration which protects most of the species: 5th percentile of the SSD (HC5)
- Very moderate sample sizes, $\sim 10 \sim 50$
- BNP describes well variability of the data, without being prone to over-fitting
- Species clustering as an outcome

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