Bayesian Nonparametric Mixtures Why and How?
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Bayesian Nonparametric Mixtures
Why and How?

Introduction
Bayesian nonparametric framework
- Massively many parameters
- Inference on curves: pdf, cdf, hazard, link...
- Mixtures, exchangeable data $X^\infty = (X_1, \ldots, X_n)$

$$X_1, \ldots, X_n \mid P \sim \left\{ \frac{P}{\sum_k k! \cdot \theta(k \cdot P(\theta))} \rightarrow 1 \right\}$$

- Natural uncertainty quantification
- Flexibility, avoids over-fitting by regularization (prior)
- Adapt to data complexity
- Underlying clustering
- Justify prior, expert
- Efficient posterior sampling
- Quantify truncation error

What prior for $P$?
- Learn about data through posterior dist.
- Discrete random probability measure prior
- Random weights $(p_i)$, and locations $(\theta_i)$

$$P = \sum_{i=1}^{\infty} p_i \delta_{\theta_i}$$

→ Dirichlet process DP($\alpha, G_0$) (Ferguson, 1973)
Predictive: Chinese Restaurant Process

$$P(X_{n+1} \in \cdot \mid X^n) = \frac{\alpha}{\alpha + n} G_0 + \frac{1}{\alpha + n} \sum_{j=1}^{n} p_j \delta_{\theta_j}$$

→ Or for varying $P(X_{n+1} \text{ new} \ldots)$

Survival Analysis
Bayesian hazard mixture (Arbel et al., 2016c)
- Data are (remission) times possibly censored
- Prior on hazard rate $h(t)$ for every time $t$
- Induces prior on survival function $S(t)$
→ Availability of post. mean, median, mode
→ Smooth estimator VS Kaplan–Meier
→ Proper uncertainty quantification

Open Questions
- How to best use underlying clustering? (Wade and Ghahramani, 2015)
- Find consistent estimator of number of clusters: posterior inconsistent (Miller and Harrison, 2014), what about posterior mode?
- Devise efficient posterior sampling, truncation error (Arbel and Prünster, 2016)

Density Estimation
Ecological risk assessment (Arbel et al., 2016b)
- Data are species critical effect concentrations (CEC), possibly censored
- Estimation of species sensitivity distribution (SSD), the density of CEC
- Safe concentration which protects most of the species: 5th percentile of the SSD (HC50)
- Very moderate sample sizes, $\sim 10 \sim 50$
- BNP describes well variability of the data, without being prone to over-fitting
- Species clustering as an outcome

Species Modeling
Data can be species, microbes, words, genes…

Discovery probabilities (Arbel et al., 2016a)
- Estimation of $\ell$-discovery
- Comparison with Good-Turing estimator
- Closed form posterior and estimators
- Uncertainty quantif., unavailable for GT
- 2nd order (fast) approximations

Diversity in ecology (Arbel et al., 2015, 2016d)
- Assess impact of pollution on microbial community via study of diversity
- $Dii = P(X_{n+1} \text{ is a species seen } \ell \text{ times})$
- Model detects an hormetic effect
- Uncertainty quantification
- Prediction across full range of covariates

References & Collaborators


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