Bayesian Nonparametric Mixtures Why and How?
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Bayesian Nonparametric Mixtures
Why and How?

www.julyanarbel.com, Inria, Mistis

Introduction

Bayesian nonparametric framework

- Massively many parameters
- Inference on curves: pdf, cdf, hazard, link...
- Mixtures, exchangeable data \( X^n = (X_1, \ldots, X_n) \)

\[
X_1, \ldots, X_n \mid P \sim \left\{ \frac{P}{\mathcal{L}(\cdot \mid \theta) P(d\theta)} \right\} \rightarrow 1 \quad \text{(1)}
\]

- Natural uncertainty quantification
- Flexibility, avoids over-fitting by regularization (prior)
- Adapt to data complexity
- Underlying clustering
- Justify prior, expert
- Efficient posterior sampling
- Quantify truncation error

What prior for \( P \)?

- Learn about data through posterior dist.
- Discrete random probability measure prior
- Random weights \( (p_i) \) and locations \( (\theta_i) \)

\[
P = \sum_{i=1}^{\infty} p_i \delta_{\theta_i} \rightarrow \text{Dirichlet process } DP(\alpha, G_0) \quad \text{(Ferguson, 1973)}
\]

Predictive: Chinese Restaurant Process

\[
P(X_{n+1} \in \cdot \mid X^n) = \frac{\alpha}{\alpha + n} G_0 + \frac{1}{\alpha + n} \sum_{j=1}^{n} p_j \delta_{\theta_j}
\]

Or for varying \( P(X_{n+1} \text{ new } \cdot \mid \cdot ) \) ↩

1. Species Modeling

Data can be species, microbes, words, genes...

Discovery probabilities (Arbel et al., 2016a)

- Estimation of \( \ell \)-discovery
- Comparison with Good-Turing estimator
- Closed form posterior and estimators
- Uncertainty quantif., unavailable for GT

Diversity in ecology (Arbel et al., 2015, 2016d)

- Assess impact of pollution on microbial community via study of diversity
- Div = \(-\sum_i p_i \log p_i\)
- Model detects an hormetic effect
- Uncertainty quantification
- Prediction across full range of covariates

2. Density Estimation

Ecological risk assessment (Arbel et al., 2016b)

- Data are species critical effect concentrations (CEC), possibly censored
- Estimation of species sensitivity distribution (SSD), the density of CEC
- Safe concentration which protects most of the species: 5th percentile of the SSD (HC5)
- Very moderate sample sizes, \( \sim 10 \) – \( \sim 50 \)
- BNP describes well variability of the data, without being prone to over-fitting
- Species clustering as an outcome

3. Survival Analysis

Bayesian hazard mixture (Arbel et al., 2016c)

- Data are (remission) times possibly censored
- Prior on hazard rate \( h(t) \) for every time \( t \)
- Induces prior on survival function \( S(t) \)

Availability of post. mean, median, mode
- Smooth estimator VS Kaplan–Meier
- Proper uncertainty quantification

Open Questions

- How to best use underlying clustering? (Wade and Ghahramani, 2015)
- Find consistent estimator of number of clusters: posterior inconsistent (Miller and Harrison, 2014), what about posterior mode?
- Devise efficient posterior sampling, truncation error (Arbel and Prünster, 2016)

References & Collaborators


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