Bayesian Nonparametric Mixtures Why and How?

Julyan Arbel

Bayesian Nonparametric Mixtures
Why and How?

www.julyanarbel.com, Inria, Mistis

Introduction

Bayesian nonparametric framework

- Massively many parameters
- Inference on curves: pdf, cdf, hazard, link...
- Mixtures, exchangeable data $\mathbf{X}^n = (X_1, \ldots, X_n)$

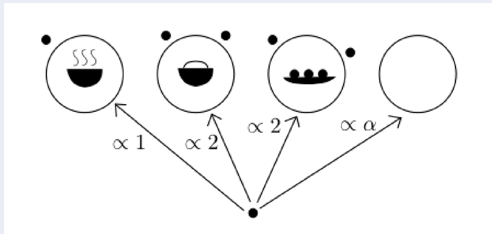
$$X_1, \ldots X_n \mid P \sim \begin{cases} P & \to ① \\ \int_\Theta k(\cdot\mid\theta)P(d\theta) & \to ②③ \end{cases}$$

- ☺ Natural uncertainty quantification
- ☺ Flexibility, avoids over-fitting by regularization (prior)
- ☺ Adapt to data complexity
- ☺ Underlying clustering
- ☹ Justify prior, expert
- ☹ Efficient posterior sampling
- ☹ Quantify truncation error

What prior for P?

- Learn about data through posterior dist.
- Discrete random probability measure prior
- Random weights $(p_i)_i$ and locations $(\theta_i)_i$

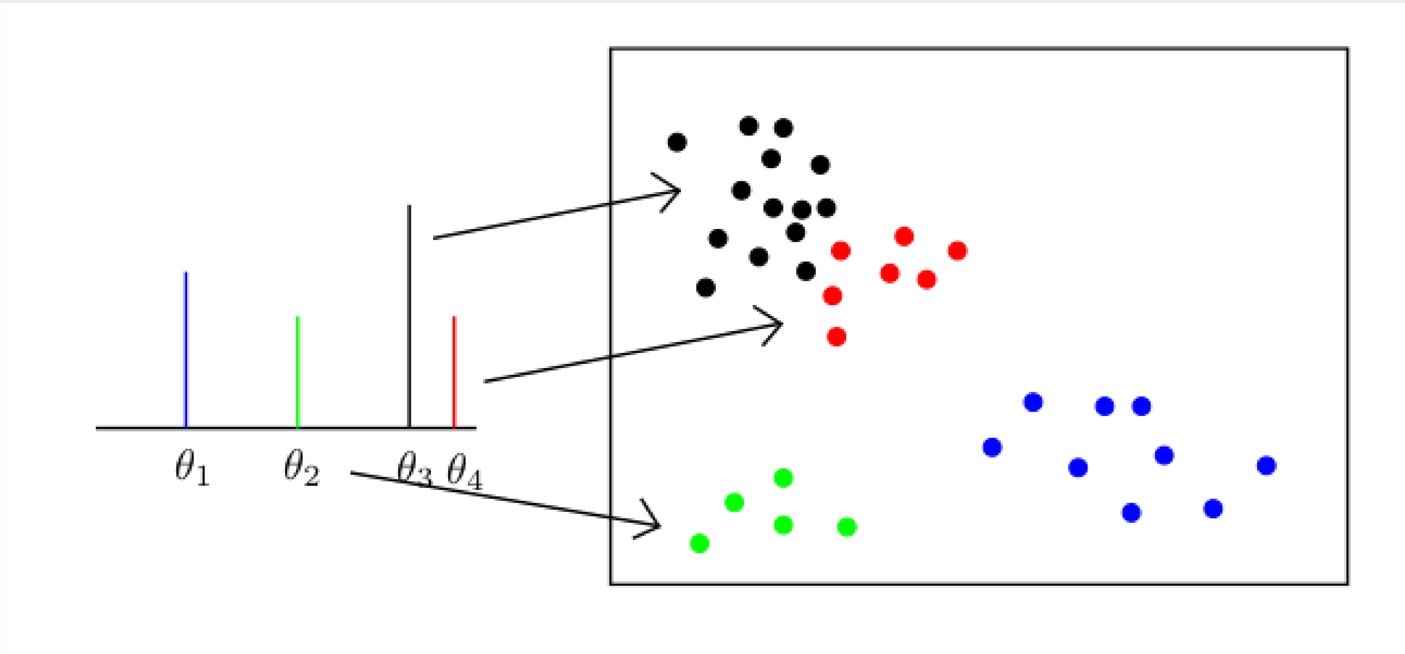
$$P = \sum_{i=1}^{\infty} p_i \delta_{\theta_i}$$

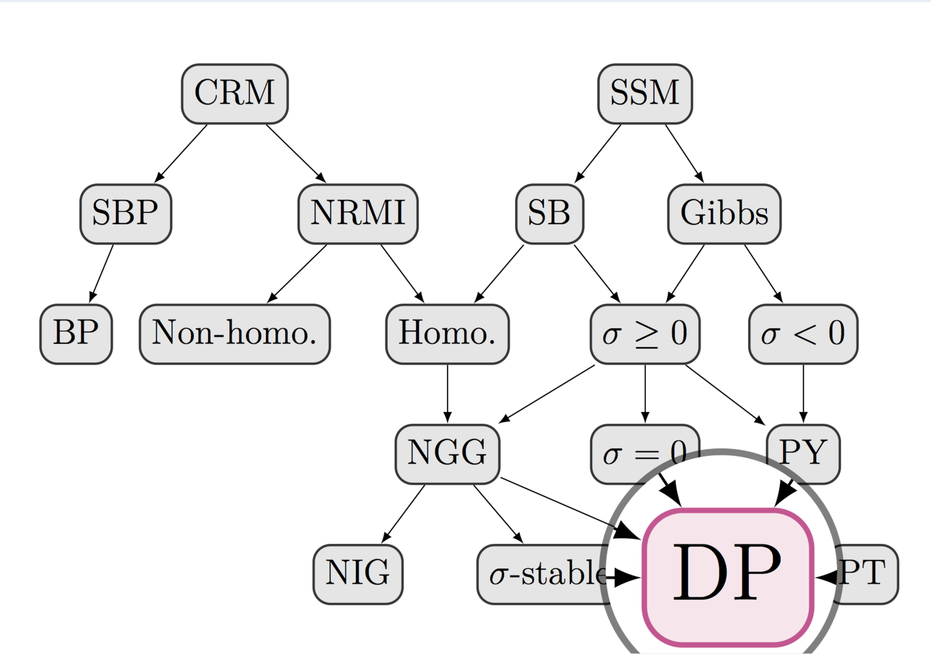


→ Dirichlet process $\mathrm{DP}(\alpha, G_0)$ (Ferguson, 1973)
Predictive: Chinese Restaurant Process

$$\mathbb{P}(X_{n+1} \in \cdot \mid \mathbf{X}^n) = \frac{\alpha}{\alpha+n} G_0 + \frac{1}{\alpha+n}\sum_{j=1}^{k_n} n_j \delta_{X_j^*}$$

→ Or for varying $\mathbb{P}(X_{n+1}\ \text{new})$ ... ↷





① Species Modeling

Data can be species, microbes, words, genes...

Discovery probabilities (Arbel et al., 2016a)

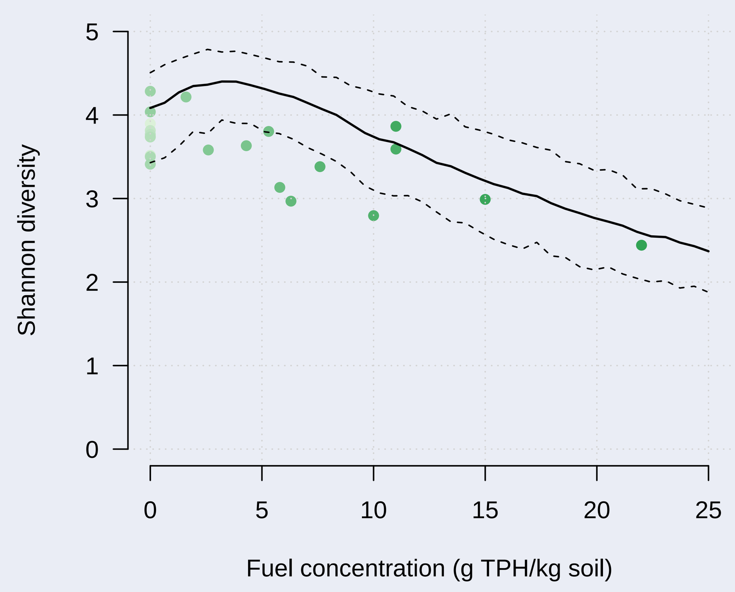
- Estimation of ℓ-discovery
  $D_\ell = \mathbb{P}(X_{n+1}$ is a species seen ℓ times)
- Comparison with Good-Turing estimator

→ Closed form posterior and estimators
→ Uncertainty quantif., unavailable for GT
→ 2nd order (fast) approximations

Diversity in ecology (Arbel et al., 2015, 2016d)

- Assess impact of pollution on microbial community via study of diversity
  $\mathrm{Div} = -\sum_i p_i \log p_i$

→ Model detects an hormetic effect
→ Uncertainty quantification
→ Prediction across full range of covariates



Funding

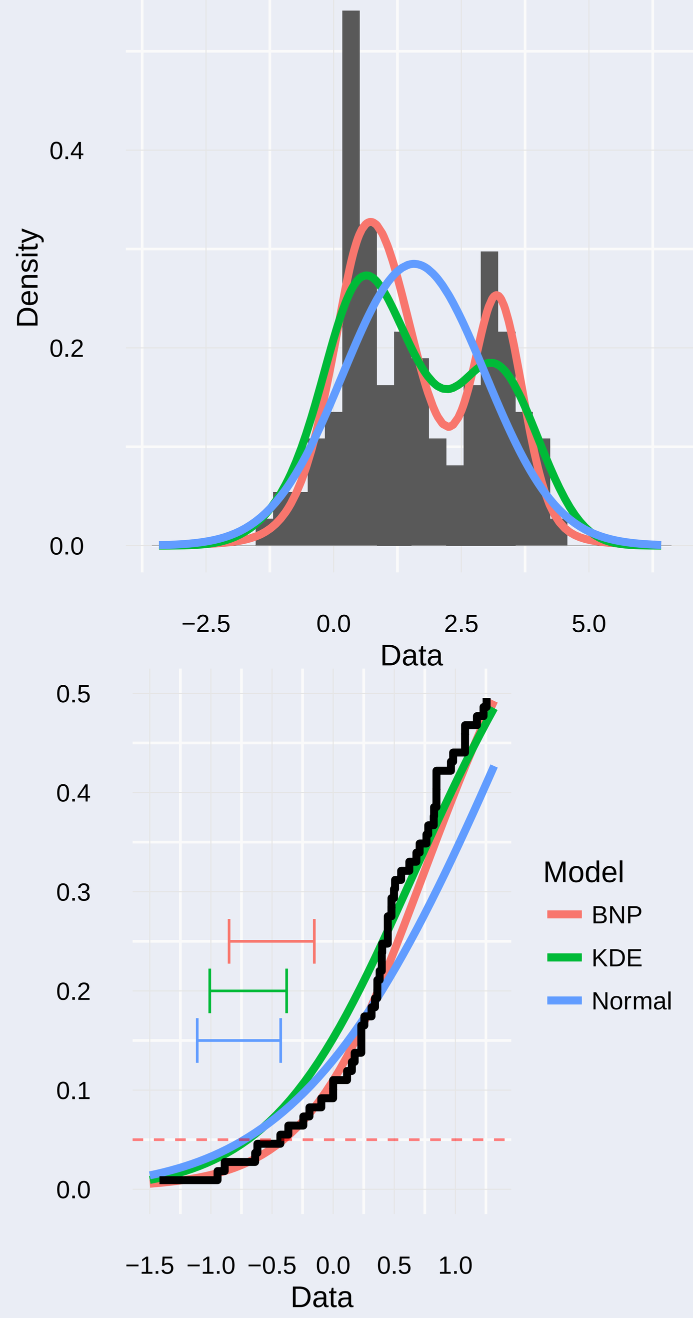
This work is supported by the European Research Council (ERC) through StG "N-BNP" 306406.

② Density Estimation

Ecological risk assessment (Arbel et al., 2016b)

- Data are species critical effect concentrations (CEC), possibly censored
- Estimation of species sensitivity distribution (SSD), the density of CEC
- Safe concentration which protects most of the species: 5th percentile of the SSD (HC_5)
- Very moderate sample sizes, $\sim 10-50$

→ BNP describes well variability of the data, without being prone to over-fitting
→ Species clustering as an outcome

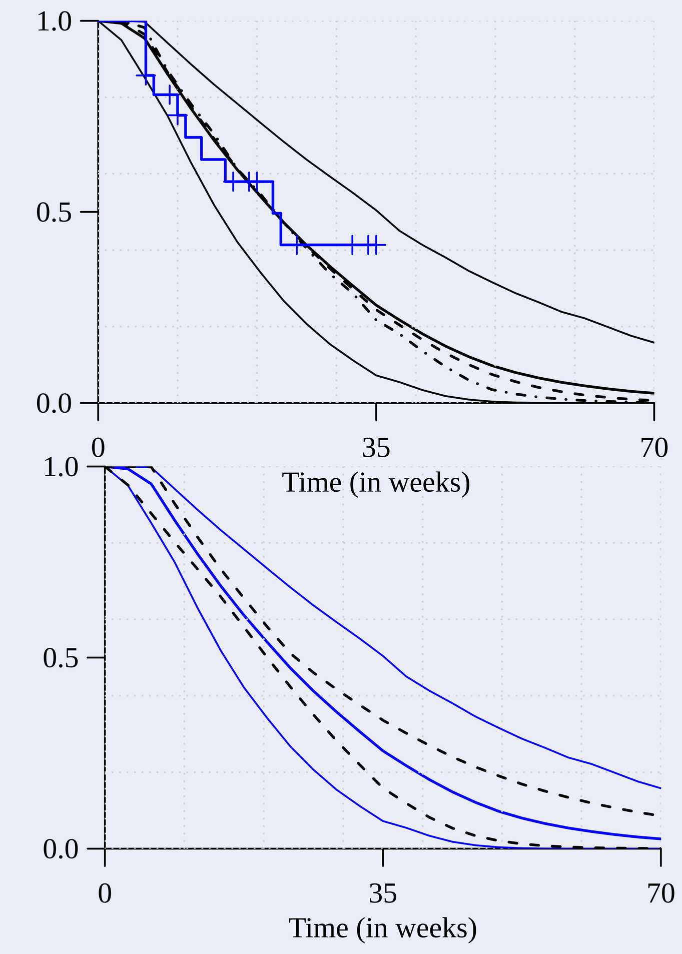


③ Survival Analysis

Bayesian hazard mixture (Arbel et al., 2016c)

- Data are (remission) times possibly censored
- Prior on hazard rate $h(t)$ for every time t
- Induces prior on survival function $S(t)$

→ Availability of post. mean, median, mode
→ Smooth estimator VS Kaplan–Meyer
→ Proper uncertainty quantification



Open Questions

- How to best use underlying **clustering**? (Wade and Ghahramani, 2015)
- Find **consistent** estimator of **number of clusters**: posterior inconsistent (Miller and Harrison, 2014), what about posterior mode?
- Devise efficient **posterior sampling**, truncation error (Arbel and Prünster, 2016)

References & Collaborators

Arbel, J., Favaro, S., Nipoti, B., and Teh, Y. W. (2016a). Bayesian nonparametric inference for discovery probabilities: credible intervals and large sample asymptotics. *Statistica Sinica*.

Arbel, J., Kon Kam King, G., and Prünster, I. (2016b). Bayesian nonparametric modelling of species sampling distributions. *In preparation*.

Arbel, J., Lijoi, A., and Nipoti, B. (2016c). Full Bayesian inference with hazard mixture models. *Computational Statistics & Data Analysis*.

Arbel, J., Mengersen, K., Raymond, B., Winsley, T., and King, C. (2015). Application of a Bayesian nonparametric model to derive toxicity estimates based on the response of Antarctic microbial communities to fuel contaminated soil. *Ecology and Evolution*.

Arbel, J., Mengersen, K., and Rousseau, J. (2016d). Bayesian nonparametric dependent model for partially replicated data: the influence of fuel spills on species diversity. *Annals of Applied Statistics*.

Arbel, J. and Prünster, I. (2016). A moment-matching Ferguson & Klass algorithm. *Statistics and Computing*.

Ferguson, T. (1973). A Bayesian analysis of some nonparametric problems. *The Annals of Statistics*.

Miller, J. W. and Harrison, M. T. (2014). Inconsistency of Pitman-Yor process mixtures for the number of components. *The Journal of Machine Learning Research*.

Wade, S. and Ghahramani, Z. (2015). Bayesian cluster analysis: Point estimation and credible balls. *arXiv*.