Teaching mathematics with multimedia-based representations-what about teachers' competencies?

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Multimedia-based representations play a major role in mathematics and mathematics education. Consequently, they are important with regard to teaching purposes, as they are supposed to be useful to represent mathematical structures and processes in different ways. Within the presented project we developed an instrument by using video-vignettes in order to assess the competencies of mathematics teachers for multimedia use in mathematics lessons. For coping with complexity we reduced the instrument's focus on two facets: cognitive load and mutual supplement of multimedia representations. As the work is still in progress, we here focus on the project's theoretical background as well as on the development of the assessment instrument based on video-vignettes.

Keywords: Multimedia-based mathematical representations, technological pedagogical content knowledge, assessing teachers' competencies, video-vignettes, secondary school mathematics.

Introduction

Since mathematical objects are conceptual and invisible, the meaning of representations plays a major role in mathematics and mathematics education. The necessity of representations for the fundamental understanding of mathematical concepts has already been postulated by Duval (2006). Therefore, it is of importance that students work with multiple representations of the mathematical content early on. Doing so they can benefit from complementary expressions and viewpoints of the subject matter and are able to improve and deepen their understanding (Ainsworth, 1999). However, as a teacher it is insufficient to simply present multiple representations to the students. It is necessary that the students build and understand the connections between different representations and gain a coherent mental model (Seufert, 2003). Schnotz and Bannert (1999) illustrate the interaction between descriptional and depictional representations in their integrated model of information processing. According to their work, these two different kinds of representations complement each other in a synergetic way to form a mental model of the represented content. With the construction of a mental model through multiple representations, students also gain in cognitive flexibility (Spiro, Coulson, Feltovich, & Anderson, 1988). According to Mayer (1997) students can also achieve better results when learning with multiple representations, however, there is practical as well as empirical evidence that this is not always the case in classroom instruction. A teacher's knowledge about multiple mathematical representations and their kind of use in the classroom can obviously not be neglected.

In this paper we first establish the theoretical background for the use of multiple, dynamically linked representations in mathematics education and the related professional competencies required of teachers in this context. After that we describe the development of a test instrument to assess these competencies from a particular theoretical point of view.

Theoretical background

Especially multimedia can offer possibilities to develop and implement learning environments containing multiple mathematical representations. When working with multimedia-based representations, mathematics teachers should, among many other things, know the benefits and pitfalls of (dynamically) linking multiple representations while also being aware of the cognitive load generated by them. Therefore a variety of technological knowledge, skills and competencies must be combined with pedagogical knowledge and content knowledge of the subject matter.

Linking multiple mathematical representations

Computer-applets based on multimedia representations are not only suitable to illustrate both descriptional and depictorial representations (Schnotz & Bannert, 1999) at the same time, but they are also useful to establish a dynamic link between them. That way it is possible to present even more information about the mathematical content than the representations could provide without being linked to each other (Kaput, 1989). The dynamic linking and the mutual supplement of the different representations provide different approaches to the mathematical content, especially because of the automatic translation of effects when changing one representation. Providing different approaches could cause synergetic effects on the construction of coherent knowledge structures (Seufert, 2003). Moreover, the automatic translation between unrelated representations could decrease the cognitive load of the learner and leaves more capacities for the process of understanding (Ainsworth, 1999). Especially in the subject of mathematics, multimedia-based representations are appropriate to demonstrate the character of mathematical processes (Vogel, Girwidz, & Engel, 2007).

However, there are also disadvantages that come along with multimedia-based representations. As much as they can encourage a deeper understanding, they could also lead to misconceptions (Hadjidemetriou & Williams, 2002), if they are misleading with regard to their external arrangement. Likewise too many multiple dynamical representations could cause a heavy extraneous cognitive load, so that the students do not have any capacity for the intended germane load (Chandler & Sweller, 1991). If the extraneous cognitive load gets too heavy, students often tend to split their attention (split-attention-effect) and focus on one form of representation only (Brünken & Leutner, 2001). Hence reducing the extraneous cognitive load is of high importance when using multiple dynamic representations in mathematics teaching.

In his work, Mayer (2009) gives different principles that should be considered in constructing multiple dynamic representations: The coherence principle, for example, states that people learn better when irrelevant material is excluded. Particularly regarding the mutual supplement of multiple representations these principles are a good guideline for constructing effective multimedia-based mathematical learning environments.

Technological pedagogical content knowledge

Apparently, the profitable use of multimedia-based representations in mathematics lessons is not only a question of mathematics education, but concerns didactics of mathematics and psychology as an interdisciplinary field of multimedia learning. Certainly, teachers first have to decide from a mathematical point of view whether the mathematical content is adequate for the use of multimedia and which aspects of the content should be presented within this use of multimedia. In the second step it is important to implement the mathematical content into a computer-applet with regard to available pedagogical and psychological insights of multimedia learning. The technological pedagogical content knowledge, that is needed for the profitable use of multimedia-based representations, is an “emerge of knowledge that goes beyond all three ‘core’ components (content knowledge, pedagogical knowledge and technological knowledge)” (Koehler & Mishra, 2009, p. 66) and requires extensive knowledge about all the aspects of multimedia learning. The TPACK-framework (Koehler & Mishra, 2009) extends the taxonomy of Shulman (1986) by adding technology knowledge which results in three new intersections: technological content knowledge, technological pedagogical knowledge and technological pedagogical content knowledge (TPACK).

Accordingly, the complexity of competencies needed to use multimedia-based representations in an effective way in mathematics lessons is high. Beyond their mathematical content knowledge teachers need an extensive knowledge about the media and technology they want to use as well as its chances and difficulties for multimedia learning. Consequently, teachers are confronted with new challenges (Koehler & Mishra, 2009) and need to develop the competencies to identify the chances and difficulties that go hand in hand with the use of multimedia-based representations (Spanhel, 1999). However, according to Koehler and Mishra (2009) many of the teachers do not feel prepared for the use of modern technologies to present these kinds of representations.

TPACK in context of multimedia learning in mathematics education

The internet provides many existing computer-applets which mathematics teachers could use in their lessons (for example see www.geogebra.org/materials). The question is if a chosen applet supports or prohibits the understanding of mathematical concepts and processes and how to determine its benefit. As far as we know there are no criteria given for evaluating an applet with regard to both mathematical and psychological aspects of multimedia learning. While several studies investigated the effect of multimedia-based representations on learning outcome in general, there is still little known on how to evaluate applets with interdisciplinary criteria of multimedia learning. Also it is little known about the competencies mathematics teachers need for an effective use of multimedia-based representations in their classrooms.

Hence it is the research goal of this study to develop a test instrument to assess competencies regarding the technological pedagogical content knowledge (TPACK, cf. Koehler & Mishra, 2009) and the interdisciplinary aspects of multimedia learning in mathematics education.

Assessing mathematics teachers’ competencies in using multimedia-based mathematical representations by video-vignettes

As functional and geometrical thinking build an essential base for the understanding of mathematics and *elementary functions* as well as *geometry* also play a major role in the german curriculum of secondary schools (Kultusministerkonferenz, 2012) we decided to focus on these two mathematical contents when starting to develop the intended test instrument. These contents deeply involve the understanding of their dynamic aspects (for instance while studying covariance of functions, transformations of geometric figures or whole families of functions or geometric objects), so multimedia-based representations could be an appropriate tool in teaching functional and geometrical thinking. Functional thinking includes mainly three aspects of functions: aspect of assignment, aspect of covariance and view as a whole (cf. Vollrath, 1989). Especially for handling the aspect of covariance dynamic representations are an appropriate tool, because changes in one variable and their effects can be directly visualized in other representations. Also for acquiring geometrical thinking, multimedia-based representations can be helpful: Geometrical thinking is based on the understanding of geometrical terms and conceptions (Ulfig & Neubrand, 2013). Young children already develop an understanding for geometrical terms, but mostly ignore the similarities (Heinze, 2002). For example they are not able to understand that a square is a special representative of rectangles. With dynamic representations it is possible to illustrate not just one example of a geometrical object, but to construct a whole class of objects by using the dynamic transformation (Kittel, 2009).

On base of our theoretical considerations it is necessary to investigate also psychological aspects beyond the mathematical ones. According to an intensive literature review, we determined eight facets of psychological aspects of multimedia learning as basis for the test instrument: *relation to the content* (e.g. Spanhel, 1999), *efficacy of the use of multimedia* (.e.g. Mandl, Gruber, & Renkl, 2002), *limitations of the representations* (e.g. Mandl et al., 2002), *misconceptions* (e.g. Mayer, 2009), *cognitive load* (e.g. Chandler & Sweller, 1991), *individual promotion of the learners* (Wauters, Desmet, & van den Noortgate, 2010), *mutual supplement of multiple representations* (e.g. Mayer, 2009) und *simplifying (mathematical) content* (e.g. Kittel, 2009). Within the development of the test instrument we conducted a multistage expert-rating in order to validate, but also to empirically support a selection of two of these eight facets for purposes of reducing complexity in this first approach. This process will be described more detailed later on.

Video-vignettes and their construction

Since video-vignettes are assumed to be an effective way of measuring teachers' competencies (Blomberg, Stürmer, & Seidel, 2011), we developed, for the time being, a pool of 36 video-vignettes, that show various situations during mathematics lessons using multimedia-based representations. Video-vignettes are short sequences of a classroom situation that show critical problems: to evaluate these situations the observing person needs special competencies (Rehm & Bölsterli, 2014). Figure 1 shows an example of a script for a video-vignette related to the psychological facet *mutual supplement of multiple representations*.

The vignettes are constructed with a closed-ended question type. Multiple statements have to be rated on a scale from one to six according to its appropriateness for the presented situation. An example of statements is shown in Figure 2.

After the development of 36 vignettes, they have been validated in a multistage expert-rating.

Validation of the constructed video-vignettes

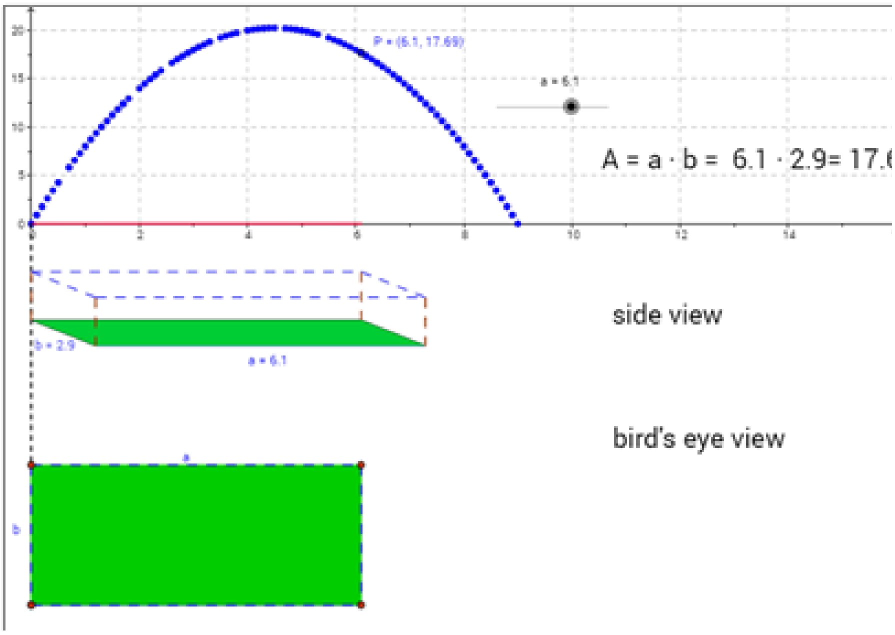
First the constructed vignettes were evaluated by nine experts in a semi-standardized qualitative interview. The aim of these interviews was to assure the relevance and the clarity of the presented situations in the vignettes. Afterwards the vignettes were rated by 104 experts in a quantitative interview. The aim here was to reassure the evaluation of the qualitative interviews as well as to analyze the distribution of the answers on the scale from one to six of each statement. Moreover, the experts could give comments on each of the vignettes. The answers and the comments of the experts from the quantitative rating were analyzed regarding four criteria: *focus regarding the mathematical content of secondary school, distribution of the answers to the statements, relevance for school and clarity of the vignette* and *comments of the experts.*

In the 10th grade of the Paula-Fürst secondary school Miss Schäfer gives the following computer-applet to her students for working in pairs:

Area of the enclosure

You got a new rabbit and now you want to build an enclosure for it.
Your parents give you 18 m of fence for the enclosure.
How long do the sides have to be so that the rabbit has the most space he could get with this fence?

Pull the slider to see the enclosure and the according function.



[In the background the following conversation can be heard:]

Miss Schäfer: *"Is everything alright?"*

Can: *"No, Timo does not understand that the function displays the area!"*

Timo: *"Yeah, I do not know why there is a round function while the area is a rectangle!"*

Figure 1: Example of a script for a video-vignette according to the psychological facet mutual supplement of multiple representations

Based on the multi-stage expert rating and psychometric properties of the instrument we chose the most appropriate vignettes and determined the two psychological facets of multimedia learning *cognitive load* and *mutual supplement of multimedia representations* as the focus for the final test instrument. Cognitive load refers to the trichotomy from Chandler and Sweller (1991): intrinsic cognitive load, extraneous cognitive load and germane cognitive load (cf. section theoretical background). So the aim is to explore if prospective teachers can estimate the cognitive load. The mutual supplement of multimedia representations refers to the interaction between two or more forms of representations of the same issue (cf. section theoretical background; Ainsworth, 1999; Kaput, 1989). Using these interactions between different forms of representations could involve many chances, but also risks. For example, it is important to link the different representations to gain an understanding of coherence (Seufert, 2003). However, as mentioned above different forms of representations could also cause the split-attention-effect (Brünken & Leutner, 2001).

Please evaluate the computer-applet with regard to the (mutual) supplement of the different representations (graph of the function, side view of the enclosure, bird's eye view of the enclosure) from your pedagogical content knowledge point of view.

		I do not agree at all					I fully agree
a)	The interdependency between the different representations is apparent.	[ ]	[ ]	[ ]	[ ]	[ ]	[ ]
b)	The computer-applet should focus on one representation only.	[ ]	[ ]	[ ]	[ ]	[ ]	[ ]

Figure 2: Example of statements for the vignette from Figure 1

Assessment

After the reduction of the vignettes as well as the reduction of the psychological facets, we revised and adapted five vignettes for each of the psychological facets as well as six to seven items for each vignette. The formulation of the items was parallelized between the vignettes in order to assure that the important aspects of each of the psychological facets are tested.

These ten vignettes were again validated in a pilot study and as the results were promising, they were used in the final assessment. In this assessment we also included covariates to prove the discriminant validity of the test instrument: pedagogical knowledge and content knowledge. Preliminary results with 261 prospective teachers in Baden-Württemberg already show evidence for the discriminant validity of the test instrument: As expected from the TPACK-framework (Koehler & Mishra, 2009) the test score of the developed vignettes shows a weak correlation with the two constructs *pedagogical knowledge* (r = .17, p = .01) and *content knowledge* (r = .29, p < .001). Furthermore we could prove expected correlations with the educational progress of the prospective teachers (r = .14, p = .03) as well as the number of attended courses addressing the use of computers in mathematics lessons (r = .17, p = .03).

Discussion and outlook

The research goal was to develop a test instrument in order to assess the competencies mathematics secondary teachers need for an effective use of multimedia-based representations in mathematics lessons. Therefore, we considered both mathematical and psychological aspects of multimedia learning and developed a test instrument for the mathematical contents of *functions* and *geometry* as well as for the psychological facets *cognitive load* and *mutual supplement of multiple representations.* With the conducted multistage expert-rating and the preliminary results of the assessment we could confirm the validity of the test instrument.

At the moment, we conduct an assessment with the final test instrument in order to research the development of the previously described competencies during the practical phase of the studies of prospective teachers. Moreover, the test instrument will be complemented with further mathematical content: *algebra* and *stochastics*. At the current stage of the project, new vignettes are developed for these two subjects which will supplement the current test instrument. The new test instrument will then be able to test a wide range of mathematical content knowledge combined with knowledge about the psychological aspects of multimedia learning.

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