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# Characterizing theories aimed at supporting teachers' mathematical classroom practices

Per Nilsson<sup>1</sup>, Andreas Ryve<sup>2</sup> and Maria Larsson<sup>3</sup>

<sup>1</sup>Örebro University, School of Science and Technology, Örebro, Sweden; [per.nilsson@oru.se](mailto:per.nilsson@oru.se)

<sup>2</sup>Mälardalen University, School of Education, Culture and Communication, Västerås, Sweden; [andreas.ryve@mdh.se](mailto:andreas.ryve@mdh.se)

<sup>3</sup>Mälardalen University, School of Education, Culture and Communication, Västerås, Sweden; [maria.larsson@mdh.se](mailto:maria.larsson@mdh.se)

*In this paper we draw upon examples from a recently published systematic literature review (Ryve et al., 2015) on productive classroom practice to contribute to the research on the nature of theories for action in mathematics education. By relating the results from the review to theories and literature on educational policy research, professional development research and implementation research we construct a framework for categorizing theories aiming at supporting teachers' actions in mathematical classroom practices.*

*Keywords: Theoretical frameworks, theories for actions, mathematical classroom practice.*

## Introduction

The development of frameworks and theories<sup>1</sup> that aim at guiding the actions of teachers have rendered much recent attention in educational research (e.g., McKenney & Reeves, 2012; Ruthven, Laborde, Leach, & Tiberghien, 2009). Within research areas such as curriculum material (Davis & Krajcik, 2005), planning and implementing whole-class discussions of cognitively demanding tasks (Smith & Stein, 2011), professional developments programs (Borko, 2004; Desimone, 2009), and improvement of mathematical instructions at scale (Cobb & Jackson, 2012) cumulative work has been conducted to establish theories for action. Further, frameworks for supporting teachers' actions and thinking such as curriculum materials, theories and models have been put forward as essential components of effective professional development (Cobb & Jackson, 2012) and for establishing productive mathematical classroom practices (Franke, Kazemi, & Battey, 2007). However, we need to know more about how theories should be designed to facilitate implementation; to be used and do real work in supporting and constructing teachers' actions (McKenney & Reeves, 2012). In this paper we relate result from a recently published systematic literature review (Ryve et al., 2015) on productive classroom practice to literature on educational policy, professional development and implementation research in order to construct a framework for understanding and facilitating the implementation of theories and research results aiming at supporting teachers' mathematical classroom practices.

## Relevant research

The approach of the present study is based on the assumption that 'theory matters' for teachers, to enhance their ability to develop rich mathematical classroom practice (Charalambous & Hill, 2012).

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<sup>1</sup> There are many ways to denote theories serving the purpose of guiding actions and this is further discussed below.

For instance, by adopting and making use of theoretical tools teachers are supposed to enhance their ability to establish productive routines in their classroom practice (Franke et al., 2007), develop and continuously adjust a learning trajectory and the means to support that trajectory (Cobb, Confrey, Lehrer, & Schauble, 2003), become more sensitive to notice instructional opportunities in the moment and be methodical without being mechanical. However, Burkhardt and Schoenfeld (2003) argue that most theories that have been applied to education are quite broad, lacking the specificity that helps teachers to guide and understand the design and analysis of learning activities. Cobb et al. (2003) adhere to this view, claiming “General philosophical orientations to educational matters – such as constructivism – are important to educational practice, but they often fail to provide detailed guidance in organizing instruction” (p. 10). So, there is this dilemma; theoretical constructs are supposed to enhance teachers’ capacity to teach but, to do such work, theories need to be of a certain kind.

### Perspectives on theories

diSessa and Cobb (2004) detail the nature of different theories relevant for research in mathematics education. They distinguish between grand theories, orienting frameworks, frameworks for action, domain-specific instructional theories and ontological innovations. Skinner’s behaviourist theory provides an example of a grand theory. Even if grand theories have a prominent position in educational research, they appear to be too general to provide guidance for explaining and supporting the learning of mathematics. Orienting frameworks, such as constructivism (Von Glasersfeld, 1995) or communities of practice (Lave & Wenger, 1991), provide general support for specifying issues of learning, teaching and instructional design whereas frameworks for actions concern analytical constructs of a more or less general *prescriptive* character (diSessa & Cobb, 2004). Domain-specific instructional theories are also of a prescriptive nature as they are typically specific to a domain or even learning trajectory of certain content and the means by which this trajectory can be supported. An ontological innovation is descriptive in nature. It is about developing analytical categories by which aspect of a phenomenon can be discerned. The framework of Socio-mathematical norms (Yackel & Cobb, 1996) exemplifies an ontological innovation.

diSessa and Cobb’s (2004) categorization not only labels the nature of different frameworks, it also points to the *descriptive*, *explanatory*, *predictive* and *prescriptive* purposes of different theories. **Firstly**, theories could be used to describe the world and many theories and frameworks within mathematics education serve such a purpose. The contribution to research in engaging in describing or characterizing objects or processes as certain phenomena could be understood in terms of new or unconventional lenses for viewing the world. **Secondly**, a further purpose of theories is to explain relations between phenomena and as mentioned above this purpose is often stressed as absolutely central for theories. A prerequisite for explaining those relations is to explicitly characterize each phenomenon. Therefore, theories used for explanatory purposes build upon or encompasses descriptive theoretical contributions. **Thirdly**, in a similar vein predictive theories necessitate explanations and clear descriptions of phenomena. Predictions include foreseeing effects of certain actions under certain conditions. **Finally**, prescriptive theories are used to identify and articulate productive ways to make decisions and performing actions. This kind of theory integrates descriptive, explanatory and predictive knowledge to guide actors in constructing and establishing

interventions. Within design research prescriptive theories are often denoted design principles but neither the term nor the nature of those design principles are settled (Ruthven et al., 2009).

### Theories for action

In this paper we are particularly interested in theories for actions and what McKenney and Reeves (2012) denote the prescriptive role of theories. Both the characteristics of theories of actions and prescriptive theories and ways of denoting them are not settled in educational research as indicated above (cf. McKenney & Reeves, 2012). As becomes apparent in (Ruthven et al., 2009), the relation between the terms used to denote theories for action is not just connected to neutral ways of denoting the same phenomenon but instead accentuates particular features and characteristics of such prescriptive theories. For instance, Ruthven et al. (2009) shortly muse about the relation between the design tools they introduce and design principles. They suggest that the conceptual set up of grand theories, intermediate frameworks and design tools introduced in Ruthven et al. (2009) stresses *theoretical* underpinnings for *sensitizing* researchers to critical issues while design principles from US often *prescribe* certain course of actions and are typically more loosely anchored in theoretical perspectives. One may ask, should theories for actions prescribe and sensitize teacher? In general, the development and understanding of design principles is weakly developed and in summarizing the most urgent issues for educational design research McKenney and Reeves (2012) suggest “a worthy challenge facing educational design researchers is to further the development of predictive and prescriptive theories” (p. 212). We want to add to this research.

### Method

The design was framed by a ten-step process for systematic literature reviews (Gough, Oliver, & Thomas, 2013): (1) Need, (2) Review questions, (3) Scope, (4) Search, (5) Screen, (6) Code, (7) Map, (8) Appraise, (9) Synthesize, and (10) Communicate. In the project we engaged in processes 1-7 and 10. Our *review questions* were: (a) What characterizes research on classroom teaching practices, teaching approaches and teaching methods in mathematics? (b) What characterizes research on teachers’ instructional strategies used to establish classroom practices in mathematics? and (c) What does research tell about teaching for the learning of mathematical competencies?

We *searched* in title, keywords and abstract in Web of Science<sup>2</sup>. Search strings<sup>3</sup> were iteratively developed while reading some abstracts. In total, we had 622 hits that we *screened* for relevance according to our *scope* defined by our inclusion criteria<sup>4</sup>. The screening was made in two steps based on: (1) title, keywords, and journal name, and (2) abstracts. Uncertain cases were discussed and decided upon collectively among the three researchers. After the two screening steps, 242 articles remained potentially relevant for the scope of the review. Simultaneously as the screening of

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<sup>2</sup> We searched Web of Science Core Collection as a way to focus on high-quality journal articles. We limited our search to the year span 2008-2014 and to the document types “article” and “review”.

<sup>3</sup> See Appendix A in Ryve et al. (2015) for the exact search strings.

<sup>4</sup> For an article to be included in the review, the article must be: (1) about mathematics teaching/learning/education, (2) related to compulsory school (grade 1-9), and (3) about the teacher’s role.

abstracts, the articles that remained relevant were *coded* on Object of study, Method, Number of participants, Context, Results, and Implications for practice. When needed, we also read other parts of the articles apart from the abstract. 201 articles remained relevant for the review based on the inclusion criteria. The next step of the *mapping* was to structure and characterize trends and interests within the discourse of research in mathematics education that focus on teaching methods, classroom practice and teacher's role in classroom practice. Therefore, we looked closer at the abstracts of the 201 articles, categorizing them in relation to object of study. See Ryve et al. (2015) for detailed descriptions of steps and rationales in the processes.

## Results

	Student knowledge		Student attributes (SA)	Practice (P) (Teaching approaches)
	Processes	Products		
Interactional strategies (IS)	3	10	4	11
Teaching approaches (TA)	9	42	8	
Learning material (LM)	5	14	2	7
Background variables (BV)	1	16	2	34
Characterization (Char) Total: 33				

**Figure 1. Categories and analytical relationships of the mapping**

Of the 201 articles, we found 168 to be structured according to an analytical relationship between an outcome variable and a design variable. In the remaining 33, the object of study did not follow such a structure. The design variables we distinguished are gathered in the left column of the matrix (Figure 1), while the outcome variables we found are categorized in the top row of the matrix. In elaborating on the mapping we intend not to go through the entire mapping. We highlight and provide examples on some specific findings, which we then follow up on in the discussion. In the discussion we construct a new framework, to be used for categorizing theories to be implemented in mathematics classroom practices. On this account, and due to space limitation, we are not backing up all claims and findings by references from the mapping.

### Elaborating on theories within the mapping

#### *Student knowledge*

Our review reveals a clear bias towards research that focuses students' product knowledge. Mathematical products relate to conventions, symbol systems, concepts and calculation techniques of mathematics. Looking at studies that emphasized a product view of mathematics and those emphasizing a process view we also notice a methodological difference. In the product view knowledge is expressed in the language of mathematical products and students understanding are profiled and ordered in accordance to the mathematics itself. Connected to such a conceptualization of knowledge, students' performances are often measured by standardized tests (Desimone, Smith,

& Phillips, 2013). In the process view, qualities and progressions are not explicitly elaborated on. These frameworks are descriptive in nature, specifying a set of analytical categories, which is used to sensitizing (Ruthven et al., 2009) the analyst on some certain characteristics of, for instance, students' ability to communicate and reason in and with mathematics.

#### *Interactional strategies*

Studies belonging to this category explicitly refer to teachers' moves and actions. The teacher takes an active role in these studies; how he/she acts in interaction with the students, is central to the investigation. The focus is on how teachers communicate and engage with their students, and what role the communication and engagement play in students' learning of mathematics. It could be about, for instance, how a teacher uses gestures and questions (Shein, 2012) and follow up on students' ideas in order to develop the mathematical classroom practice (Akkus, 2013).

#### *Teaching approaches*

Teaching approaches refer to studies taking a broad perspective on classroom teaching in mathematics. The teachers' actions and interactional behavior are not the main object of investigation. The teacher may be important, but it is the more general and overall structures of teaching that are the object of study. In our review we found different examples of teaching approaches, such as technology-based teaching, mathematical games, problem-based teaching, and contrasting ways of using textbooks in mathematics teaching.

#### *Learning material (task design)*

Some studies focus on how a specific artifact or design principle can support or challenge students' learning in mathematics. In these studies, the teacher takes a passive role. Focus is on the students' interaction with the learning material and the role of the teacher is basically to execute the lesson. Studies belonging to this category may investigate the role of visualization or simulation in the learning of mathematics (David & Tomaz, 2012). In this group we also include issues of task design; types and sequences of tasks (Hattikudur & Alibali, 2010) and instructions for solving mathematics tasks (Orosco, 2014).

#### *Background variables*

Several studies did not connect classroom practice and students' learning to any didactical design variable. These were studies giving accounts of personal attributes such as teachers' beliefs, attitudes and knowledge in order to explain classroom practice and students' performance.

#### *Characterization*

In 33 articles of the articles, the object of study did not follow the structure of an analytical relationship between two didactical variables. In this group of studies, to describe a certain practice or teaching approach is the focus in itself. The goal is to provide *descriptive* accounts of analytical categories of a teaching/learning phenomenon, which can be used to sensitizing researchers and teachers to critical issues of the phenomenon in question (Ruthven et al., 2009). It may concern the characterization of curriculum material (Sherin & Drake, 2009), the orchestration of math-talk with interactive whiteboards (Beauchamp, Kennewell, Tanner, & Jones, 2010), mapping the mathematics in classroom discourse (Herbel-Eisenmann & Otten, 2011), or profiling students' understanding or strategies of specific subject matter content (Wagner & Davis, 2010).

## Discussion

By relating the results of our mapping to literature on educational policy, professional development and implementation research we construct a framework for understanding and facilitating the implementation of theories and research results aiming at supporting teachers' mathematical classroom practices.

In studying the papers it is apparent that teachers are ascribed different roles in different research studies. While quite a few studies within the category of learning material position teachers as administrators of tasks and computer programs other studies highlight the role of expert and orchestrator of classroom practices. Within the latter categories of articles, the role of teachers is central in asking questions, explaining content and acting formatively to support and challenge students' mathematical thinking. In understanding the implementation of theories and results aimed at improving classroom practices and students' mathematical learning it seems essential to consider how **theories construct the role of teachers in classrooms**.

Ruthven et al. (2009) notice the distinction between theories that prescribe teachers' actions and theories that aims at sensitizing teachers to essential aspects of classroom practices. In a similar vein, while some theories and studies in our review are clearly prescriptive towards teachers (e.g., theories belonging to Instructional strategies and Learning material) and what they should do in classroom practices, others aims at sensitizing and empowering teachers (theories belonging to the Characterization category). We are not normative about these different ways and suggest that both could be productive for different teachers in different context. Further, we hypothesize that these two strands are correlated with research methodologies in that many studies within mathematics education taking an educational design perspective aims at empowering teachers while studies taking a stricter experimental approach prescribe and praise clear prescriptive instructions. However, to what extent and in which ways **theories prescribe or sensitize teachers** seem relevant to consider for anybody collaborating with teachers.

Cobb and Jackson (2012) stress that tools and frameworks within educational policies play a prominent role. When it comes to designing and using tools Cobb and Jackson suggest that it is important that the tools can be used by agents immediately in that they are easy to access, but at the same time harmonize with the planned reorganization of the practices. In addition, in developing frameworks, theories and tools it is essential to consider the amount and type of learning that are required for teachers to develop in order to use them in an appropriate and reliable way. Of course, such using requires good mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008). However, there is also reason to believe that certain types of frameworks are easier then other to use and apply in a mathematical teaching practice. For instance, in our survey we notice how the product perspective dominates research focusing on student knowledge. That the product perspective has a long tradition in the field is probably the major reason for this. However, taking a closer look to the mapping we can also understand this dominance as if there is a higher degree of transparency in how to use product knowledge theories compared to process-knowledge theories. From this we learn that, in implementing theories to school practice, we need to consider how user-friendly, accessible and transparent different types of theories are to teachers. In other words, we need taking into account to **what extent and which kind of teacher learning is necessary** for productively implementing theories or frameworks to mathematical classroom practices?

Desimone's (2009) put forward *coherence* as a critical feature of professional development programs. The concept of coherence refers to the relation between the PDP and teachers' knowledge and beliefs. This raises questions about the extent to which theories should be coherent with teachers' knowledge and beliefs. In other words, should theories strengthen teachers' knowledge and beliefs or should it challenge their knowledge and beliefs? Should theories aim at strengthening classroom practice or should theories aim at reorganizing classroom practices (Cobb & Jackson, 2012)? The reorganization of practice could include working with new types of mathematical problems, new roles for students and teachers, and the establishment of new classrooms. Hence, in examining and choosing theories, frameworks and models, mathematics educational researchers working with teachers should consider whether the aim is to **strengthen or reorganize ongoing practices** and, consequently, consider how frameworks are supportive for such endeavors.

To conclude, as a complement to categorize theories for actions in terms of content areas we suggest it is productive for researchers working with teachers to consider theories in terms of: the positioning of teachers in classroom practices; the positioning of the teacher as a receiver of the theory; the amount and type of teachers' learning required; and if theories primarily function to strengthen or to reorganize practices.

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