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Motivated and talented mathematics students are not always convinced about the relevance of mathematics. More insight into applications of mathematics can be beneficial for students in terms of preparing them for their future study and career. Using design research a particular intervention had been developed which differentiated by student interest, in order to improve students' beliefs about the relevance of mathematics. The students selected were those studying advanced mathematics at upper secondary school in the Netherlands. The intervention had been designed to teach differential equations through tasks with science-, medicine-, or economics- related contexts. The results show that the students appreciated the context-rich tasks, which provided them with insights into how mathematics can be applied in other sciences and contributed to the improvement of their beliefs about the relevance of mathematics.

Keywords: Mathematics education, task design, relevance of mathematics, differential equations, differentiation by interest.

Introduction

Students' beliefs about the relevance of mathematics has been a topic of much research over the last decades. However, in most studies the main reason for studying students' views on the relevance of mathematics has been the assumption that a positive notion of the importance of mathematics contributes to a positive attitude towards learning mathematics (Schoenfeld, 1989). Hence, most research studies conducted in this area have aimed at understanding and improving students' attitudes towards mathematics, and ultimately improving their performance in mathematics at school (Farooq & Shah, 2008; Mohamed & Waheed, 2011).

However, we start from the premise that already motivated and talented students can also benefit from a clear view on the usefulness of mathematics for their future education and career. Dutch secondary school students in an advanced mathematics course (aimed at improving students' algebraic skills and elaborating the connection between mathematics and other sciences) praised the course as being challenging and fun, but they also mentioned that it was not clear to them how the mathematics would be useful for their future study (Van Elst, 2013).

In our research project a design research approach has been used to develop and intervene with tasks aimed at simultaneously teaching a new (for students) mathematical concept (differential equations) and improving students' beliefs about the relevance of mathematics in general, and in particular with respect to their future study and career ambitions. The students sampled were those taking the advanced mathematics course (in their final year before university entry) in Dutch upper secondary schooling.

The mathematical topic of the designed intervention was the theory of analyzing, solving and interpreting first order differential equations. Differential equations are an important subject in any undergraduate university curriculum in a broad range of domains, such as engineering, physics, biology and economics, which makes it a suitable topic for an intervention with a focus on the relevance of mathematics.

To improve the odds that the real-life problems posed in the designed tasks would appeal to the students, the designed intervention differentiated by student interest. Students were offered the opportunity to choose between different real-life problems, whilst ensuring that regardless of their choice they learned the same mathematical concepts of differential equations.

The research question of the study was the following: How does a learning strategy based on differentiation by interest for teaching ordinary differential equations (to upper secondary students in a “strong mathematics” course) improve the view of students on the relevance of mathematics for their future study and career?

In the subsequent section, we provide an overview of the relevant literature. Next, the research design including the context of the study and the data collection strategies used are described. Finally, we provide a discussion of the results and our conclusions.

Literature

Students’ beliefs about the relevance of mathematics is considered as one of the factors that can play an important role in their attitude and motivation. Several questionnaires measuring the attitude of students towards (learning) mathematics use a scale for measuring students’ beliefs about the usefulness of mathematics: for example, the Attitude scale towards Math (Martinot, et al., 1988) contains a scale named Relevance of Mathematics.

Most studies using these surveys do not emphasize the improvement of students’ beliefs about the relevance of mathematics, as they are aimed at students’ attitudes towards mathematics. However, a recent study of first year university students in engineering focused on improving perceptions of the relevance of mathematics in engineering. In this study Flegg (2012) has described the use of context-based learning by applying mathematics to real-life problems as a promising approach.

The teaching of differential equations has undergone some major changes over the past decades in favor of more contextualized, problem-based education, and a less traditional, analytical approach (Boyce, 1994). This was in line with our planned design to incorporate real-life problems in the assignments. At university level several initiatives have reported good results using this new teaching approach (e.g. Huber, 2010) and the development of new course material using Realistic Mathematics Education (e.g. Rasmussen & King, 2000). In a comparison of a traditional textbook with a textbook that incorporated discipline-specific perspectives to teach the mathematical knowledge to engineering students, Czocher and Baker (2010) conclude that a contextual approach is also more in line with recommendations from the research literature.

Task design is widely recognized as an important, albeit complex activity that is at the core of mathematics education. The term ‘task’ is used to describe a wide variety of student activities aimed at learning mathematics (Watson & Ohtani, 2012). In our research tasks are guided group assignments about a real-life problem.

The study

The context

The Dutch education system consists of eight years of primary education, and 4-6 years of secondary education (depending on the level of education). The highest level of secondary education is the pre-university education called VWO (voorbereidend wetenschappelijk onderwijs) with a duration of six years, which provides students access to university. For every student at VWO level mathematics is a mandatory course. However, in the last three years students can choose between two different mathematics courses: “wiskunde A” (mathematics A) and “wiskunde B” (mathematics B), the latter being the more mathematically demanding course, which is obligatory for technical and engineering studies at university.

In 2007 an advanced mathematics course called “wiskunde D” (mathematics D) was introduced to offer challenging and engaging mathematics, and where the relevance of mathematics and its connection to other sciences should become clearly visible (cTWO 2007). This course is aimed at students with an interest in sciences and engineering, and it includes mathematical topics, which are part of every first year university curriculum: e.g. complex numbers; analytic geometry; and differential equations. Students with ‘wiskunde B’ are offered the opportunity to take this advanced course in mathematics in addition to their regular course.

It might be expected that students who take this advanced course in mathematics are convinced about the relevance of mathematics, which is likely to fuel their obvious motivation to learn mathematics by taking this advanced course. However, studies on the implementation of ‘wiskunde D’ tell a different story. In a study by Van Elst (2013) students praised the course as being challenging and fun, but they also mentioned that it was not clear to them why the mathematics in the course was useful for their future study and career. According to a study by Cheung (2012) teachers of ‘wiskunde D’ stated that the course was well suited as a preparation for a future study in a technical or engineering environment, but they also stated that the curriculum did not emphasize enough the applications of mathematics and the connections to other sciences (Cheung, 2012).

To be able to understand the theory of ordinary differential equations, secondary school students require almost all basic mathematical knowledge taught in secondary education as pre-knowledge. Hence, this topic is scheduled near the end of the final (6th) year, to make sure all major mathematics/’wiskunde B’ and mathematics/’wiskunde D’ topics have been covered. It can also be expected that at that time the students have a good idea of their intended future study. This might motivate the students to choose tasks with real-life problems associated with their future study.

The real-life problems in the study by Flegg (2012) all had an engineering context, which was in line with students’ study (of engineering at university). However, from our experience secondary school students taking the ‘wiskunde D’ course are not all interested in engineering. Students typically also enroll for medical or economics studies after graduating with ‘wiskunde D’. Hence, only focusing on real-life problems in engineering would have been a too narrow approach. To give students a good view of applications of mathematics to solve real-life problems, the learning strategy should offer differentiated instruction, based on students' interest.

Beside this "practical" reason to give students a choice of which problems they wanted to investigate, research shows that differentiated instruction based on students' interest supports their autonomy and improves students' motivation for the task at hand (Katz & Assor, 2006). Differentiation is commonly used to accommodate the different learning styles of the students; however, it can also serve to accommodate other differences between the students, such as their interests and plans for their future study and career (Tomlinson et al., 2003).

Research design and data collection strategies

For the development of the intervention, a module consisting of tasks on real-life problems, a design research approach was chosen. The design process comprised of three phases: a preliminary phase; a iterative development phase; and a final evaluation phase. The preliminary phase included a context analysis and a literature review. The learning goals for teaching differential equations were defined, based on the five strands of mathematical proficiency (Kilpatrick et al., 2001).

In the second phase a partial prototype of the module was developed. In March 2015, a small pilot test was conducted, with a prototype consisting of only three tasks. In a second design cycle a module was developed, which comprised of 14 tasks covering five different types of differential equations. Validation of separate tasks was done (1) by expert appraisals from university experts in mathematics education, focusing mainly on relevance and consistency of the design; and (2) by secondary school teachers focusing on the consistency and expected practicality of the design.

The designed intervention was carried out from January to March 2016. Three classes of 'wiskunde D' students of two different schools in the Netherlands, in total 49 students, participated. The three classes all had a different teacher, one being a researcher in this study.

The data collection strategies for the intervention included

- two student surveys (before and after the intervention) asking the students about their future study plans and their views on the relevance of mathematics, using the 8 question scale Relevance of Mathematics (Relevance scale) from The Attitude scale towards Math (Martinot et al., 1988);
- a student survey after the intervention to evaluate the module and the tasks;
- student interviews after the intervention about the module and the relevance of mathematics in general, and in particular for their own future study and career;
- the video recording of a teacher meeting about their experiences during the intervention;
- the collection of exam results after the intervention at one school. These results were compared to the results of the 'wiskunde D' students of the previous exam years, who were taught the same theory of differential equations but in the traditional classroom setting.

The intervention

The intervention module comprised of five tasks, each covering a different type of differential equation. Of each task up to three different versions were developed, which applied the same mathematical concept to entirely different contexts. Prior to the first task students were given time to read short descriptions of each problem and were given the opportunity to make, for each of the five subsequent tasks, a choice which context (real-life problem) appealed to them most.

The different contexts for each of the tasks consisted of: a science/engineering related problem; a biological/medical problem; and an economical/social problem. Regardless of students' choices of contexts for the five tasks, the students worked with and learned the same mathematical concepts during their work on these parallel tasks. The students were guided through the process of modelling the problem and exploring the mathematical model analytically, graphically and numerically. After solving the mathematical problem the students were asked to interpret the results within the context of the chosen assignment. Table 1 gives an overview of the 14 tasks.

Task	Differential equation	Scientific/engineering problems	Biological/medical problems	Economic/social problems
1	$y' = \alpha y$	Nuclear disaster	Bacterial food poisoning	Forged paintings
2	$y' = \alpha y + \beta$	Mixing water problem	Intravenous infusion	Advertising effect
3	$y' = \alpha y + f(t)$	CO poisoning	Estimating time of death	Price indexing
4	$y' = \alpha y^2 + \beta y$	Oil production	The Ebola epidemic	Population growth
5	$y' = f(y,t)$	Skydive	Blood alcohol content	

Table 1: Overview of the real-life problems used in the module

Results

40 out of the 49 participating students filled in the survey about the module. Responses to most questions were measured on a five-point scale, ranging from "I strongly disagree" to "I strongly agree". The students were generally quite positive about the whole module. Asked to grade the whole intervention on a scale of 1 to 10, they rated the module 6.5 on average. Interestingly, this mean score differed greatly between the three groups, with one group scoring surprisingly lower than the other two (see Table 2). Additional comments in the survey from this group suggest that they lacked proper guidance of their teacher. Hence, the practicality and effectivity of the intervention can substantially benefit from a good description of the role of the teacher as coach, helping the students to advance in their assignments. In the teacher meeting the same recommendation was made.

Overall, 28 out of 40 students agreed (of which 6 strongly) with the statement "The tasks gave me a good view on how mathematics is applied in other sciences", and only 3 students disagreed with this statement. The negative statement "I think that these tasks were not very realistic" was disagreed by 29 students (of which 12 strongly) and only two students agreed with this statement.

Schools	Group	Group size	#Respondents survey	Rating (mean)	#Respondents Relevance scale
School A	1	19	18	7.1	18
	2	17	14	5.5	7
School B	3	13	8	6.9	8
Total		49	40	6.5	33

Table 2: Respondents and rating of the module by group

The survey also contained open questions asking students what they liked about the module and what in their opinion should be improved. Positive points were working in groups (11 times) and the application of mathematics in other sciences (15 times) with statements like: "it was fun to experience that math is useful", "clear applications of math" and "you get a better view on how mathematics can be applied".

Some negative comments were about ICT problems encountered during the intervention (e.g. "slow laptops"). However, the majority of the feedback was about the way the mathematical concepts of differential equations were introduced to them. For example, they complained that having to read the theory by themselves made it harder to grasp the concepts. Some also missed the "traditional" teacher-led instruction, where the "basic" theory is explained by the teacher before the students start working on the assignments.

The student interviews were conducted by the teacher/researcher (one of the authors). In the interviews the students voiced the same concerns about the lack of teacher instruction and the ICT problems. However, they were positive about the tasks and the application of mathematics to real-life problems, as the extract shows.

Interviewer: Did the tasks have some added value?

Student 1: Yes, I think so. I think it adds quite a lot. It helps…

Interviewer: It helps? In what way?

Student 1: To give a lot of people the idea what can be done with mathematics. That there is mathematics behind everything. Not a lot of students would have realized that before, I think.

Comparison of the 'wiskunde D' exam results of the 36 students from school A with the results on a similar exam by the 27 'wiskunde D' students of the previous year did not show a significant change in the grades. As can be seen in Table 3, the students of 2016 scored slightly better than their peers in 2015 but that was to be expected as their average grade before the exam was also better.

The intervention was not intended to improve the exam results, and the findings showed that the dual focus on both the mathematical concepts of differential equations and the relevance of mathematics to other sciences did not affect the grades of the students in a negative way.

Exam results school A	Group size	Average grade before exam	Average exam score
Students 2016 (intervention)	36	75.2 %	72.6 %
Students 2015 (traditional course)	27	74.6 %	71.4 %

Table 3: Exam grades of the students in 2015 and 2016

From the Relevance of Mathematics scale, conducted before and after the intervention, we obtained some promising results. Only 33 out of the 49 students completed both the pre and post survey, as shown in the last column of table 2.

A one-sided paired-samples t-test was conducted to compare the answers of the 33 students on 8 items of the Relevance of Mathematics scale before and after the intervention. The result of this test for the whole group of 33 respondents did not show a significant improvement with a p-value of 0.19. But the same test on the 18 results from the high response group gave a p-value of less than 0.02 indicating a significant positive change in their response to the questions about the relevance of mathematics. Due to the low response rate of the two other groups we were not able to get any significant results from these separate groups.

We also conducted the paired t-test on the answers of the 17 students who had the lowest scores on the pretest. They had scored relatively low on their beliefs about the relevance of mathematics, all with an average score between 2.38 and 3.63 for the 8 items on the five point Likert scale. On the same test after the intervention their average scores ranged between 2.63 and 4.63. 12 of these 17 students scored higher on the test after the intervention resulting in a p-value of 0.002, which indicates a strong significant positive change in these students' beliefs about the relevance of mathematics. This result indicates that students who do not already have strong beliefs about the relevance of mathematics benefit most from the context-based tasks.

Conclusion

The intervention had positive effects on selected student views about the relevance of mathematics without affecting the examination results. Providing students with purposefully designed tasks where mathematics is applied to real-life problems cannot only challenge their assumptions about the relevance of mathematics, but also improve their awareness of its usefulness.

Teacher professional development is a topic for further study that will be the focus of a new design cycle of the module. Applications like a teacher meeting, where the goals of the module are explained and an extension of the teacher manual with guidelines how to introduce the module and how to coach the students during the group assignments, are expected to contribute to a better understanding of the role of the teacher during future interventions.

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