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Teachers’ voices related to the mathematical meaning constructed in the classroom

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Research on classroom teaching practices is mainly focused on teachers’ knowledge, beliefs and practices paying limited attention to a crucial aspect of the instructional activity, that is, on the mathematical meaning constructed in the classroom. The present study examines three highly motivated and professionally active primary teachers’ instructional practices and their reflections on them in an attempt to identify critical elements shaping classroom mathematical meaning construction. The results show that all three teachers, intentionally or not, make instructional choices, which tend to restrict the mathematical meaning under negotiation. These choices could be attributed to their desire to provide children with an ‘easy’, ‘safe’ and ‘pleasant’ learning environment.

Keywords: Teacher practices, teachers’ reflections, epistemological features

Introduction

Significant research has been carried out on classroom teaching practices. An important part of this research examines teachers’ mathematical knowledge, beliefs and practices employing different theoretical as well as empirical lenses and aiming to understand its impact upon students’ learning. Although the importance of the mathematical meaning constructed in the classroom is widely recognized and implicitly implicated in all the different approaches, relatively little attention has been paid to a detailed analysis of this construction.

In previous studies, we attempted to systematically examine this important aspect of mathematics teaching and learning, analyzing teaching episodes of different teachers, mathematical topics or age students. Lately teachers’ reflections on their teaching decisions and practices drew our attention, because of the interest in understanding the value teachers attribute to the nature of the mathematics knowledge shaped in the classroom (Linares and Krainer, 2006). Thus, in this study, after analyzing three primary teachers’ classroom practices with respect to the mathematical meaning constructed, we focus on their reflections on this construction. Our aim is to examine each teacher’s decisions and practices related to the mathematical meaning construction process.

Classroom mathematical meaning construction

We first attempt to define “classroom mathematical meaning construction” and then present framework related to the epistemological features of this meaning used to analyze teaching practices.
Mathematics is a very special discipline and the nature of the mathematical knowledge and the way it operates are among the main objectives of the subject matter curriculum worldwide. The mathematical way of developing (ideal, theoretical) objects and processes reveals ideas’ attributes and relations as well as definitions and theorems to identify objects, to produce new, to relate them and to justify properties and relations. One of the most significant aspects of mathematics education is the understanding of this mathematical way of operating to which students gradually become acquainted by the way teachers manage mathematics within the classroom. Thus, the mathematical meaning construction concerns the significance that teachers’ classroom management attaches to different mathematical contents and procedures. How does this construction appear to be omitted from different studies?

The considerable body of research focusing on the so called ‘mathematical quality’ of teaching employs terms such as ‘connection to worthwhile mathematical ideas’, ‘richness of the mathematics’, ‘accuracy’ and so on (Ball, et al., 2008) to describe various dimensions of the mathematical meaning shaped in the classroom. Kilday and Kinzie (2009) report on different dimensions related to the ‘quality of mathematics instruction’ for classroom observation verifying the absence of clear designation for these terms. They report on various tools used to examine this quality summarized along the following dimensions: teaching aspects (roles, strategies, classroom setups, tasks, time, etc.), not necessary related to mathematical content; teachers’ knowledge (e.g., mathematical content); teachers’ and students’ functioning (for example, interactions, behavior, engagement, expectations, etc.) and learning aspects (such as cognitive demands). Only one rather unclear term can be identified among the above referring to the mathematical meaning constructed in the classroom that could be seen as relevant, ‘clarity and correctness’, without though any specific relation to the mathematical content.

Within our perspective (Kaldrimidou et al, 2008; 2013), the term ‘mathematical meaning construction’ is oriented to the epistemological features of mathematics. These features concern ‘definitions’ to identify and differentiate the theoretical mathematical objects, ‘attributes and relations’ to study them and special ‘processes’ for the management of these objects and relationships. Students are expected to gradually approach these ideas, objects and properties through the meaning assigned to them by the teacher’s classroom management. This aspect of the classroom ‘mathematical meaning construction’ is only partly approached by the research, which examines the mathematical quality of teaching.

**Theoretical Perspectives**

Most of the studies examining the mathematical knowledge at play in the classroom focus indirectly on this knowledge, often adopting a teachers’ knowledge perspective. This section discusses some of these studies with respect to the way these examine the shaped mathematical meaning.

Ball et al (2008) studied teachers’ mathematical knowledge for teaching (MKT) arguing that this special knowledge, in addition to other, like the knowledge of content and students, content and teaching, content and curriculum and so on determine the quality of teaching and thus the learning outcome. They propose a framework for examining this quality including features like richness and rigor of the lesson, presence of mathematical explanation and justification, mathematical representation, etc. Their approach allows the study of certain features of the mathematical
knowledge present in teaching practices but not the exact mathematical meaning shaped in the classroom.

Turner & Rowland (2010) focus on teaching practices and examine teachers’ mathematics knowledge based on their instruction. Four categories of situations revealing teachers’ mathematics knowledge are identified: ‘foundation’ referring to the knowledge, beliefs and understanding acquired during teachers’ education, ‘transformation’ and ‘connection’ explaining the ways teachers present the relevant mathematical topic and ‘contingency’ related to the ways teachers react to ‘unanticipated events’. Their framework named Knowledge Quartet (KQ) mostly relates the mathematical knowledge in teaching to teachers’ mathematical expertise or to principles of classroom management in different types of situations. Thus, it is less connected to the mathematical meaning shaped in the classroom as a result of teachers’ management of this meaning.

Some studies attempted to examine teachers’ knowledge based on teaching practices, errors and teachers’ reflection on these, the use of representations and examples (Lin & Rowland, 2016) without deepening, however, into the impact of this knowledge in the classroom management of the mathematical meaning. Also others examined teachers’ knowledge from a cognitive point of view, without concentrating on specific contexts and the nature of mathematics (Davis & Simmt, 2006).

From a decision making perspective, Schoenfeld (2013) proposed a framework for classroom observations related to effective instruction analyzed along three basic dimensions: access, accountability and productive dispositions. Here the focus is on mathematics and opportunities for their learning, thus on mathematical meaning construction: “Students are given a chance to learn mathematics… This requires making mathematics learning practices explicit and accessible … Mathematical exploration and discussion should be accurate. Reasoning and justification should be tied to mathematics” (p. 611). Terms like ‘mathematical reasoning’, ‘mathematical accuracy’, ‘richness and integrity’ are used to describe the mathematical character of the knowledge built. However, although the framework is thoroughly and accurately presented, it leaves unclear the meaning of each term and its connection to the epistemology of mathematics.

Mathematical meaning and the understanding of the nature of the mathematics constructed in the classroom have been also seen as an important aspect of classroom management encountered as a complex multi-dimensional phenomenon and studied in varied ways. Relevant research indicates that teachers make decisions based on multiple perspectives, often less mathematical and mostly pedagogical or didactical (Bednarz & Proulx, 2009).

For a number of years, our studies have been looking at teachers’ classroom management, teaching practices in various mathematical contexts and students’ learning in relation to the epistemological status of the knowledge under construction in the classroom. The results indicate that “in most cases the activity developed in the classroom had none of the epistemological features characterizing mathematics, thus affecting students’ mathematical understanding” (Kaldrimidou, Sakonidis & Tzekaki, 2013, p. 306). Below, an episode from our data is analyzed within different perspectives to exemplify the aspects of the mathematical meaning under consideration. In this episode, a primary teacher offers an introduction to fractions and deals with definitions:

T(eacher). … Tell me, what is the difference between fractions and natural numbers? … How do they differ? … Are they the same numbers?
S(tudent). The fractional numbers ... can b... That is, we have a cake and we cut it in six pieces and take one. This is 1/6. The natural numbers are 1, 2, 3, … up to infinity!

T. Good! …

The student presents fractions using a specific example making reference to descriptive characteristics and then simply names natural numbers; the teacher accepts his answer (although a description rather than a definition is provided) and even praises him. What is the meaning of definitions constructed by the students? Is at all connected to the mathematical meaning of definition? The teacher’s urge to offer a familiar context to the students destroys the accuracy of the definition, and, thus students’ understanding of it.

What could we detect examining this episode through the mathematical quality lenses? Error, richness, rigor, or presence of mathematical explanation (Ball, et al., 2008)? There is no error, while the student’s explanation (accepted) has no rigor or any other mathematical quality. Similarly, using Schoenfeld’s framework we could identify less opportunities for mathematics learning. However, both analyses cannot explain the meaning constructed by the students. In an analogous manner, the KQ lenses would examine the connection between the initial question, the specific response, the descriptive explanation and the teacher’s decision to accept it, but wouldn’t explain the meaning constructed by the students. Examining the episode from the teacher’s knowledge perspective (Davis & Simmt, 2006), her management provides no hints about this knowledge, because her decisions are consciously aiming to create a familiar environment for the students.

The above suggest that studies examining teachers’ classroom management of the epistemological features of mathematics as well as the ways in which they understand and interpret this management play a central role in the improvement of mathematics teaching and learning. In the present study we look at teachers’ reflections, interpretations and justifications related to the teaching decisions shaping the mathematical meaning constructed in the classroom.

**Methodology**

The data presented here come from a large project following the development of a new mathematics curriculum promoting mathematical literacy through active learning in social contexts. Here the focus is on three primary teachers, members of a small group chosen on the basis of their substantial teaching experience and promising professional development record, implementing units of the new syllabus over a school year. There were all females with more that fifteen years of experience each, teaching in an experimental primary school in the northern part of the country. Over the year, they discussed, designed, implemented and evaluated a series of lessons in collaboration and under the supervision and support of an advisor/consultant. The lessons, the meetings as well a number of interviews were taped and transcribed providing the data for the study. For the purposes of this report, three transcribed lessons and a follow up semi-structured interview on certain aspects of the teaching session for each teacher are considered.

The research problem pursued was to explore different meanings constructed in these teachers’ classrooms related to mathematical objects, their definitions, attributes and relations to other mathematical objects based on their teaching actions/ management as well as reflections on them.
A combination of content analysis and grounded theory techniques were used to analyze the transcribed lessons and the discourse developed in the interview. In particular, we first identified critical episodes in the teachers’ practices related to the above mentioned features and then analyzed their reflections on these, seeking to identify teachers’ acknowledgement of the mathematical nature of this knowledge.

**Results - Analysis**

In this section, the results for each teacher participant according to the aforementioned scheme of analysis are presented. Due to the limited space, for each teacher, a critical episode is first provided and commented and then her reflection on it is discussed.

**1. Teacher A (5th graders):** The episode below concerns the notion of percentage. Classroom activity concentrates on the completion of a 2x2 table, its rows representing games and its columns the number of students out of 100 voting favorably for each game, in three forms (fraction, decimal or percentage), partly completed. Teaching management focuses on the calculation procedure needed to move from one number representation to the other, especially on division, paying no attention to the equivalence of these representations.

Teacher: Because, 100 divided by 4 makes 25!! Hence, we have 25 out of 100! ... Do you agree?... She had the fraction ¼ and wanted to turn it to decimal… Because here we have 100 students.

T: Danae said before that the decimal fraction is what?
Danæ: A fraction with denominator 10, 100, 1000 …!
T: And since I want 100 as denominator, what am I going to do?
Thanassí: I will multiply it by …
S(tudent): By hundred!!
T (to Thanassí): By what? (She writes on the board simultaneously)
Thanassí: … (noise increasing in the room) … By 25! I will do the same with the numerator
T: (She writes on the board) That is, 25/100! ...
T: How did you come up with this 0,25? Thano?
Thanos: We got 0,25 from the fraction 25/100
T: It was very easy for you to do the decimal number from the decimal fraction …
Adriana: We will perform the division of 1 by 4 and we will find 0,25!
Teacher: Why shall I divide 1 by 4?
S: Because, if I divide the numerator by the denominator, I make decimal!
T: Because it is very easy to make decimal fractions, but it suits me to get numerator with denominator, to divide them, because I am very good at division! … The percentage! ... Have we met the percentage only in graphs so far, eh? We haven’t really worked with percentages … What does 25% mean? This is the new idea that came up there!

Teacher A claims that the mathematical focus of her session is on % and then on pupils becoming able to see the three different number representations (fraction, decimal, percentage). However, the way in which she manages group work and outcomes (dominance of question - answer practice and vague transitions between representations) destroys the mathematical equivalence between representations envisaged by the task. Nevertheless, in her reflection on this she appears unaware that this equivalence should be at the heart of her teaching. At the end of the interview, explaining why
children tended to ‘calculate the decimal to be able to deal with the number’s, she even argues that this might be her fault as she also does this in everyday life.

(2) Teacher B (2\textsuperscript{nd} graders): The episode comes from an introductory session on fractions. The task here focuses on fair/even sharing of certain objects and materials depicted on paper, including a loaf of bread, a lolly-pop, some candies and a pizza. Children adopted strategies of folding and measuring, with the latter being mostly praised by the teacher.

Fotini: I took the ruler and I measured it! I found its half!
T: That is, how much is the biscuit … Take first the biscuit … Let your co-students see… The way the biscuit is, what did you measure? (She shows) … Aaaa! You measured this side from above! And how much did you find that the biscuit is?
Fotini: Twelve centimeters!
T: Oh, you found that the biscuit is 12 centimeters!
Fotini: I cut it into six! …
T: Ah! Go and bring your notebook and show us how you shared your loaf? … Because I haven’t seen many to share this way! … Look, please how did George share his loaf!! Do you agree?
S: Yes!
T: Did he share into two equal pieces?
Students: Yes!
Joanna: Yes! And then I cut it in the middle!
T: How did you cut it in the middle? What did you think? …
Joanna: … I cut it! …

Teacher B explained in her interview that she wanted pupils to ‘explore and discover’ for themselves how this even sharing is carried out, almost forgetting that this was all about fractions. Although we tried to draw her attention to the interesting strategies pupils came up with while trying to share, she insisted in the interview on the importance of children getting familiar with the ‘sharing procedure’ which they had recently discussed in the class. She was stuck with material and kinesthetic activities promoting no connections and generalizations related to the idea of fractions, because “they were familiar, manageable by the children” and therefore appropriate. This interpretation is apparently context-specific, that is, concerns this particular occasion of the teacher’s management. Nonetheless, it is difficult to deny that this occasion can frame students’ conceptions specifically and possibly inappropriately with respect to the mathematical meaning under construction. Even if this is seen as an ‘effective’ introduction to the concept of fractions, it is possible to keep both the teacher and the students stuck to this action driven approach in the future, which allows for a partial conceptualization of the concept at hand.

(3) Teacher C (5\textsuperscript{th} graders): The episode selected comes from a teaching session on comparing fractions. The teacher is closing the lesson by attempting to help students generalize and draw a conclusion. However, both she and the students remain faithful to referring to pizzas and to the quantity “we eat”.

T: But there must be something in order to be able to compare! What have you noticed? How did I place the fractions in order to be able to compare? What is common in each case?
S: Either the denominators or the numerators are the same!
T. When the numerators are the same, which fraction is larger?
Spyros: When the numerators are the same, you eat more when there are fewer pieces!
T: Listen … When the denominators are the same, when do you eat more?
S: When the denominator is smaller!
T: Smaller!! Whereas, when the denominators are the same, when do you eat more?
S: You look whether the numerator is bigger!

The episode above is typical of what we would call a classroom ‘destruction of the mathematical meaning’ case. While working on ordering fractions, teacher C (a Mathematics degree holder, actively involved with research) keeps holding on to pieces and pizzas. Reflecting on this in her interview, she appears aware of the epistemological issues related to the knowledge managed in the classroom, but she is prepared to “sacrifice them”, to deal loosely with these, because of her priority to motivate students, to allow them accessing the mathematical idea to “any cost really" even though distorted (Kaldrimidou, Sakonidis & Tzekaki, 2013, p. 309).

**Discussion and concluding remarks**

We presented the cases of three professionally active and highly motivated teachers with different, however, mathematics education background and varied awareness related to the nature of the mathematical meaning emerging in the classroom during instruction. The first of these teachers seems to be unaware of this aspect, while the second attempts to allow for mathematical elements to emerge, but through teaching practices of practical and partial character. Teacher C knows the importance of the mathematical content but prioritizes accessibility and manageability. In other words, all three teachers, intentionally or not, make choices concerning tasks, elements to highlight and approaches to manage which tend to reduce the mathematical meaning under teaching negotiation. These choices could be attributed to their desire to provide children with an ‘easy’, ‘safe’ and ‘pleasant’ learning environment. Their reflections on their teaching practices indicate that these decisions are strongly influenced by their own experience, regardless of their training and involvement with the pilot project and in accordance with Ponte & Champan’s (2006) position that “teachers eventually develop their own PCK … shaped by their own experiences” (p.469).

In concluding, we would highlight two issues. First, that an analysis revealing classroom construction of the mathematical meaning requires, in addition to the study of teachers’ knowledge, of the mathematical content elaborated and of the tasks employed (Ball, et al., 2008) as well as of the management of students’ actions and thinking (Turner & Rowland, 2010; Davis & Smitt, 2003), a detailed analysis of the epistemological features of the content under construction (Kaldrimidou Sakonidis & Tzekaki, 2013). Teachers need to be aware of the importance of such an analysis of the mathematical meaning construction because they tend to either ignore or underestimate it. It is imperative to become aware that their management of the mathematical objects within the classroom connects or dissociates students from what should be at the heart of their instruction, that is, learning epistemologically legitimate mathematics.

**References**


