Introduction to the papers of TWG17: Theoretical perspectives and approaches in mathematics education research

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Line of thought of the previous working groups in CERME5 to CERME9

Since CERME4 in 2005, theoretical approaches and perspectives have been the topic of an ongoing CERME working group. The idea of the "networking of theories" emerged at CERME 4 and was explored in the subsequent conferences. At CERME 5 (Arzarello et al., 2007), the diversity of theories in the field of mathematics education was regarded as a source of richness, and the networking of theories as a multi-theoretical approach which preserves theoretical identity but also while allowing to bridge the boundaries of theories for a better understanding of teaching and learning. Thereby, the effort was made to make hidden assumptions and relationships of theoretical approaches visible. Principles and heuristics of handling the diversity of theories in empirical research were explored as a new possibility to better grasp the complexity of empirical situations of teaching and learning mathematics, such as the interplay between the individual and the social. Typical heuristics to network theories were to relate different approaches based on research: bottom-up on the one hand and starting from theoretical views top-down on the other, but also mixed types were presented. One interesting result was that not only theoretical principles may be hidden in the use of theories but also the view on the nature of mathematics can be an implicit but relevant feature in the specific theoretical approach.

The central theme of TWG 9 of CERME 6 (Prediger et al., 2010) was investigating how the use of networking strategies may lead to a more comprehensive understanding of the empirical world, what kind of limits have to be faced, and what kind of difficulties have to be considered. In this respect, the questions of commensurability and complementarity of theories came into play. Radford's conceptualizing of theories (2008) as a triad of principles, methodologies and research questions built as a cultural entity of research practice in the semiosphere, a cultural-semiotic space of research as activities, was applied to structure the way networking strategies were used as guiding heuristics which link different aspects of theories. Examples showed that through the networking of theories new questions of 'balance' can be posed, concepts at the boundary of theories may become relevant to solve problems, and theoretically "zooming-in and zooming-out" can be a strategy when theories of different grain size are coordinated. The discussion in this TWG was captured by a dynamic view on theory "as a 'living entity' embedded in the researchers' social, cultural and institutional heritage" (Prediger et al., 2010, p. 1533).

Two years later, the TWG 16 at CERME 7 (Kidron et al., 2011) re-addressed meta-theoretical views on the networking of theories and recognized the need for a meta-theoretical frame for the networking of theories. Projects began to implement the networking of theories as a research practice following the new research aim of building a relationship between corresponding concepts of different theories. Besides the semiosphere (Radford, 2008) which was re-used in the paper presentations as a space where the networking of theories may be conducted, Artigue, Bosch and Gascón (2011) applied the Anthropological Theory of the Didactics to investigate the networking of theories as a research praxeology leading to specification of the relevance of problems and phenomena. Monaghan (2011) described Theoretical Genesis as an analogy to instrumental genesis on the part of the researcher adopting a theoretical view. It is a meta-view on the process of theorizing through the practice of "writing, learning, engagement with research and other voices" (ibid., p. 2498). The interesting point was the insight that also meta-theoretical views on theories largely depend on the cultural-semiotic way (meta-)theories are considered in the specific community. The contributions and discussion of this TWG substantiated the view that the networking of theories may advance the quality of research and lead to more linked and comprehensive results of research.

Whereas teaching and learning mathematics has been the main focus before, in TWG 16 of CERME 8, teacher education provided new directions of considering theories that involve new ways of theorizing on new research objects. This new topic also renewed the understanding of theories as epistemological tools: "the theoretical approaches need to be considered by what they enable researchers and practitioners to do, the questions raised, the regularities identified and described, that is, in a sense the results obtained" (Kidron et al., 2013, p. 2788). Besides addressing goals and practices of networking theories in research, the aspect of time as an additional dimension was emphasized; for example, because the networking strategy of coordinating theories may be executed as an intermediate step in time when theoretical elaborations have reached a particular status, or because coordinating theories may be executed fruitfully in a sequential way.

Although the networking of theories has also been an ongoing topic in TWG 17 of CERME 9, the main focus of this working group was on the notion of theories (Bosch et al., 2015). The discussions once more emphasized that theories are living entities that develop through processes of theorizing in research, beginning with local models, and developing towards more global entities dependent on the requirements of research. These processes result from exploring specific research questions, which may or may not broaden the theories' scope in mathematics education and beyond.

The Thematic Working Group 17 of CERME 10

The TWG 17 of CERME 10 continued the discussion on multi-theoretical approaches (the use of more than one theory in research, see the paper of Chan and Clark in the proceedings), particularly on the networking of theories, but also shifted its attention towards multi-theoretical approaches in design research and the problem of transfer inherent in the tension between home-grown and borrowed theories. The latter aspects have been an ongoing theme, already addressed by Steiner in the conferences on Theory of Mathematics Education of 1985 (Steiner, 1985). The description of the call of TWG 17 of CERME10 showed what kind of contribution was expected:

> This networking of theories approach is also addressed in the TWG 17 of CERME 10. With this working group we want to build on previous work of the group but this time we also want to address more specific topics: theories as prerequisite and result of design research, theorizing in research which involves technology, theories involved in interdisciplinary research with mathematics education. We want to explore how theories are used and built to better understand their role in and beyond mathematics education and the use of theories to inform practice.

Twelve papers and five posters were presented in the TWG 17. All but one poster abstract are published in the proceedings. They are grouped in three topics, the essentials of which will be extracted in the subsequent summary.

Networking of theories approaches

As in previous CERMEs, the discussion within this TWG has addressed the question of how to deal and work with theories, particularly concerning multi-theoretical approaches, which respect the theories' identities and at the same time are able to connect them fruitfully to solve problems in the field and understand the complexity of teaching and learning mathematics better. In terms of the networking of theories approach, the subsequent contributions witness a growing methodological maturity of handling the diversity of theories in research. This maturity is strongly related to deepening and broadening insight about the complex nature of the teaching and learning settings on two intertwined levels, the level of data analysis as well as the level of methodological and theoretical considerations and decisions, both being intertwined.

For example, Tabach, Rasmussen and Dreyfus conduct research to understand how learning in inquiry-based classrooms takes place individually and collectively and how these two learning planes are linked. They *coordinate* two theories, namely Abstraction in Context and Documenting Collective Activity, in a way that represents an innovative methodological step of research within the networking of theories strand that allows to identify how specific ways of coordinating may lead to in-depth insights into the functioning of inquiry-based learning, individually and collectively.

The effect of using networking strategies is directly investigated by Shinno, who has undertaken two case studies following two consecutive networking strategies; namely, coordinating and locally integrating. His research reveals: While coordinating preserves the meanings of the concepts involved as parts of theories, locally integrating theory elements changes the meanings of concepts. The reason for this seems to be that the concepts were *integrated* into a new theoretical framework, with new kinds of issues, questions, and aims. This result substantiates the fact that the meaning of a concept is deeply determined by the theory to which it belongs.

The mathematical workspace (MWS) presented by Nachache and Kuzniak even *requires* to be networked with further theory elements. The MWS originates in practical work with teachers, preserving its pragmatic character in linking semiotic, epistemic and cognitive genesis. Kuzniak et al. illustrate the plasticity of the model by connecting it to several theories or models for teaching mathematics. The reason why this connection is possible is the empirical load: The mathematical workspace model is empirically empty, and therefore allows models with high empirical load to complement it according to the three components offering ways to navigate through them.

Chan and Clarke's purpose in using a multi-theoretical approach is to explore the notions of complementarity and commensurability in an empirical way, a theme that has repeatedly been

addressed in previous working groups. They have instantiated a research project allowing to clarify the concepts of complementarity and commensurability based on analysis of the same data sets of problem solving activities from three different theoretical perspectives. Thus, the common data sets function as boundary objects (Star, 2010), objects that can function in different practices for different purposes, even without the need for consensus (Star, 2010).

In the fifth example, Behrens and Bikner-Ahsbahs add the perspective of the indexicality of signs to their theoretical framework for analyzing gestures related to speech, representations, and a technological tool. This choice is driven by the need to better understand the development of gestures from hand movements on the iPad's digital place value chart towards epistemic gestures, contributing to build knowledge. They show that the process of conceptualizing decimal fractions proceeds as an epistemic shift from *gesture-of* towards *gesture-for*, thus justifying their choice by a methodological result: The indexical nature of signs is a fruitful theoretical perspective for the analysis of epistemic processes as it allows tracing these processes back to their origins.

(Multi-)theoretical approaches in design research

The call explicitly asked for examples of theory use; specifically, in design research. This is particularly challenging because the purpose of theory use in this area is different from that in studies considered before in the networking of theories cases in the previous CERME-TWGs and the previous section. What is special about design research is that the justification of an educational goal requires normative theories, and the ways in which means are implemented to reach the goal – for example in design principles – require prescriptive theories. Finally, there is also a need for theoretical tools to analyze the empirical data of the implementation of the design, using descriptive or explanatory theories (Prediger, 2015). The normative and prescriptive theories developed, for example in the form of design principles, conjecture maps or hypothetical learning trajectories, raise the issue of methodology in relation to theory (cf. Radford, 2008). Kelly (2004) challenged design researchers to come up with what he calls an argumentative grammar – the reasoning from methods via analysis to warranted conclusions, which in the case of randomized controlled trials largely relies on the structure of argumentation.

Bakker takes up the challenge and argues that in design research, as in many other qualitative and mixed methods approaches, scholars cannot rely purely on the structure of argumentation. They need to account for the content too (content of the learning goals, content of core concepts used, context etc.). Bakker argues that design research may need several argumentative grammars and proposes elements of an argumentative grammar that he proposed to experts in design research during an interview study.

Given the multi-theoretical focus of the TWG, it was interesting to see how theories could play different roles in the design of curriculum or learning activities. For example, Johnson and colleagues used different theories for the design of their learning activities and for analyzing the resulting learning processes. The authors show how making theories of different grain sizes — grand theories (Piagetian theory), intermediate theories (Marton's variation theory), and domain specific theories (Thompson's theory of quantitative reasoning) — interact with each other allows designing effective dynamic computer environments and tasks to promote students' learning. Kouropatov and Dreyfus, on the other hand, used two theories for the design of a task-based

curriculum and for analyzing resulting learning processes to feed back into improved design. The two theories – Abstraction in Context and Proceptual Thinking – were of different grain size and had different focuses, which made them complementary. The authors argue that in the process of designing learning units, the different theories have been interwoven while keeping different roles from stage to stage.

Simon and his colleagues, building on constructivist theory and their empirical research, developed an elaboration of Piaget's construct of reflective abstraction for the purpose of undergirding an instructional design theory for promoting mathematical concepts. In conjunction with this elaborated construct, they have articulated an instructional design approach that fosters reflective abstraction of particular concepts. In doing so, they have afforded a change in design research (i.e., teaching experiment methodology) from a focus on students' mathematical reasoning and operations to a focus on the conceptual learning process and designs for promoting that process.

Transfer of theory elements: The tension between home-grown and borrowed theories

The tension between home-grown and borrowed theories was one question previous working groups have dealt with by several contributions. In this working group, the discussion focused on two main interrelated issues. On the one hand, home-grown concepts may bear with them meanings specific to the social and cultural context or the field in which they have been elaborated, and that raises the question how to transfer them into a foreign context or field. Research is needed to address the viability of adapted concepts. On the other hand, theoretical tools or perspectives which are borrowed from other fields must either be adapted to mathematics education or particularized or complemented with content-related theoretical tools in order to be fruitfully put to work. Some of the contributions presented in the working group faced one of these two issues.

For example, Roos' contribution shows how a home-grown concept which emerged in specific cultural context may be difficult to transfer or translate into a different one. More specifically, Roos presents an overview about the concept of *Grundvorstellungen*. This concept emerged in German-speaking countries as a practical tool for teaching. It is impossible to translate and even difficult to explain in English, even if one can recognize the existing links with the notion of *concept image* or with Vergnaud's *theory of conceptual fields*. This difficulty raises the question of how ideas, or even entire theories, which emerge within a specific cultural context and therefore bear cultural-historical meanings, can be communicated on an international scale.

In other cases, home-grown concepts seem to have the potential to be more easily tranferred and adopted in foreign contexts. This is shown in Liljekvist's contribution. She uses the concept of *prosumer*, which stems from sociological research, to understand mathematics teachers re-sourcing and using social media in a Web 2.0 world, linking the two activities of consuming and producing. Even if the concept of *prosumer* bears meanings and values from its native context and has to be futher developed for mathematics education purposes, it has the potential to be easily translated and spread outside as it carries its 'origins' in the term itself.

Adapting borrowed concepts and theories from other fields is not only a question of translation. Mathematics and mathematics education have their own specificities, which must be taken into account when borrowing theoretical tools and concepts from other fields. How that can be done is at the core of the tension between home-grown and borrowed theories. For example, Haspekian and

Roditi faced the issue of adopting general concepts from the field of assessment research in education to a specific research study in mathematics education. The authors developed a methodological tool for analyzing teacher-student interactions in mathematics classes as an adaptive dynamic process. The discussion on their uses of concepts from the assessment field illustrates a way of locally connecting research areas via a shared methodological tool.

Similarly, Georget and Sabra draw on general sociocultural theories to investigate the professional development process occurring in mathematics teachers' communities; however, their study emphasises the need to resort to a complementary theoretical focus addressing specifically the place of mathematics in such communities in order to account effectively for the dynamics which take place in the community. The same tension is also present in the contribution of Zerafa. Zerafa developed an intervention programme addressed to children experiencing mathematics learning difficulties. The design of this programme relies upon the adoption of a large number of borrowed theoretical tools. That raises the question of how to complement borrowed theoretical framework in order to take into account the specific mathematical content at stake.

Researchers can also meet problems concerning the adaptability of theories in context or for purposes different from those in which and for which theories have been developed, even within a given specific field. For example, Benedicto, Gutiérrez and Jaime faced such a problem when applying an existing model, developed to analyse cognitively demanding tasks in the areas of arithmetic and algebra, with the aim to analyse tasks in different mathematical topics. In fact, the original model revealed not to fit adequately their research needs. Their contribution illustrates the processes of analysing the model and the difficulties emerged. Thus, they adapted the model to the new needs and obtained an improved model that did not lose its core meaning while being more widely applicable.

Issues to be considered in future meetings

For future development, several participants expressed the wish to make progress on broader themes that superseded individual presentations. One way to do so is by proposing themes that participants commit themselves to for the next TWG, as was done in previous groups. This would allow the working group to continue working by themes, and discuss the studies in relation to a central theme (say one per day). This can make each author's contribution a case of a more general issue, and allows us to do cross-case analyses in the working group. However, the challenge of this theory group is to balance concreteness and generality in the discussions to make the huge number of theories (48 theories in TWG17) being presented in this working group accessible to all the participants even if they are familiar with some of them. Suggestions for central themes are:

1. **Progress** and **quality**: On what grounds can we decide whether the networking of theories is a contribution to the field? Concepts often used are: complementarity, commensurability, consistency, usability, and fit to purpose. What **methodologies** for research with networking theories are suitable? What **criteria** are suitable for selection and adaptation when networking theories? Criteria may be different for researchers who have a fundamental interest than for educators who work with models that teachers need to work with. Theories can be placed in a framework of different levels (diSessa & Cobb, 2004); are there any heuristics we can derive on good practices of how to coordinate multiple theories; and what

do disciplines outside mathematics education have to offer us in this respect (history and philosophy of science?)

2. What **work** do you do with theories to be able to use them for your purposes? In what respect do you have to adapt a theory or combine it with others? What is the nature of theories used: Describing and explaining learning processes versus offering design heuristics or guidelines? The incompleteness of theoretical models (discussed by Kuzniak et al.) can be an advantage because generality or emptiness can make a model or construct easy to transport (transportable). However, there are also risks when there is a lack of specificity. An issue raised was how theories are insensitive to differences that may matter.
3. What are appropriate **argumentative grammars** for types of research that explicitly have a normative and/or prescriptive element, such as design research? How do we ensure that design embodies theoretical ideas, and how to study the resulting learning as a consequence of the design?
4. How can we deal with concepts that are hard or even impossible to **translate** into English (*milieu*, *Grundvorstellung*, *Stoffdidaktik*, types of participation in Asian countries, …)? The Lexicon Project will have a lot to offer in this respect.

To deepen the understanding of theory and methodology in European research, this thematic working group of CERME should in the future address the issues of quality of the networking of theories in research practice, of the specificities of theories, of identifying different argumentation grammars and scientific ways of communicating culturally bound concepts and theories on the international plane.

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The Thematic Working Group 17 of CERME 10: Grouped topics of TW17 of CERME10

Networking of theories approaches