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A new location-scale model for conditional heavy-tailed distributions

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1. Abstract
We are interested in a location-scale model for heavy-tailed distributions where the covariate is deterministic. We first address the nonparametric estimation of the location and scale functions and derive an estimator of the conditional extreme-value index. Second, new estimators of the extreme conditional quantiles are introduced. The asymptotic properties of the estimators are established under mild assumptions.

2. Model
Let \( Y = a(x) + b(x)Z \),

where \( x \in \mathbb{R} \) is a nonrandom covariate, \( Y \in \mathbb{R} \) a random variable that depends on \( x \) and \( Z \) an independent random variable of \( x \). The location function \( a(\cdot) \) and the scale function \( b(\cdot) \) are unknown. We assume that the survival function of \( Z \) denoted by \( F_Z \) belongs to the class of regulary varying functions at infinity \( F_Z(z) = z^{-1/\gamma}(\gamma > 0) \), where \( \ell \) is a slowly-varying function at infinity and \( \gamma \) is the conditional extreme-value index.

4. Asymptotic results
Assume \( n \) large enough so that \( h = h_n < 1/2 \), the following results are obtained under some regularity conditions on the probability density function of \( Z (f_Z) \) and \( F_Z \) and a Lipschitz condition on the kernel \( K \).

Proposition 1 : Estimation of classical conditional quantiles
Let \( (t_i) \) be a sequence in \([h_n, 1 - h_n]\) and \((\alpha_i)_{i \in \mathbb{Z}} \) a strictly decreasing sequence in \((0, 1)\). If \( n h_{\alpha_n} \to +\infty \) and \( n h_{\alpha_i} \to 0 \) as \( n \to +\infty \), then

\[
\frac{\sqrt{n h_{\alpha_n}}}{b(t_{\alpha_n})} q_{\alpha}(t_{\alpha_n}) - q_{\alpha}(t_{\alpha_n}) \to A_k \text{ with } (k, l) \in \{(1, \ldots, \ell)\}^2 \text{ and } \sqrt{\cdot} \text{ (resp. } A_k) \text{ denotes the maximum (resp. the minimum).}
\]

Proposition 2 : Estimation of location and scale functions
If \( n h_{\alpha_n} \to +\infty \) and \( n h_{\alpha_i} \to 0 \) as \( n \to +\infty \), then for all sequence \((t_i)\) in \([h_n, 1 - h_n]\),

\[
\frac{\sqrt{n h_{\alpha_n}}}{b(t_{\alpha_n})} \left[ a(t_{\alpha_n}) - a(t_{\alpha_i}) \right] \to 
\frac{d}{d\rho} \left[ b(t_{\alpha_n}) - b(t_{\alpha_i}) \right] \text{ where } B \text{ is a given symmetric matrix.}
\]

Proposition 3 : Estimation of extreme conditional quantiles
Let \( (t_i) \) be a sequence in \([h_n, 1 - h_n]\) and \((\alpha_i)_{i \in \mathbb{Z}} \) a positive and strictly decreasing sequence, \((t_{\alpha_i})\) a sequence such that \( t_{\alpha_i} \to 0 \), \( nh_{\alpha_i} \to +\infty \) and \( nh_{\alpha_i} \to 0 \) as \( n \to +\infty \). Then

\[
\frac{\sqrt{n h_{\alpha_n}}}{b(t_{\alpha_n})} q_{\alpha_n}(t_{\alpha_n}) - q_{\alpha_n}(t_{\alpha_n}) \to A_k \text{ with } (k, l) \in \{(1, \ldots, \ell)\}^2 \text{ and } \sqrt{\cdot} \text{ (resp. } A_k) \text{ denotes the maximum (resp. the minimum).}
\]

6. References

5. Conclusion
The study of the problem in fixed design, which we have made, reveals interesting theoretical results. As a perspective, we are considering the conditional extreme-value index estimation and a validation of our results on simulations. We also plan to extend these results to the random framework.

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