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A new location-scale model for conditional heavy-tailed distributions
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1. Abstract
We are interested in a location-scale model for heavy-tailed distributions where the covariate is deterministic. We first address the nonparametric estimation of the location and scale functions and derive an estimator of the conditional extreme-value index. Second, new estimators of the extreme conditional quantiles are introduced. The asymptotic properties of the estimators are established under mild assumptions.

2. Model
\[ Y = a(x) + b(x)Z, \]
where \( x \in \mathbb{R} \) is a nonrandom covariate, \( Y \in \mathbb{R} \) a random variable that depends on \( x \) and \( Z \) an independent random variable of \( x \). The location function \( a(\cdot) \) and the scale function \( b(\cdot) \) are unknown. We assume that the survival function of \( Z \) denoted by \( F_Z \) belongs to the class of regularly varying functions at infinity
\[ F_Z(z) = z^{-\gamma} f(z), \quad \gamma > 0, \]
where \( \ell \) is a slowly-varying function at infinity and \( \gamma \) is the conditional extreme-value index.

4. Asymptotic results
Assume \( n \) large enough so that \( h = h_n < 1/2 \), the following results are obtained under some regularity conditions on the probability density function of \( Z (f_Z) \) and \( F_Z \) and a Lipschitz condition on the kernel \( K \).

Proposition 1: Estimation of classical conditional quantiles
Let \((t_n)\) be a sequence in \([0,1] - [0,h] \) and \((\alpha_j), j \geq 1 \) a strictly decreasing sequence in \((0,1)\). If \( n h \rightarrow +\infty \) and \( n h^2 \rightarrow 0 \) as \( n \rightarrow +\infty \), then
\[ \frac{\sqrt{n}h}{b(t_n)}\hat{q}_n(a_j|t_n) - q_v(a_j|t_n) \rightarrow_d \sqrt{Q_2(u)}1_{|1|}K^2(u)du A_i, \]
where \( A_i = \frac{\alpha_n(1-\alpha_n)}{f_Z(q_z(a_i))(q_z(a_i))} \) for all \((k,j) \in \{1, \ldots, J\} \) and \( \sqrt{v(\alpha)} \) denotes the maximum (resp. the minimum).

Proposition 2: Estimation of location and scale functions
If \( n h \rightarrow +\infty \) and \( n h^2 \rightarrow 0 \) as \( n \rightarrow +\infty \), then for all sequence \((t_n)\) in \([0,1] - [0,h] \),
\[ \frac{\sqrt{n}h}{b(t_n)}\hat{a}(t_n) - a(t_n) \rightarrow_d \sqrt{Q_2(u)}1_{|1|}K^2(u)du B, \]
where \( B \) is a given symmetric matrix.

Proposition 3: Estimation of extreme conditional quantiles
Let \((t_n)\) be a sequence in \([0,1] - [0,h] \) and \((\alpha_j), j \geq 1 \) a positive and strictly decreasing sequence, \((\gamma_k)\), a sequence such that \( \gamma_k \rightarrow 0 \), \( nh \gamma_k \rightarrow +\infty \) and \( nh \gamma_k^2 \rightarrow 0 \) as \( n \rightarrow +\infty \). Then
\[ \frac{\sqrt{n}h\gamma_k}{b(t_n)q_z(a_i)}\hat{q}_n(\gamma_k|t_n) - q_v(\gamma_k|t_n) \rightarrow_d \sqrt{Q_2(u)}1_{|1|}K^2(u)du C, \]
where \( C_{ij} = \frac{1}{\gamma_{ij}}, \) for all \((k,j) \in \{1, \ldots, J\} \).

6. References

5. Conclusion
The study of the problem in fixed design, which we have made, reveals interesting theoretical results. As a perspective, we are considering the conditional extreme-value index estimation and a validation of our results on simulations. We also plan to extend these results to the random framework.

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