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A new location-scale model for conditional heavy-tailed distributions

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1. Abstract
We are interested in a location-scale model for heavy-tailed distributions where the covariate is deterministic. We first address the nonparametric estimation of the location and scale functions and derive an estimator of the conditional extreme-value index. Second, new estimators of the extreme conditional quantiles are introduced. The asymptotic properties of the estimators are established under mild assumptions.

2. Model
\[ Y = a(x) + b(x)Z, \]
where \( x \in \mathbb{R} \) is a nonrandom covariate, \( Y \in \mathbb{R} \) a random variable that depends on \( x \) and \( Z \) an independent random variable of \( x \). The location function \( a(\cdot) \) and the scale function \( b(\cdot) \) are unknown. We assume that the survival function of \( Z \) denoted by \( F_Z \) belongs to the class of regualry varying functions at infinity
\[ F_Z(z) = z^{-1/\gamma}(\gamma), \quad \gamma > 0, \]
where \( \gamma \) is a slowly-varying function at infinity and \( \gamma \) is the conditional extreme-value index.

3. Estimators

Let \( \{(x_i, y_i) : i = 1, \ldots, n\} \) be a \( n \)-sample of observations such that \( Y_i = a(x_i) + b(x_i)Z_i \), \( i = 1, \ldots, n \), where \( Z_i \) are independent and identically distributed. For simplicity, the design points are of the form \( x_i = i/n, i = 1, \ldots, n \) and \( x_0 = 0 \) by convention.

Notations
- \( \hat{F}_y(\cdot | x) \) : the conditional survival function of \( Y \) given \( x \).
- \( \hat{F}_y(\alpha | x) \) : the conditional \( \alpha \)-th quantile of \( Y \) given \( x \).

Kernel estimator of \( \hat{F}_y(\cdot | x) \):
\[ \hat{F}_y(\cdot | x) = \frac{1}{n} \sum_{i=1}^{n} K_{\alpha}((x_i-x)/\hat{b}(x)) dt, \quad (x, y) \in [0, 1] \times \mathbb{R}, \]
where the bandwidth \( h = \hat{h}_n \) is a nonrandom sequence such that \( h \rightarrow 0 \) as \( n \rightarrow \infty \).

Kernel estimator of \( \hat{F}_y(\cdot | x) \):
\[ \hat{a}(\cdot | x) = \frac{1}{n} \sum_{i=1}^{n} \alpha - \hat{a}(x), \quad \text{for all } (k, l) \in \{1, \ldots, J\} \text{ and } v (\text{resp. } \alpha), \]
denotes the maximum (resp. the minimum).

4. Asymptotic results
Assume \( n \) large enough so that \( h = h_n < 1/2 \), the following results are obtained under some regularity conditions on the probability density function of \( Z \) and \( F_Z \) and a lipschitzian condition on the kernel \( K \).

Proposition 1: Estimation of classical conditional quantiles
Let \( (t_{ik})_i \) be a sequence in \( [h_n, 1 - h_n] \) and \( (\alpha_{ik})_i \) a strictly decreasing sequence in \( (0, 1) \). If \( nh_n \rightarrow +\infty \) and \( nh_n^d \rightarrow 0 \) as \( n \rightarrow +\infty \), then
\[ \sqrt{nh_n} \left( \frac{a(t_{ik})}{b(t_{ik})} - \hat{a}(t_{ik}) \right) \rightarrow \mathcal{N}(0, \alpha t_{ik}, 1_{1/\gamma} K^2(u)du A_i), \]
where \( A_i = \frac{\alpha}{f Z(a(t_{ik})) f Z(a(t_{ik}))} \) for all \( (k, l) \in \{1, \ldots, J\}^2 \).

Proposition 2: Estimation of location and scale functions
If \( nh_n \rightarrow +\infty \) and \( nh_n^d \rightarrow 0 \) as \( n \rightarrow +\infty \), then for all sequence \( (t_{ik})_i \) in \( [h_n, 1 - h_n] \),
\[ \sqrt{nh_n} \left( \frac{\hat{a}(t_{ik})}{b(t_{ik})} - a(t_{ik}) \right) \rightarrow \mathcal{N}(0, \alpha t_{ik}, 1_{1/\gamma} K^2(u)du B), \]
where \( B \) is a given symmetric matrix.

Proposition 3: Estimation of extreme conditional quantiles
Let \( (t_{ik})_i \) be a sequence in \( [h_n, 1 - h_n] \) positive and strictly decreasing sequence, \( (\alpha_{ik})_i \) a sequence such that \( \alpha = 0, nh_n(\alpha) \rightarrow +\infty \) and \( nh_n^{\alpha_{ik}} \rightarrow 0 \) as \( n \rightarrow +\infty \). Then,
\[ \sqrt{nh_n^{\alpha_{ik}}} \left( \frac{\hat{a}(t_{ik})}{b(t_{ik})} - a(t_{ik}) \right) \rightarrow \mathcal{N}(0, \alpha t_{ik}, 1_{1/\gamma} K^2(u)du C), \]
where \( C_i = \frac{1}{(\hat{a}(t_{ik}))^2} \) for all \( (k, l) \in \{1, \ldots, J\}^2 \).

6. References

5. Conclusion
The study of the problem in fixed design, which we have made, reveals interesting theoretical results. As a perspective, we are considering the conditional extreme-value index estimation and a validation of our results on simulations. We also plan to extend these results to the random framework.

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