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Henrik P. Bang[1], Niels Grønbæk[2] and Claus R. Larsen[3]

[1]Christianshavns Gymnasium, Denmark; hb@cg-gym.dk

[2]Department of Mathematical Sciences, University of Copenhagen, Denmark; gronbaek@math.ku.dk

[3]Christianshavns Gymnasium, Denmark; cl@cg-gym.dk

We outline a model for analyzing the use of ICT-tools, in particular CAS, in teaching designs employed by 'generic' teachers. Our model uses the business economics concepts "out-" and "in-sourcing" as metaphors within the dialectics of tool and content for the planning of teaching. Out-sourcing is done in order to enhance outcomes through external partners. The converse concept of in-sourcing refers to internal sourcing. We shall adhere to the framework of the anthropological theory of the didactic, viewing out- and in-sourcing primarily as decisions about the technology component of praxeologies. We use the model on a concrete example from Danish upper secondary mathematics to reveal what underlies teachers' decisions (deliberate or spontaneous) to incorporate instrumented approaches.

Introduction

Has use of computers in schools resulted in better education? With the steadily growing take-up of technology throughout the world, this question is as important as ever. The role and importance of technology has undergone phases from initial excitement to, more recently, a mixture of cautious optimism, moderate skepticism, and the stance that the use of computers might forfeit the true values of educational discipline. A recent, rather extensive international report (OECD, 2015) indicates countries' improvements in learning by a number of measures against their investment in ICT. The foreword summarizes the implications for educational policy:

Mere embracement of ICT in itself is at best harmless. Access to ICT does not automatically improve learning, "The results also show no appreciable improvements in student achievement in reading, mathematics or science in the countries that had invested heavily in ICT for education".

In person teacher-learner contact is essential, "One interpretation of all this is that building deep, conceptual understanding and higher-order thinking requires intensive teacher-student interactions, and technology sometimes distracts from this valuable human engagement".

There is a need for alignment of technology and learning: "Another interpretation is that we have not yet become good enough at the kind of pedagogies that make the most of technology; that adding 21st-century technologies to 20th-century teaching practices will just dilute the effectiveness of teaching".

A deeply rooted trust in the progressive power of ICT (but with a somewhat unimaginative scope to traditional learning material), "Why should students be limited to a textbook that was printed two years ago, and maybe designed ten years ago, when they could have access to the world's best and most up-to-date textbook?".

A tall order on teachers to meet the expectations (by pedagogy rather than by subject discipline), “Perhaps most importantly, technology can support new pedagogies … it is vital that teachers become active agents for change” (OECD 2012).

Other meta-studies (Higgins, Xiao, & Katsipataki, 2012; MBUL, 2015) point to similar conclusions, “There is at most a weak positive correlation between bulk use of computers and learning outcome.”

In contrast, there are numerous reports on very fruitful and insight-giving use of computers (Böhm, Forbes, Herweyers, Hugelshofer, & Schomacker, 2004; Heid, 2003; Nabb, 2010). These are often the result of computer focused teaching designs that are part of didactic research or teaching development, carried out by dedicated teachers. Therefore, the question is not how much, but how, about what, and by whom.

The Danish landscape

In the OECD report Denmark is ranked second in use of computers. From the mid 1990’s there has been rapidly growing CAS-use in Danish high schools, starting with graphing calculators and accelerated through the extensive use of PC’s from around 2005. The situation now is that most high schools use Maple, TI-Nspire, Geogebra, and/or a CAS-tool specially developed for Danish high schools (WordMat, a CAS engine integrated within Microsoft Word). Students bring their own PC to the classroom, and use of PC is required at examinations. Initially, the transformation was carried through by progressive and CAS-curious teachers, many of whom were inspired by reform pedagogic ideas that supported a shift from abstract mathematics towards applications and more intuitive conceptual understandings. There was (and is) also an element of believing in diffusion: If CAS helps advanced (university) students to solve advanced problems, we might as well use CAS to help less advanced students solve less advanced (but to them difficult) problems. The educational system eagerly supported the development. Mathematics has been a vehicle for use of PC in other subjects, and examinations using CAS could (possibly) help more students achieving higher levels in math. Since 2005, CAS has gone from being a tool for enthusiastic teachers to a tool for everyone, including teachers with less interest and less competency in CAS. There has been no essential change in the standard curriculum (only minor ones allowing time for, say, χ^2-test) – and standards for CAS use have not been introduced. On the contrary, the curriculum endorses the use of CAS in mathematical modeling and concept building, but without any indication of how, and in connection with what topics, to carry this out. In this landscape, many teachers have developed templates that students are allowed to use in exams, and the preparation of students to use these has become an important activity during normal lessons. Students of teachers, who for one reason or another disfavor such, may find templates on the internet or borrow from friends. Most of such templates have little *epistemic value* and a rather narrow *pragmatic value* in the sense of (Artigue, 2002) towards solving (standard) problems. With CAS at the national tests these tendencies of trivialization are even more pronounced, as problems must be formulated to be equally solvable on different CAS platforms,

Denmark’s extended use of computers in education reflects of course a trend in society but is also as described above to a large degree the result of explicit educational policies. Hence, a teacher has to find his/her pathway through the affordances, constraints, possibilities etc. stipulated by official guidelines, curriculum and instruction plans. As indicated, successful use of computers does seemingly not scale up (MBUL, 2015). In order to understand the reasons for this better we propose an analysis model to help understand teachers' decisions on use of computers in mathematics teaching.

Theoretic framework

As our proposal aims at elucidating teaching in an institutional context we find the anthropological theory of the didactics (ATD, Chevallard, 1999) well suited. We start by briefly recalling the most important concepts of ATD that we shall use. Mathematics as an enterprise (educational as well as scientific) is seen as human activity composed of two blocks each with two parts, a *praxis block* comprising types of problems, *tasks,* with *techniques* to accomplish these and a *theory part* comprising *technology* and *theory*. Together such two blocks are termed a *praxeology* (praxis + logos). Tasks are the immediate goals of the activity, i.e. finding the slope of a graph of a function at a given point. A task can be accomplished by several techniques, i.e. plotting the graph on a computer, zooming in on the point in question until the graph appears linear and reading off the slope. The technology part concerns the discipline discourse of the technique and its relation to the tasks, i.e. the scope and limits of computer rendering of graphs (in relation to variation of functions). The theory part is a discourse on the technology part and its relation to the praxis block, i.e. on the concept of linear approximation that the sketched approach leads and on how it is related to a larger body of mathematical knowledge and practice, for instance that of the theory of differentiation.

We would like to stress a couple of points. A given task can be unfolded in many praxeologies. To choose, detail and organize such unfoldings is the essence of teaching design. Any praxeology has underlying praxeologies, i.e. praxeologies aimed at slope of a linear function, and is itself related to/part of other praxeologies. A praxeology always comprises all four parts. This is one key point of the analysis in (Barbé, Bosch, Espinoza, & Gascon, 2005).

Praxeologies take place within organizations of mathematical practice and knowledge. In ATD such organizations are formed by two components, a mathematical body consisting of a totality of objects, concepts, statements, interrelationships, procedures, etc., termed the *scholarly knowledge* and an *institution* of society within which this body is taught, manifesting possibilities and constraints for acquisition of learning. The passage from scholarly knowledge to its institutional version (which has more components than indicated) is in ATD conceptualized as *didactic transposition.*

In (Artigue, 2002, p. 271) it is noted that didactic transposition in its first version was described with respect to rather traditional scholarly mathematics. In order to underline a computerized setting Artigue has singled out the term *instrumented techniques.*

The model

In ATD the teacher is considered as the director of the learners' didactical processes (Barbé et al., 2005), that is, responsible for the establishment of relations between learners and organizations of knowledge within institutions. We shall take this a step further, viewing it as production of learning outcomes through production activities, the praxeologies. For this production, the teacher has at his/her disposal a palette of resources, typically in terms of techniques (along with their theoretical block) to solve tasks. The 'employees' who use the techniques towards the production are the students.

This setup is very similar to a business economic model of the production of a corporation. In order to enhance the outcome a corporation director makes sourcing decisions on the allocation of resources. In modern terminology, one speaks of outsourcing, insourcing, backsourcing[1], “outsourcing involves allocating or reallocating business activities (both service and/or manufacturing activities) from an internal source to an external source” (Schniederjans, Schniederjans, & Schniederjans, 2005, p.3). Insourcing can be viewed as an allocation or reallocation of resources internally within the same organization. Any business activity can be outsourced or insourced (dichotomy), but this decision is crucial to the success of the corporation. The basic idea of outsourcing is old, essentially, it is the dictum ‘buy or make’. However, in the last few decades, outsourcing has grown almost explosively. A main reason for this is the development of ICT. But outsourcing is risky. It is reported (Schniederjans et al., 2005, p. 12) that half of all outsourcing agreements fail due to lack of appropriate analysis, and the necessity of *strategic planning* has become evident. There seems to be general acceptance (Schniederjans et al., 2005, p. 9) that such starts with an analysis to identify the strengths of the corporation, in terms of *core activities* (‘core competencies’ in (Schniederjans et al., 2005). Loosely, a core activity is what the corporation does better than its competitors and possible outsourcing providers. Core activities must be insourced, non-core activities are candidates for outsourcing and a balanced decision to achieve the strategic goals must be made. Key advantages of outsourcing of inspiration for didactic equivalents are: focus on core activities, gain of outside technology, enhancement of capacity and lower cost, whereas some key disadvantages are loss of control, increased costs, negative impact on employees’ morale and difficulties in managing relationships with outsourcing provider.

In the didactic version, the *client* is a didactically transposed knowledge organization along with the teacher(s) to direct the didactic processes. The *outsourcing provider* is an external knowledge organization, typically within a CAS. In the business model, external/internal refers to ownership. For our purpose the fundamental feature of ownership is that it allows for control of processes, i.e. outsourcing implies loss of control. We shall take this as the defining property. Hence *outsourcing a mathematical activity means allocating it to a resource at the price of giving up control of processes*. A blunt example could be a teacher encouraging students to find solutions to homework on the internet; a more elucidating example is employment of instrumented techniques in the form of black-box applications of CAS. As pointed out, any activity can be outsourced or insourced, that is full praxeologies, be it punctual, local or regional, or just parts of praxeologies, typically the praxis block. To be more precise, the starting point of CAS outsourcing is typically a problematic task to be solved by the outsourcing provider’s technique thereby creating a transformed or new praxeology. We stress that a CAS such as Maple is not solely a provider. To the extent that a teacher exercises control over CAS processes, these are considered insourced. Outsourcing to CAS is a more restrictive concept than mere use of CAS. (Teacher control must be distinguished from student control as the latter is the result of the first, and perhaps of other competencies, acquired without the influence of the teacher.)

[1] Backsourcing means reallocating tasks from external sources to internal. This could be in order to regain control of the production process but could also be imposed by outside regulatives. In an educational context such could be new stipulations of use of CAS at national tests.

A simple example (an object of many controversies) illustrates the concepts. Arithmetical computations require a careful analysis of what are core activities that accordingly should be insourced. Depending on (long-term) learning goals, these could be the systematics of paper and pencil algorithms, skills of mental arithmetic with “nice numbers”, etc. On the other hand, multiplication of many-digit numbers is hardly a core activity and is therefore a candidate for outsourcing to calculators[2]. This does not mean that tasks, which can be solved by mental computation, should not be insourced by calculator techniques. The point is that the core activity of mental computing may give control also of some calculator computations. Note that a calculator praxeology is completely different from its non-instrumented equivalents, for instance its theory part may involve representation of numbers in a finite memory.

The very decision to use CAS (or other *instrumented techniques*) involves, regardless of its specific use, outsourcing. The teacher has no control of the coding that underlies the CAS, the syntax, the defaults, the library of routines, etc. Most CAS-tools are developed with teaching in mind, at least partly. Perhaps most importantly, the CAS design may have didactic intensions, which the teacher may surrender to if not disable. Maple’s ‘clickable math’ is a good example of this. The relationship between non-instrumented mathematics and computerized mathematics resembles that of a *strategic partnership* with mutual outsourcing. This relationship is dialectic in nature. The potential of CAS in mathematical praxeologies needs non-instrumented mathematics to be redeemed.

There is of course nothing new in the very idea of strategic planning. Mathematical activity has at all times involved use of ‘non-controlled’ components and didactic considerations have always had this as a condition. The modern aspect of CAS is the magnitude of impact, calling for a much clearer elaboration of such planning. The addition of the concepts of out- and insourcing to ATD offers a model for reflection on crucial choices between instrumented and non-instrumented praxeologies on basis on insight in the CAS-tool and in possible mathematical activities. On one hand, the model gives a framework for investigation of ordinary teachers’ undertakings and perhaps more importantly, of what is not undertaken. On the other, it provides a strategic planning scheme for the teacher cf. (Schniederjans et al., 2005, Figure 1.3), where II+III are the crux of the matter:

I. Establishment of content and learning goals of the mathematical organization to be taught
II. Detailed analysis of subject matter and activities of possible praxeologies.
III. Identification
    a. Core activities
    b. Non-core activities
IV. Sourcing decisions
    a. Core activities are insourced
    b. Non-core activities are candidates for outsourcing.

[2] A business equivalent of the historically initial excitement about the freeing potential of calculators and the afterthought concerning (permanent?) loss of core activities: The reservation system of the flight company TWA was superior to those of its competitors, i.e. a core activity, but was outsourced in the 90’s. TWA never regained its market share and went out of business in 2001.

How do teachers decide on what is a core activity? The dialectics of pragmatic and epistemic value (Artigue, 2002) seems inevitable, but is not directly reflected in the dichotomy of out- and in-sourcing. The computational power of several thousand digits, obviously to be outsourced, may have epistemic value in relation to approximation by decimal expansions. The pragmatic value of graphing of polynomials may be an asset of outsourcing in order to study whether polynomials have desired properties, which are considered epistemic of certain mathematical models. In other praxeologies graphing, by hand or by CAS, may be insourced.

Methodology for prospective work with the model

We aim at a fully-fledged model to give a general description of ordinary teachers' implementation of CAS and through this, an insight in the scaling-up question mentioned previously. Our first step is to analyze a rather extensive material of reports on teaching designs with CAS, succeeded by reflections on further development, modification and refinement of our model. These reports are produced by project participants at Center for Computer based Mathematics Education (CMU[3]), University of Copenhagen, as the last step of a reflective practitioner process. The mission of CMU is to support use of CAS in Danish high schools respecting core mathematics qualities in order to reverse the trivialization tendencies described above. The only condition for participating is a moral subscription to this mission. Thus, teachers have been free to choose subject, CAS platform (within CMU's coaching expertise), design of teaching, etc. This first round of analysis data is collected in contemplation of dissemination, rather than evidencing answers to research questions, but in a systematic way that allows for a grounded theory approach.[4]

Having an elaborated model, we intend a large-scale investigation on Danish mathematics high school teachers' choices and rationale for outsourcing to CAS.

A sketch of an analysis: a praxeology of finding derivatives

The so-called 3-steps method of finding the derivative, $f'(x_0)$, of a function is the canonical approach to differentiation in Danish high schools, explicitly mentioned in official guidelines. We recall: (1) With $\Delta y = f(x_0 + \Delta x) - f(x_0)$ form the fraction $\frac{\Delta y}{\Delta x}$; (2) reduce the fraction to make $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ accessible (3) find the limit $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ (if it exists). These are tasks in three praxeologies with rather separate theory blocks involving algebra, topology and geometry. In a CMU – project (Differentialregning, tretrinsreglen), the teacher, in the sequel L, wants to improve on students' understanding of the method by CAS-outsourcing "to give the students hand-on experience of secant and tangent slope and limits through experimentation with CAS sheets" (our transl.). An outsourcing strategy like this is rather common in Denmark. L has worked on the teaching problematics of the 3-steps method for many years 'without really understanding why students find it so difficult' (our

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[3] The Danish Industry Foundation, Department of Mathematical Sciences at University of Copenhagen, The Danish Ministry of Education, and Maplesoft Inc. sponsor CMU.

[4] For further details about the CMU material, we refer to the CERME 10 poster of TWG 15 (Bang, Grønbæk, & Larsen, 2017)

translation from Danish taken from the project report). This time L starts with a thorough analysis of prerequisites ending in 12 points. L decides to use CAS in the case of $f(x) = x^2$ on three points of the 12: '(5) computing slope of a straight line; (7) understanding what tangent and slope are; (12) understanding (the concept of) limit' (our transl.). A few observations: (A) L is by his very wish to understand reasons for learning difficulties led to in- and outsourcing considerations. (B) There is a tendency to regard pragmatic and epistemic values as separate features: (5) is pragmatic and (7) & (12) are (by L phrased as) aiming at epistemic value. (C) Some core activities are recognized as such and insourced, i.e. algebraic reduction of polynomial expressions such as Δy for $y = a\,x^2 + b\,x + c$ - partly insourced to paper and pencil, partly to CAS. Other core activities are outsourced, i.e. use of sliders on the graph $y = x + 2$ to find $\lim_{x \to 0} x + 2$, as last step in the 3-step method with x replacing Δx; (D) non-core activities are not spelled out. What is it that sliders can do for secant-tangent considerations without sacrificing core activities? (E) Affective aspects are outsourced: 'CAS tools should … activate students and challenge their desire to … explore mathematical problems' (our transl.) From L's reflections, it appears that the outsourcing (D) is indirectly motivated by the textbook treatment of the subject. Textbooks rarely have core activity considerations, but rather bold instigations to CAS use. This risk of dilution of mathematical competency is pinpointed by the concept 'outsourcing core activities'.

Further use and development

L is an example of a teacher with neither desire nor reputation to be a front-runner, but navigating resourcefully and dedicatedly under post-modern circumstances of mathematics teaching. Our observations (A), … (E) apply to many teachers (CMU, 2015; CMU, 2016) so the sketch of an out-/in-source analysis of L's project is testimony that our approach may have potential for shedding light of the kind of decisions, with shortcomings and potentials, that ordinary teachers make. The business metaphor seems confluent with natural praxis of resource considerations, thus providing a framework for large-scale investigations much similar to studies of business economics forces that govern trade and production. One may fear a risk of introducing yet another business corporation model to education. Outsourcing is growing in business due to the incitement of fierce competition. While perhaps tempting, a flat educational interpretation of this is misleading. The rooting in a business model is motivated within a local or regional mathematical organization through its level of didactic co-determination. Even though mathematics may be seen as a productive force, learning outcome is not a commodity. It cannot be bettered simply through optimization tactics.

Our sketch has focused on director decisions, i.e. the teacher's planning. Further development must include employees, i.e. students, that is, the last step of the didactical transposition: matter taught → matter learned.

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