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This paper presents results from a survey exploring the kind of resources that engineering undergraduates (N=201) use when studying for their mathematics modules. By using Factor analysis, we were able to produce a typology of these resources. The resulted typology was further analysed by combining Leontiev's version of Activity Theory and Wartofsky's hierarchy of artefacts. This helped us to draw links between the tools that students use and infer about their learning actions.

Keywords: Blended learning, educational resources, student centered learning, activity theory, Wartofsky.

Introduction

Historical records suggest that using tools has always been inseparable from expressing and doing mathematics (Roberts, Leung, & Lins, 2012) with numerous examples demonstrating the capacity of tools to influence the development of mathematics itself as a scientific discipline (Laborde & Sträßer, 2009)[1]. Since the end of the nineteenth century many different types of tools have been used for the teaching and learning of mathematics (Kidwell, Ackerberg-Hastings, & Roberts, 2008) however for many the term "technology" corresponds to electronic or digital in nature tools (e.g. calculators) something that manifests a kind of historical amnesia (Roberts et al., 2012). Students nowadays have access to a plethora of digital/online resources that they could use alongside more "traditional" ones (e.g. textbooks, lecturers or their own notes) and thus blend their learning. As a matter of fact, Masie (2006) asserts that this has been always the case as students were always combining resources in order to support their learning. The notion of blended learning (BL) has been introduced almost 30 years ago however, the term has not been clearly defined yet (Bos & Brand-Gruwel, 2016; Graham, 2013; Torrisi-Steele & Drew, 2013). The three main definitions of BL used in education are (Sharma, 2010): BL as a *combination of face-to-face and on-line teaching*; BL as a *combination of technologies* and; BL as a *combination of pedagogical methodologies*. In this paper, we are mainly referring to BL as a combination of technologies or better as a combination of tools since each era has its own technologies not only digital ones. Despite what its name implies, BL has received criticisms and some authors argue that it should be rather called *blended teaching* because current views adopt a teacher-centred and not a student oriented perspective by focusing on the resources that instructors choose for their students (De George Walker & Keeffe, 2010; Oliver & Trigwell, 2005). If students indeed blend their learning by mixing different resources not just digital or the ones provided to them by their institution, *what kind of* resources *do they blend* and *how can these* resources *be classified*? In terms of the resources that students use, we have found a lack of empirical studies exploring the kind of resources that undergraduates *themselves choose and use*, with previous studies focusing

[1] Although many authors consider the terms "tools", "artefacts", "instruments" or "resources" to be different, for the purposes of this paper we treat them as having the same meaning.

mostly on digital/online or institutionally-led resources and thus neglecting the "blended" nature of their learning (for a short review see Anastasakis, Robinson, & Lerman, 2016). In effect, our sense of the field aligns with comments from authors suggesting that the student perspective has not been taken into account in the BL literature (e.g. Ituma, 2011; López-Pérez, Pérez-López, & Rodríguez-Ariza, 2011). Having previously identified the kind of resources that a sample of engineering undergraduates uses when studying mathematics (Anastasakis et al. 2016), our aim in this paper was to propose an empirically based typology of these tools.

Theoretical framework

Long before the dominance of the world wide web and computers, many researchers had emphasised the significance of tools in our everyday activities; a well-developed theoretical account of human praxis which emphasises the role of physical tools is second generation Activity Theory (AT) (Leontiev, 1981). From an AT perspective, our relationship with the "objective" world is mediated by tools: the central role of tool mediation within this framework is due to the fact that tools shape the ways we interact with reality and they reflect past people's experiences and practices (Kaptelinin & Nardi, 2006). In AT, activities have a hierarchical structure and they consist of three layers (*Ibid*.): at the top is the activity itself directed towards the object of the activity (our overarching goal); in the middle lie actions (what we do) directed at goals (what we want to achieve) and; finally, at the lower layer we find operations (non-conscious, routine processes) which are directed to conditions (non-subjective factors that affect our actions). In sum, AT asserts that tools are the means by which subjects are trying to achieve their goals and in this sense tools are bounded with a subject's practice. Despite the central position that tools hold in AT, little is written from this perspective in regards to how they can be categorised; among the little accounts found in the literature, is Wartofsky's typology of artefacts. Wartofsky (1973) considered tools as the genes of our cultural evolution and proposed that they can be classified into *primary*, *secondary* and *tertiary* artefacts. As primary are considered the tools themselves (Engeström, 1990); secondary artefacts represent "modes of action using primary artefacts" (Cole, 1996, p. 121) and they "synthesise the ways and procedures of using instruments and materials" (Miettinen, 1999, p. 189). Finally, tertiary artefacts are those which emphasise creativity (Kaptelinin & Nardi, 2006) and "transcend[s] the more immediate necessities of productive praxis" (Wartofsky, 1973). As an example of what constitutes a primary and a secondary artefact, Bussi and Mariotti (2008) refer to the abacus: the abacus itself is a primary artefact and the ways of using abaci for counting, keeping records or making computations represents a secondary artefact. Engestrøm notes that Wartofsky's typology is closely related to Leontiev's levels of activity (Petersen, Madsen, & Kjær, 2002): primary artefacts correspond to the level of operations/conditions, secondary to the level of actions/goals while tertiary to the level of the activity (Engeström, 1990; 2015). Our focus in this paper are primary and secondary artefacts.

Method

This study is part of a doctoral project that aims to identify the kind of tools that undergraduates use when studying mathematics, how these tools are used and the reasons for using them. During the autumn term, a paper-based questionnaire was administered to four different groups of second year engineering students in Loughborough university and in total 201 completed it. Loughborough has one of the largest cohorts of engineering students (over 3000 undergraduates) in the UK and a well established provision of Mathematics Support (http://www.lboro.ac.uk/departments/mlsc). It has also led on significant projects producing high quality printed material (e.g. the HELM project: http://helm.lboro.ac.uk). The questionnaire consists of three main parts and its design was guided by Activity Theory (AT) (Leontiev, 1981). Here we report only on the part related to the resources (tools) that undergraduates use. In this, students were explicitly asked to identify how often they use a list of 14 resources on a 6-point semantic scale (1/Never, 2, 3, 4, 5, 6/Always) with two additional open ended items for other resources not listed in the questionnaire. The list was based on our literature review, five in depth interviews with undergraduates conducted in 2015 and the resources that Loughborough University offers to students e.g. the Learn website (university's VLE). The list of resources was carefully generated and encompasses a great variety of tools available to students; in this way it reflects -to a certain degree- students' reality as learners when it comes to the resources they use. Students were also asked to identify which five of these 14 resources they use the most (top-5) and rank them in a descending order (not reported here).

Analysis and results

Summary statistics

Results for the tools that students use are presented in Figure 1. These results have been already presented elsewhere (Anastasakis et al., 2016) but we include them here for clarity. By using each tool's mean, we categorised them into three main groups: tools with a mean greater than or equal to 4.5 were characterised as *high-use*, those with a mean between 3 and 4.5 were assigned into the *mid-use* group while resources with a mean between 1.5 and 3 were put into the *low-use* group.

Factor Analysis

Exploratory Factor Analysis (EFA) is a statistical method aiming at grouping variables which have something in common (i.e. they correlate with each other). This enables researchers to identify latent constructs in the data that cannot be measured otherwise directly. Each group (or cluster) of variables is then called a factor and the variables consisting each factor are thought to be measuring the same underlying/latent construct i.e. each factor represents one underlying construct. An initial EFA was performed on all the 14 variables for tools. We used an oblique rotation because the underlying constructs sought in our data were expected to be related (all variables are related to tool-use after all). Our sample's adequacy was measured by the Kaiser-Meyer-Olkin measure of sampling adequacy and found to be above the minimum value of .5 (KMO=.711). Bartlett's test of Sphericity also showed that the correlations between variables are not 0 i.e. the correlation matrix is not an identity matrix: this is true when the significance value for this test (p) is less than .05 and in our case, it was $p<.001$.

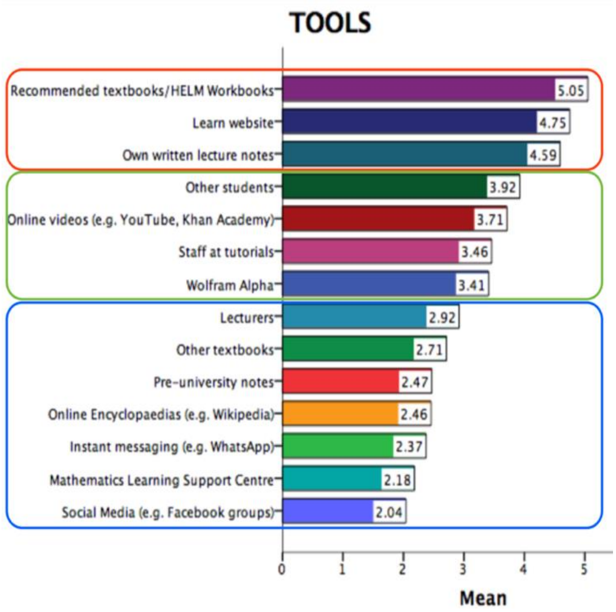

Figure 1: Tools and their grouping based on their mean (high-use: red, mid-use: green, low-use: blue)

Pattern Matrix[a]

	Factor		
	1	2	3
Mathematics Learning Support Centre	.733		
Other textbooks	.643		
Lecturers	.638		
Pre-university notes	.479		-.31
Staff at tutorials	.367		
Instant messaging (e.g. WhatsApp)		.733	
Other students		.710	
Social Media (e.g. Facebook groups)		.495	
Online Encyclopaedias (e.g. Wikipedia)			.523
Online videos (e.g. YouTube, Khan Academy)			.390

Extraction Method: Principal Axis Factoring.
Rotation Method: Oblimin with Kaiser Normalization.[a]

a. Rotation converged in 6 iterations.

Table 1: Pattern matrix of the final EFA (please note that values below .3 are omitted)

Structure Matrix

	Factor		
	1	2	3
Mathematics Learning Support Centre	.698		.313
Lecturers	.644		
Other textbooks	.643		
Pre-university notes	.521	.334	
Staff at tutorials	.374		
Instant messaging (e.g. WhatsApp)		.740	
Other students		.724	
Social Media (e.g. Facebook groups)		.497	
Online Encyclopaedias (e.g. Wikipedia)			.537
Online videos (e.g. YouTube, Khan Academy)	.327	.302	.435

Extraction Method: Principal Axis Factoring.
Rotation Method: Oblimin with Kaiser Normalization.

Table 2: Structure matrix of the final EFA (please note that values below .3 are omitted)

The scree plot was used as a criterion for determining how many factors should be kept. Two lines that best summarise the scree plot were drawn with the intersection of these lines (called point of inflection) indicating how many factors are present in our data, excluding the factor on the point of inflection (in our case 3 factors). We additionally examined both the pattern and structure matrices and decided to exclude the variables “own written lecture notes”, “HELM workbooks”, “Learn website” and “Wolfram Alpha” from our subsequent analysis. This was done because these variables were either having factor loadings below the cut-off value of .364 that we used based on our sample’s size (see Stevens, 2002, p. 374) or because only one variable was present on a single factor (the goal of EFA is to group similar to something variables). Based on this analysis, we run a second EFA on the 10 remaining items with an oblique rotation (KMO = .711, $p<.001$) by requesting a 3 factor solution. Both the pattern and structure matrices were interpreted (Tables 1 and 2). Factor loadings (numbers at structure and pattern matrices) can be thought as the correlations between each variable and the factors and they represent how well a variable "fits" into a factor. The final obtained factors included the following variables:

- **Factor 1** (5 variables): "Mathematics Learning Support Centre", "other textbooks", "lecturers", "pre-university notes" and "staff at tutorials"
- **Factor 2** (3 variables): "other students", "instant messaging" and "social media"
- **Factor 3** (2 variables): "online videos" and "online encyclopaedias".

At this point, we decided to treat the 4 variables not loading on any Factor ("HELM workbooks", "own written lecture notes", "Learn website", "Wolfram Alpha") as latent constructs too. This decision was made for two reasons: first, these variables did not load on any Factor (i.e. not relating with other variables); and second, the nature of each resource is different and unique when compared with the other resources, thus they can be thought "measuring" something on their own. By adopting a descriptive approach (Rummel, 1970) we named the 7 identified types of resources as follows:

1. **The "official" mathematical textbook**: "HELM workbooks"
2. **Students' lecture notes**: "own written lecture notes"
3. **University's VLE**: "Learn website"
4. **The calculator**: "Wolfram Alpha"
5. **Teaching staff**: Factor 1
6. **Peers and communication tools**: Factor 2
7. **External online tools**: Factor 3

From the above types of resources, "teaching staff" (Factor 1) contains 5 variables which they seem not fitting together; is it reasonable to interpret together different in nature variables such as "pre-university notes", "other textbooks" and "lecturers" for example? In our opinion, it makes good sense since they correspond to students' direct interactions with university's teaching staff ("Mathematics Learning Support Centre", "lecturers", "staff at tutorials") and ways that students interact indirectly with teaching staff; this includes the use of resources probably suggested by teaching stuff ("other textbooks") or resources which are the product of prior interactions with a person holding a teaching position e.g. A-levels tutor ("pre-university notes").

Discussion

Our aim for this paper was to produce a typology of the resources that engineering students in our sample reported using. By performing an Exploratory Factor Analysis on our data, we were able to identify 7 different types of resources that undergraduates in our sample reported using (Table 3, left column). From a Wartofskian point of view, all the tools used by undergraduates when examined separately are primary. On the other hand, secondary artefacts represent "*modes of action* using primary artefacts" (Cole, 1996) and they "synthesise the ways and *procedures of using instruments* and materials" (Miettinen, 1999, p.189, our emphasis); this means that our proposed typology corresponds to the different *secondary artefacts* that undergraduates use. When examined from an AT perspective, the typology of tools corresponds to *thematically related actions* that students undertake when studying mathematics. This is because actions in AT are the "...specific interactions that people have with artefacts and other people..." (González, Nardi, & Mark, 2009) i.e. as actions we account the processes of *using* a tool (types 1, 2, 3, 4 and 7) and/or *interacting* with other subjects (types 5 and 6). Our Wartofskian and AT-based interpretations are also consistent from a statistical point of view: students were asked how frequently they use a resource thus, the nature of each variable is related to using a tool or interacting with a person i.e. an action from an AT perspective (*Ibid.*) or the ways of using primary artefacts. Thus, from both a Wartofskian and AT perspective, the 7 different types of resources that undergraduates use, correspond to the following secondary artefacts/actions (Table 3, right column):

1. Studying the mathematical textbook
2. Taking notes during a lecture
3. Accessing institutionally provided material (online)
4. Performing (complex) calculations
5. Interacting with teaching staff
6. Interacting with peers (in-person or virtually)
7. Searching for external/alternative material online

Typology of Tools	Secondary Tools (Wartofsky) - Actions (AT)
(1) The "official" mathematical textbook	Studying the mathematical textbook
(2) Students' lecture notes	Taking notes during a lecture
(3) University's VLE	Accessing institutionally provided material (online)
(4) The calculator	Performing (complex) calculations
(5) Teaching staff	Interacting with teaching staff
(6) Peers and communication tools	Interacting with peers
(7) External online tools	Searching for additional/alternative resources of information

Table 3: Proposed typology of tools (left) and their representations as secondary artefacts and actions

Conclusion

In this paper, we analysed survey data from a cohort of second year engineering students (N=201) about the kind of tools they use when studying for their mathematics modules. In contrast with common approaches found in the literature, we did not focus only on digital/online or institutionally provided resources but rather we incorporated a variety of resources that students have at their disposal. An Exploratory Factor Analysis of our data allowed us to identify 7 different types of tools that students in our sample reported using: the "official" mathematical textbook ("HELM workbooks"); their own written notes; university's VLE ("Learn website"); a sophisticated calculator ("Wolfram Alpha"); university staff (Factor 1); peers and social apps (Factor 2); and non-institutional online tools (Factor 3). By adopting Wartofsky's hierarchy of tools these dimensions represent secondary artefacts i.e. 7 different ways that students use primary artefacts or students' modes of action when studying mathematics: studying the mathematical textbook, taking notes during a lecture, accessing institutionally provided material, performing calculations, interacting with teaching staff, interacting with peers and searching online for additional/alternative sources of information. This interpretation is consistent with AT because these dimensions represent students' actions when studying mathematics. One important implication of our analysis is that although some resources are different (e.g. people, digital) they may be used by students in a *similar* way: this was the case of Factor 1 which contained different in nature resources. This result contradicts our common assumptions when categorising resources (e.g. people, digital, online etc.) and adds an empirical basis for the argument that the way we usually classify resources does not necessarily reflect the ways these resources are used. Because of the nature of our data, we could only infer about the nature of the 7 types of tools that undergraduates use by only examining the resources included in each type. However, our preliminary analysis of 14 interviews suggests that our interpretation aligns with these resources' actual use: for example, students who interviewed reported using Facebook for communicating with peers when having an issue with mathematics (either by using Messenger or by posting a question on Facebook groups created by undergraduates). Our intention for the future is to complement the survey data with data gathered with other methods (interviews and diaries). Finally, we are of the opinion that the results of our analysis (students' learning actions), highlight the temporal nature of all primary tools used in learning and suggest that our future research foci in mathematics education should be on the ways that these tools are used rather than the tools themselves.

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