A branch-and-cut algorithm for the generalized traveling salesman problem with time windows

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The generalized traveling salesman problem with time windows (GTSPTW) can be defined as follows. Given a directed graph, the vertex set is partitioned into clusters with one cluster including only the depot. Arcs are only defined between vertices belonging to different clusters. Each vertex is associated with a time window (TW) and each arc has a traveling cost and a traveling time. The objective of the GTSPTW is to find a minimum cost tour starting and ending at the depot such that each cluster is visited exactly once and time constraints are respected. It reduces to the well-known Generalized Traveling Salesman Problem when TWs are not considered.

Several valid inequalities are proposed for the GTSPTW, dividing into two types, i.e., polynomial families and exponential families. Polynomial families : (1) Arc orientation inequalities. An arc can only be oriented in one way, it either enters or leaves a vertex. This family of inequalities can be generalized considering clusters instead of vertices. (2) Arc-or-vertex inequalities. If a vertex cannot be visited either before or after an arc due to TWs, then only one of them can be chosen in any feasible solution. Exponential families : (1) Generalized subtour elimination constraints (GSEC). (2) Sequential ordering polytope constraints (SOP). SOP inequalities are based on the precedence relationships between pairs of vertices which are inferred by TWs. (3) Clique inequalities. If there is no feasible path passing through all the vertices in a certain set, then the number of vertices in this set that can be visited in all feasible solutions is strictly less than the size of the set.

We develop a branch-and-cut algorithm for the GTSPTW. Here we describe some important procedures included in this algorithm : (1) Preprocessing. The size of the problem can be reduced by eliminating arcs and vertices that cannot be a part of any feasible solution. (2) Initial heuristic to calculate upper bound. Given a fixed cluster sequence, a feasible solution can be obtained by solving a shortest path problem with TWs. (3) Separation algorithms to tighten lower bound. Heuristic separation for SOP inequalities and exact separation for GSEC inequalities are developed to iteratively improve the lower bound. Besides, other valid inequalities are stored in a cut pool and they are taken into account only when violation is found. (4) Branching priorities. To enhance the performance of the branch-and-cut algorithm, we branch on priority on variables that fix the visiting sequence of the clusters.

The algorithm is implemented in C++ using CPLEX 12.6 and Concert framework. For testing our algorithm, we propose new instances for the GTSPTW derived from GTSP instances. Results show that the proposed valid inequalities are effective to improve the lower bound. Instances with up to 24 clusters and 120 vertices can be solved to optimality within one hour.