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A branch-and-cut algorithm for the generalized traveling salesman problem with time windows

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1 Problem description

The generalized traveling salesman problem with time windows (GTSPTW) can be defined as follows. Let $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ be a directed graph, where the set of vertices $\mathcal{V} = \{0, 1, ..., N\}$ is partitioned to $\mathcal{C}_0 = \{0\}, \mathcal{C}_1, ..., \mathcal{C}_K$ clusters. $\mathcal{K} = \{0, 1, ..., K\}$ denotes the cluster index set. Cluster C_0 contains only the depot. Arcs are only defined between vertices belonging to different clusters, that is, $\mathcal{A} = \{(i, j) : i \in \mathcal{C}_k, j \in \mathcal{C}_l, k \neq l\}$. Each arc $(i, j) \in \mathcal{A}$ is associated with a traveling cost C_{ij} and time T_{ij}. Each vertex is associated with a time window (TW) $[E_i, L_i]$ with $[E_0, L_0] = [0, T]$. A visit can only be made to a vertex during its TW. The objective of the GTSPTW is to find a minimum cost tour starting and ending at the depot such that each cluster is visited exactly once and time constraints are respected. Three variables are introduced: binaries y_i and x_{ij} represent if vertex i and arc (i, j) are chosen or not in the tour; real variable t_i is the arrival time at vertex i. The GTSPTW can be modeled as follows.

$$
\begin{aligned}
\min \ & \sum_{(i,j)\in\mathcal{A}} C_{ij}x_{ij} && && (1)\\
\text{s.t.} \ & \sum_{i\in\mathcal{C}_k} y_i = 1 && k \in \{0, 1, ..., K\}, && (2)\\
& \sum_{(i,j)\in\delta^+(i)} x_{ij} = \sum_{(j,i)\in\delta^-(i)} x_{ji} = y_i && \forall i \in \mathcal{V}, && (3)\\
& E_i y_i \leq t_i \leq L_i y_i && \forall i \in \mathcal{V}, && (4)\\
& t_i - t_j + T_{ij}x_{ij} \leq L_i y_i - E_j y_j - (L_i - E_j)x_{ij} && \forall (i, j) \in \mathcal{A}, j \neq 0, && (5)\\
& t_i + T_{i0}x_{i0} \leq L_0 && \forall i \in \mathcal{V} \setminus \{0\}, && (6)\\
& y_i \in \{0, 1\} && \forall i \in \mathcal{V}, && (7)\\
& x_{ij} \in \{0, 1\} && \forall (i, j) \in \mathcal{A}. && (8)
\end{aligned}
$$

The objective function (1) minimizes the overall costs. Constraints (2) ensure that exactly one vertex from each cluster is visited. Constraints (3) are flow conservation constraints. Constraints (4) ensure that a vertex is visited during its TW. Constraints (5) ensure that the arrival and traveling time are consistent, meanwhile eliminating subtours. Constraints (7) and (8) are variable definitions.

The GTSPTW reduces to the well-known Generalized Traveling Salesman Problem [1] when TWs are not considered. For the multi-vehicle case, the problem is named Generalized Vehicle Routing Problem with TWs [2].

2 Methodology

2.1 Polynomial families of valid inequalities for GTSPTW

- Arc orientation inequality. $x_{ij} + x_{ji} \leq y_i \quad i, j \in \mathcal{V}, i \neq j$.
  In any feasible solution either (i, j) or (j, i) is used, but not both.
- Arc-or-vertex inequality. $x_{ij} + \sum_{h \in \mathcal{C}_k^{ij}} y_h \leq 1 \quad i, j \in \mathcal{V}, i \neq j, k \in \mathcal{K} \setminus \{0\}$,
  where $\mathcal{C}_k^{ij} = \{h \in \mathcal{C}_k | E_h + SP_{hi} + T_{ij} > L_j$ or $E_h + SP_{hi} > L_i, E_i + T_{ij} + SP_{jh} > L_h$ or $E_j + SP_{jh} > L_h\}$.
  SP_{ij} represents the shortest traveling time from vertex i to vertex j. When the triangle inequality is not satisfied, the shortest path to go from i to j can include the visit of other vertices. These constraints detect if a vertex and an arc can not be simultaneously selected in a feasible solution due to TWs.

2.2 Exponential families of valid inequalities for the GTSPTW

- Generalized subtour elimination inequality (GSEC).
  $\sum_{(i,j) \in \delta^+(\mathcal{S})} x_{ij} \geq 1 \qquad \forall \mathcal{S} = \cup_{h \in \mathcal{H}} \mathcal{C}_h,\ \mathcal{H} \subset \mathcal{K},\ 2 \leq |\mathcal{H}| \leq K - 1$.
  GSEC avoids tours visiting a subset of clusters.
- Clique inequality. $\sum_{i \in \mathcal{S}} y_i \leq |\mathcal{S}| - 1,\ \mathcal{S} \subset \mathcal{V}$.
  If no feasible path passing through all the vertices of set $\mathcal{S} \subset \mathcal{V}$ exists (due to TWs), then the vertices of $\mathcal{S}$ that can be visited in any feasible solution are less than its size .

2.3 A branch-and-cut scheme

We develop a branch-and-cut algorithm for the GTSPTW. We include at the root node of the branch and bound tree all the inequalities defined in Section 2.1. On the other side, due to their exponential cardinality, we separate the GSEC inequalities using a classical approach based on the resolution of a max-flow problem using Gomory-Hu algorithm.

Clique inequalities on subsets S of $\mathcal{V}$ with cardinality up to 3 are directly introduced at the root node of the branch and bound tree. In addition, clique inequalities on subsets S of $\mathcal{V}$ with cardinality of 4 are dynamically added whenever a fractional solution violates one of them.

Please note that GSECs and clique inequality are satisfied for any feasible integer solution of the GTSPTW and they are not problem defining. However, they are helpful to improve the linear relaxation lower bound during the branch and bound algorithm.

3 Conclusions and perspectives

The branch-and-cut algorithm is tested on the instances we propose for the GTSPTW created from instances for the GTSP. Instances with up to 16 clusters with 76 nodes can be solved to optimality. Undergoing work is on the development of heuristic algorithms to obtain initial feasible solutions. Meanwhile new families of cuts based on infeasibility detection are under development.

References

[1] Fischetti Matteo, Juan José Salazar González, and Paolo Toth. A branch-and-cut algorithm for the symmetric generalized traveling salesman problem. *Operations Research*, 45(3):378–394, 1997.

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