
Geometric focusing of internal waves: linear theory

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Abstract

Alongside two-dimensional mechanisms such as the intersection of separate beams (e.g. Smith & Crockett 2014) or between an incident beam and its reflection at a slope (e.g. Grisouard et al. 2013), a specific three-dimensional mechanism is susceptible of amplifying internal gravity waves: the geometric focusing of waves emitted by horizontally curved forcing. Appleby & Crighton (1987) and Simakov (1993, 1994), for simple oscillating objects, then Bühler & Muller (2007) and Grisouard & Bühler (2011), for internal tides over circular ocean topography, demonstrated its occurrence theoretically. The first laboratory experiments were conducted at LEGI by Flór (1997, unpublished results), generating internal waves and turbulence with a horizontal circular torus in a stratified fluid; the same approach was repeated by Duran-Matute et al. (2013) for inertial waves in a rotating fluid, experimentally and numerically. Focusing has been identified by Buijsman et al. (2014) in the Luzon Strait in the South China Sea, in the form of enhanced interference between the internal tides over two curved parallel ridges. We present a linear theory of internal wave focusing from annular forcing, for a slender annulus of negligible local curvature. The analysis is applied first to a torus, either complete or partial. Focusing is seen to arise in all cases and yield significant isopycnal slopes, close to overturning, even at low oscillation amplitude. The slopes get higher as the Stokes number increases, changing the beam structure from unimodal to bimodal. Gaussian ocean topography is considered next, either circular or horseshoe-shaped. Focusing is seen to arise as well, though its efficiency is lower owing to the weak topographic slope and oscillation amplitude.

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