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Teachers' beliefs about mathematical modelling: An exploratory study

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Drawing upon a theoretical framework based on beliefs, learning and teaching of mathematical modelling as well mathematical modelling itself, this paper explores mathematics teachers´ beliefs about these themes. Based on responses to an online questionnaire, teachers' beliefs, experiences, and mathematical modelling lessons were shared from the teachers' perspective; several similarities with the cyclical process of modelling emerged, as well as sharing a new point of view of the aspects that teachers consider as mathematical modelling for example, real-life situations and processes behind mathematics problems.

Keywords: Teacher beliefs, mathematical modelling, mathematical model, learning and teaching mathematical modelling.

Introduction

Mathematical modelling involves the development of models to explain real-world situations. Such models allow for making predictions, explaining phenomena, making decisions, and disseminating knowledge (Schichl, 2004). In addition, learning mathematical modelling is a cyclical process in which pupils' study a problem derived from the real world and create a mathematical model to explore, predict and explain in order to provide a solution to that problem (Mason & Davis, 1991).

Mathematical modelling may also be seen as an approach to learning that uses elements of reality to create models with mathematics. In this approach, students work together in a cyclical process that involves different stages, such as formulating a mathematics problem based on a situation in real life, setting up a mathematical model that explains the problem, attempting to find a mathematical solution for the problem, explaining the model and interpreting the solution, and comparing the solution with the original problem in real life (Mason et al., 1991; Blum & Borromeo, 2009; Lawson & Marion, 2008). It is important to note that while it is true that the students are engaged in the modelling process in a lesson, it is the teachers who initially implement the strategy, for example, by choosing the modelling task.

In this context, previous studies have suggested that, when applying mathematical modelling, teachers should consider, for example, 'teachers have to know ways how to support adequate student strategies for solving modelling task' (Blum et al., 2009, p. 54). Complementary to this idea, Tekin Dede and Bukova Güzel (2016, p. 1) suggest that 'some researchers indicate the teacher are not be sure about how they should act in this implementation process. Especially the teacher who are novice or have not enough experience in modelling can have difficulties in this process'.

Considering that the role of the teacher appears to be a crucial part of the development of the cycle of mathematical modelling; the question then arises of what beliefs teachers have about mathematical modelling that they could share with their students. As Kaiser (2006, p. 399) notes, 'teachers and their beliefs concerning mathematics must be regarded as essential reasons for the low realization of applications and modelling in mathematics teaching'.

Bearing these factors in mind, this study seeks to examine mathematics teachers' beliefs about mathematical modelling through an online questionnaire about teachers' backgrounds as well as their opinions about mathematics classes, mathematics models, and modelling. This study is the first part of an ongoing process of a doctoral research project; the aim of this paper is thus to examine teachers' beliefs and practices related to mathematical modelling in order to discuss future implications when conducting mathematical modelling.

Theoretical framework

Thompson (1992) discussed the idea that teacher beliefs about the nature of mathematics should be considered – for example, concepts, meanings, and rules, among others – as well as teachers' beliefs about teaching and learning mathematics. In addition, as Stipek, Givvin, Salmon, and MacGyvers noted in their study on evaluating teacher beliefs, we should consider (2001, p. 213):

(1) the nature of mathematics (i.e., procedures to solve problems versus a tool for thought), (2) mathematics learning (i.e., focusing on getting correct solutions versus understanding mathematical concepts), (3) who should control students' mathematical activity, (4) the nature of mathematical ability (i.e., fixed versus malleable), and (5) the value of extrinsic rewards for getting students to engage in mathematics activities.(6) Teachers self-confidence and enjoyment of mathematics and mathematics teaching.

Handal (2003, p. 47) states that teacher beliefs can be related to 'what mathematics is, how mathematics teaching and learning actually occurs, and how mathematics teaching and learning should occur ideally'. It would appear that beliefs about mathematics share common themes with the field of mathematics itself, as well as with teaching and learning mathematics. Indeed, many researchers currently study beliefs about mathematics, practices, and teaching, so it is natural that many researchers would study beliefs within specific areas of mathematics (for example, mathematical modelling), since mathematical modelling is part of mathematics.

When examining beliefs about mathematical modelling, we should consider that 'beliefs in the context of mathematics education can be classified as beliefs about mathematics (as a science), beliefs about the learning and teaching of mathematics [,] and teacher self-efficacy beliefs' (Mischo & Maaβ, 2013, p. 22). According to those authors, the first aspect – mathematics as a science – refers to the formal aspect of mathematics as theorems, rules, problem-solving, and applications as subjects to be learnt. In this context, mathematical modelling is part of this frame, since it is a process that involves a task that takes elements from reality to be explained mathematically. The second aspect – learning and teaching mathematics – refers to constructivist and socio-constructivist views as a way of teaching modelling. Finally, teacher self-efficacy refers to teachers' beliefs that should be carried out and activities that should be implemented in order to reach the teacher's goals within a lesson.

Ärlebäck J. (2009, p.2100) posits that in order to understand the beliefs from teachers about mathematical modelling and models it is necessary to take account of 'beliefs about the nature of mathematics, real world (reality), problem solving, school of mathematics and beliefs about applying, and applications of, mathematics'.

Previous studies from the literature review, however, do not seem to have taken into account the 'perceptions and beliefs about mathematics [that] originate from past experiences' (Mutodi &

Ngirande, 2014, p. 432); perhaps these studies would have had more to offer if they had included relations with past experiences, because if teachers have a history with mathematics (in particular mathematical modelling, because it is part of mathematics), then their beliefs about the field can be related to the background and experiences they have lived: for example, when they teach or learn mathematics.

According to the literature, we may observe that teachers' mathematics beliefs may be classified into different topics; these topics stem from the experiences that teachers have had with mathematics, either through teaching or learning or when they themselves have studied the subject. In this study, considering the idea that mathematical modelling is part of mathematics, teachers' beliefs about mathematical modelling will be classified into three dimensions on the basis of the literature review: (1) mathematics in itself, in particular considering mathematical models and modelling; (2) beliefs about learning and teaching mathematics modelling, considering students, behaviours, lesson planning, and task design; and (3) real-life experience with mathematical modelling, which means that the history described by each participant will be taken into account in relation with mathematical modelling and models, since our beliefs about mathematics stem from our past experiences. It is important to note that this third dimension cannot be separated from the first two, because the experience gained from any context will be enriched by the mathematics itself as well as by the experiences of teaching and learning mathematics.

Methods

The aim of this study was to explore teachers' beliefs about mathematical modelling, therefore an exploratory research was adequate because this attempts to 'seek new insights' (Robson, 2002, p.59) on teaching mathematical modelling in light of my considerations in the introduction, consequently gaining familiarity with the beliefs of mathematics teachers.

Bearing in mind that beliefs are related with mathematics on different dimensions, the selection of the participant was through an activity that is related to mathematics, such as teaching mathematics, conducting research in mathematics education, or studying mathematics itself. Consequently, three of the participants were Chileans who worked at a university in Chile in the mathematics faculty where they are training future mathematics teachers; six of the participants were from the United States, where they worked at high-need schools[1]. The richness of these participants, helps me to have an international overview of the beliefs in this area.

In order to research the beliefs of mathematics modelling, an online questionnaire was designed, comprising ten structured and open-ended questions based on the literature review about mathematical modelling with a focus on the teachers' relationships with the field of mathematics education, mathematical modelling and models. In addition, there are similarities between online questionnaires and structured interviews, in that the researcher has the same direct pre-established questions for each participant without giving interruptions among questions. Those similarities,

[1] 'The school is located in an area in which the percentage of students from families with incomes below the poverty line is 30 percent or more; or in an area with a high percentage of out-of-field teachers; is in an area in which there is a high teacher turnover rate; or is in an area in which there is a high percentage of teachers who are not certified or licensed; is within the top quartile of elementary schools and secondary schools statewide, as ranked by the number of unfilled, available teacher positions at the schools' (No Child Left Behind Act of 2001, 2002, p.115, STAT.1656).

made me consider and exploring the online questionnaire as a way to approach at teacher beliefs. Furthermore, taking into account the limitations of distances between countries and accessibility, the online questionnaire was adequate.

The online questionnaires were distributed between December 2015 and January 2016. The answers were transcribed and analysed based on categories described above; the way that I used to analyse the answers was considering all of them, highlighting common factors that emerged and after that observing how these related to the theoretical framework described previously. It would have been preferable to include the questions, but this was not possible due to space limitations.

Because this is an initial study on mathematics teachers' beliefs about mathematical modelling, it is impossible to generalise from this point of view, but the study can present an opportunity to explore what occurs in beliefs of teachers about this subject on persons related with mathematics but not necessarily those currently working with mathematical modelling and thus provide insights within this large and expanding field, for example, when an implementation is carried out by teachers.

Results

A few representative teacher examples are provided below to illustrate their responses; the responses were transcribed whole, to prevent loss of fluency. In the transcripts, certain parts of the texts have been put in italics font below for emphasis, showing the common factor highlighted on the analysis process. In addition, 'TUS' refers to teachers from the United States, while 'TC' refers to teachers from Chile. The number next to the initials indicates the person who has answered the question.

Mathematical modelling

Real-life situations and processes behind mathematics problems

According to teachers' responses about mathematical modelling, for example, what comes to mind when you think of mathematical modelling, teachers' beliefs may be categorised in two ways: (1) real-life situations or real-life problems and (2) the process involved in solving mathematics problems. Both considerations also take mathematics itself into account: more specifically, the real world and the application of mathematics, as Ärlebäck (2009) has noted.

TC1: *Real situations*, math representation, solving, and interpretation.

TC8: It came to my mind to think of applied math, i.e., to *relate math with nature* or a *daily situation.*

TUS2: I explain to the students that we use mathematics to *model situations in real life* to be able to understand them better and, if possible, to find a solution to the situation … I think of it as visuals that will simplify the situation I am reading. I think of equations or systems of linear equations that will allow me to find solutions.

TUS4: I think mathematical modelling means *showing the students the thought process that is involved in solving a math problem.* It's the problem-solving techniques in regard to a given (abstract or *real-world*) math situation – explaining which

method, formula, etc., is going to work and why. That's what I think mathematical modelling is.

TUS5: The teacher needs *to model the way a problem should be processed* and thought through in order to come up with a valid solution.

As the reader may have noted, there is a link between the beliefs about mathematical modelling and the 'real situation' which can be interpreted, for example, with real life and is related to the modelling cycle mentioned previously. However, there is also a distinction between these beliefs because to others 'modelling' means modelling through a 'step by step' approach showing the process behind a mathematics problem.

Mathematical models

In response to the question 'what do you think about mathematical models?', some of the teacher beliefs about modelling are related with the use of models within mathematics; in some of the teachers' responses, the teachers' beliefs were related to the nature of mathematics itself (Ärlebäck, 2009; Thompson, 1992; Handal, 2003; Stipek et al., 2001; Mischo & Maaβ, 2013) as well as with mathematics learning (Handal, 2003; Stipek et al., 2001; Mischo & Maaβ, 2013). In accordance with Mutodi and Ngirande's work (2014) about experience and beliefs, some of the teachers mentioned their past experience with models, when they were responding to 'what is your opinion about the statement: 'Modelling is everywhere'' (Mason & Davis, 1991, p. 9).

TUS7: Taking real-world data and using graphing tools and computers to assist in *obtaining models*.

TUS6: I think a mathematical model is a key factor in *describing measurement that is located in space*. [Models are] essential for describing the composition of matter in our universe.

TUS2: During college, we used mathematical models in *calculus and geometry* as well as *numerical calculus and number theory* … I agree with 'modelling is everywhere'. I believe some of the models are more elaborate than others, but in general, *I can see everything being a model of concepts*, including numbers and graphs.

TC9: *Visuals* are necessary; hands-on [teaching] is almost vital for understanding.

From the transcript, it is possible to infer that the teachers viewed mathematical models as ways to understand mathematics with a factor of utility of model, using words such as assist, essential or necessary.

Learning and teaching mathematical modelling

Experience with mathematical modelling

In terms of experience in modelling, some of the teachers mentioned that they had studied modelling at their universities (i.e., dynamic systems and geometry, among others); they also discussed their experience in teaching (or not having experience in mathematical modelling). This situation relates to what Mutodi and Ngirande (2014) discuss in their work: our beliefs about

mathematics come from our experience with mathematics. In addition, some of the teachers recognized that they had limited experience with mathematical modelling, or they lacked the time to do it; one of the teachers, however, also mentioned her intention to learn more about modelling.

These ideas could help to explain why teachers are often unsure of how to act when they work with mathematical modelling, which results in necessary (and time-consuming) planning during the implementation stage, as mentioned by Tekin Dede and Bukova Güzel (see the introduction to the current paper).

TUS2: As a school teacher, I would love to be able to *have the time to read and understand more about mathematical modelling*.

TUS3: *I have not spent nearly enough time* doing mathematical modelling.

TC8: My experience with *mathematical modelling is with dynamical systems*. In this area, you can find many examples of different situations where modelling is present. In the classroom, you can use simple examples and particular cases of this area to show the students.

Mathematical modelling class

After the question, 'how do you imagine a mathematical modelling class?', several teacher beliefs have presented several similarities when using elements from reality yet only one teacher mentioned focussing on models, as has been mentioned by several authors in this paper (Mason & Davis, 1991; Blum, 2009; Lawson & Marion, 2008); In addition, teachers' beliefs as to how mathematics teaching and learning can occur as Handal (2003) described previously were present. For example, some of the teachers' responses in the present study suggested that mathematical modelling lessons could be very creative, active, and didactic, where the students could take part in the learning process.

TUS2: I imagine [that mathematical modelling class] *focusses on models* more than a specific area of mathematics.

TUS6: Let's take, for example, the concept of geometry. *The setting within a classroom could be used as mathematical modelling*. Without going outside the four walls [of a classroom], one could introduce to students the concept of angles, lines, planes, perpendicular and parallel lines, congruent angles and similar figures, and so on … I think of [mathematical modelling class] *as very creative*; it is an atmosphere where *students are launched into using their minds* so that they will become creative, inquisitive, and analytical… Mathematical modelling is one of the most *intriguing, creative, and thought-provoking subjects that one can teach*. It blends into other subjects, such as art, physics, and chemistry.

TC8: I imagine a mathematical modelling class as being very *active and didactic – students working in groups and discussing the problem that has been assigned*.

Discussion

Through this study, we can observe that the beliefs are quite similar in both countries. Teachers link mathematical modelling with situations to be modelled; this situation stems from real-life

experiences. This belief shows a similarity with mathematical modelling as a subject that can involve the development of models to explain real-world situations (Schichl, 2004). One teacher, however, said that mathematical modelling was how 'a problem should be processed'; in this case, the teacher's beliefs were not related to real-life situations but to the process involved in solving mathematics problems. Perhaps this particular belief about mathematical modelling could be understood in light of some of the teachers' limited experience with mathematical modelling or, as Tekin Dede and Bukova Güzel (2016) have stated, because the teachers were unsure how to act when working in a class that involves mathematical modelling.

On the other hand, teachers' beliefs about mathematics modelling classes in general showed that they felt that classes could be very creative, intriguing, and thought provoking; as one teacher said, they could be taught with the participation of the students. One teacher did recognise, however, that it can be difficult to break down some students' beliefs. Even so, other teachers stated their intention to learn more about mathematical modelling.

In terms of mathematical models, sometimes teacher's beliefs referred to the utility of using a model as a way to demonstrate mathematics concepts. In this sense, mathematical model beliefs are related to the nature of mathematics (Mischo & Maaβ, 2013; Stipek et al., 2001; Thompson, 1992; Handal, 2003; Ärlebäck, 2009) as well as with the experiences that they have had in the past (Mutodi & Ngirande, 2014). Then the questions that naturally arise include, In which ways can the utility beliefs of a model influence the implementation of mathematical modelling? What kind of decision does the teacher take during an implementation? What types of feedback can the teacher give to the students?

Finally, through this particular study and in consideration of teachers' beliefs about mathematical modelling, more questions and insights have emerged; for example, how to lead an implementation of the mathematical modelling cycle that would bear teachers' beliefs in mind, and how can the usefulness and reliability of online questionnaires be linked to carry out an exploratory study on beliefs.

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