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Arturo Mendoza[1,2], Stéphane Roux[1], Julien Schneider[3], Estelle Parra[2]

[1]LMT (ENS Paris-Saclay/ CNRS / Univ. Paris-Saclay), 61 avenue du Président Wilson, 94235 Cachan, France,
e-mail: {arturo.mendoza.quispe, stephane.roux}@lmt.ens-cachan.fr

[2]Safran Tech, Rue des Jeunes Bois, 78772 Magny les Hameaux, France,
e-mail: {arturo.mendoza-quispe, estelle.parra}@safrangroup.com

[3]Safran Aircraft Engines, Rond-Point Réné Ravaud, Réau, 77550 Moissy-Cramayel, France,
e-mail: julien.schneider@safrangroup.com

Abstract
Digital Volume Correlation is an appealing technique for establishing a new and powerful characterization based on full field kinematic measurements for structured materials such as 3D woven composites. This approach provides a quantifiable description of the weaving distortions on woven composites. Therefore, this technique offers applications ranging from the design of new weaving architectures for composite structures, up to nondestructive testing of composite parts. The method is validated on a real 3D woven composite part with satisfactory results.

Keywords: Tomography, Woven Composites, Digital Volume Correlation

1 Introduction

Composite materials are fast becoming key in many industries such as aeronautical and aerospace mainly due to their very attractive specific properties (e.g. strength to weight ratio), especially those of woven composites. These are conformed by yarns (reinforcement phase) woven after a pattern and held together by a resin (matrix phase). In particular, 3D woven composites (three-dimensional weaving pattern) have appeared in new applications requiring high mechanical properties such as the engine fan blade [1].
Evidently the increasing interest in these materials has generated a high demand for nondestructive testing (NDT) techniques as well as for proper characterization methods [2], accurate FE simulations and adequate visualization techniques. In this context, X-ray computed tomography has opened new horizons.
It is the aim of the present study to investigate the potential of (relative) 3D displacement measurements obtained from Digital Volume Correlation (DVC) [3, 4] on pairs of tomographic volumes of 3D woven composites. Such kinematic measurements allow for the quantitative evaluation of continuous deformations (e.g. stretching and bending of yarns) that may define acceptable/unacceptable weaving distortions, as well as the retrieval of weaving abnormalities. The latter are called "topological differences" as they cannot be eliminated by a continuous deformation of the medium. This powerful *quantitative* characterization drastically improves over current *qualitative* NDT tools since they do not provide enough information to detect subtle deformations and even not so subtle ones such as the aforementioned topological differences. Thereby, DVC may be turned into a NDT technique for the manufacturing of high technology composite materials.

2 Digital Volume Correlation

As an extension of standard Digital Image Correlation (DIC), Digital volume correlation (DVC) is a technique that measures the internal displacements field from a pair of volumes (3D images) generally obtained with the same acquisition technique, for example with high resolution X ray tomography as in the present study.
DVC is a fast growing technique [5] and can be either local [3] or global [6]. Usually DVC relies on the *natural* texture of the studied material as a basis for correlation since most procedures used to improve the contrast of the material texture could also modify its mechanical properties (which may be of interest). Moreover, as a true three-dimensional technique, DVC is distinct from the 3D *surface* deformation methods that rely on two-angle planar images (stereo-correlation [7]).
Thus, given the pair of 3D continuous images $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$, DVC is based on the brightness conservation assumption:

$$f(\boldsymbol{x}) \approx g\big(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})\big) \equiv \tilde{g}(\boldsymbol{x})$$

where $\boldsymbol{u}$ is the Lagrangian displacement vector field. Then, it is sought to minimize the residuals $\eta = f - \tilde{g}$ defined over the whole region of interest Ω using the following cost function:

$$\tau = \int_{\Omega} (f - \tilde{g})^2 \, \mathrm{d}\boldsymbol{x}$$

Given that the problem is ill-posed, a better conditioning can be obtained when the displacement field is restricted to a space of low dimensionality. A convenient choice is the decomposition of $\boldsymbol{u}$ over a set of well chosen kinematic fields $\psi_i(\boldsymbol{x})$, such as those used in the framework of the finite element method [8]:

$$\boldsymbol{u}(\boldsymbol{x}) \approx \sum a_i \psi_i(\boldsymbol{x})$$

The superposition of these different displacement fields makes the sought amplitudes a_i interdependent, hence the term "global". Finite element shape functions —such as T4 (tetrahedron) or H8 hexahedron— are particularly attractive because of the interface they provide between the measurement of the displacement field and its corresponding numerical modeling based on a constitutive equation. Finally, a classical Newton-Raphson routine (iterative linearized optimization procedure) is used to solve for $\partial\tau/\partial\boldsymbol{a} = 0$, which leads to the linear system,

$$\frac{\partial^2\tau}{\partial\boldsymbol{a}^2}\,\Delta\boldsymbol{a} = -\frac{\partial\tau}{\partial\boldsymbol{a}}$$

which can be written as

$$\boldsymbol{M}\Delta\boldsymbol{a} = \boldsymbol{b}$$

with

$$\boldsymbol{M}_{ij} = \int_{\Omega} \left(\Phi_{\mathrm{i}} \cdot \Phi_j\right) \mathrm{d}\boldsymbol{x}$$

and

$$\boldsymbol{b}_i = \int_{\Omega} \left(\Phi_{\mathrm{i}} \cdot \eta\right) \mathrm{d}\boldsymbol{x}$$

where

$$\phi_{\mathrm{i}}(\boldsymbol{x}) = \boldsymbol{\nabla} f(\boldsymbol{x}) \cdot \psi_i(\boldsymbol{x})$$

As such, the vector $\boldsymbol{a}$ containing all the unknown degrees of freedom is updated with $\boldsymbol{a}^{k+1} = \boldsymbol{a}^k + \Delta\boldsymbol{a}$ until convergence.
Furthermore, regularization techniques can be used in order to better condition the problem, as well as to reduce the measurement uncertainty. While a classical soft Tikhonov regularization [9] could be used, it is preferred to introduce a penalization based on the "distance" between the estimated displacement field and that of the solution to a homogeneous elastic problem [10]. This distance is known as the *equilibrium gap* [11, 12]. It is important to note that this strategy does not require the studied volume to strictly obey linear elasticity. Rather it can be seen as a filter that *locally* dampens abrupt displacement gradients in order to guarantee a smooth and differentiable displacement field. Such filter can be tuned through the use of a regularization length ξ.

3 Multiresolution isotropic approach

The hierarchy of information is an important notion in image analysis. This happens due to a natural nesting of several objects or features (of different sizes) within a single image. Developing robust methods capable of dealing with fine details —which usually are not a highly discriminant source of information— while also coping with broad definitions and still remain numerically efficient, proves to be a complex task. Coarse-to-fine resolution strategies often provide a simple yet effective solution. These include the traditional (Laplacian) pyramidal approach [13] or the more recent scale space theory [14], both being strongly related. The pyramidal method uses the results obtained on a lower resolution image to guide the processing on a higher resolution image, repeatedly until the original image is reached. Thus, the coarser levels will help to capture the biggest displacements, which will be iteratively refined on each finer resolution level.
Moreover, when dealing with displacements, the notion of anisotropy should be addressed in the image analysis. For the present case study, the yarn cross-sections are of great interest. Though initially (in the manufacturing process) approximately circular in shape, the yarn cross-sections on the final part are greatly flattened, thus obtaining horizontally elongated structures. In fact, they are commonly modeled after ellipses, power ellipses (special case of a super ellipse) or lenticular shapes [15, 16]. As such, similar displacements on different directions will not have the same impact. For example, horizontal yarn displacements can be of bigger magnitude than vertical ones before having to deal with yarn inter-penetration. A straightforward solution is to make the image isotropic, where yarn cross-sections approximate a circular shape.

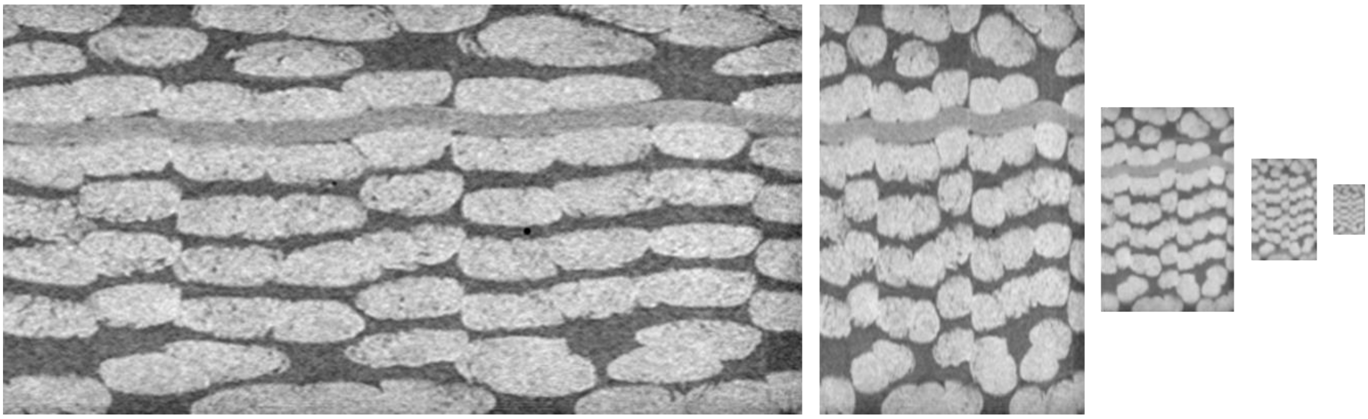


Figure 1: Slices (y-z) for the images obtained using the multiresolution isotropic approach

Finally, the multiresolution isotropic approach is pursued using a Gaussian pyramid in which the kernel radius is set to be half of the scaling factor. A first step is to remove the anisotropy using a convenient scaling factor. This one is quickly determined by roughly measuring some cross-sections in each of the main axes (as defined by the warp and weft yarns). Then the pyramid levels are constructed with four successive steps using a scaling factor of $2 \times 2 \times 2$. Additionally, the calculations are only performed on the four isotropic levels of the pyramid, thus considering sequentially images as shown from right to left in figure 1. Hence, the obtained results can be extrapolated to the anisotropic case if needed.

4 Topological differences

Woven composite materials hold a key intrinsic information: the underlying weaving pattern. Such weaving pattern is represented in a unit cell, which is then tessellated (tiled) to form a full scale textile. Hence, this procedure of correlation on woven composite parts is based on the assumption of a constant topology: properties are preserved through deformations, twistings, and stretchings; tearing, however, is not allowed. So, in principle, different composite parts made with the same weaving pattern can *always* be set into correspondence using a continuous displacement field. And whenever such assumption is invalidated, so-called "topological differences" will occur.
Evidently the procedure is only valid between regions of interest containing the same sample of the weaving pattern between regions of interest (of arbitrary size and shape) but separated by a multiple of the spatial period, as shown in figure 2.

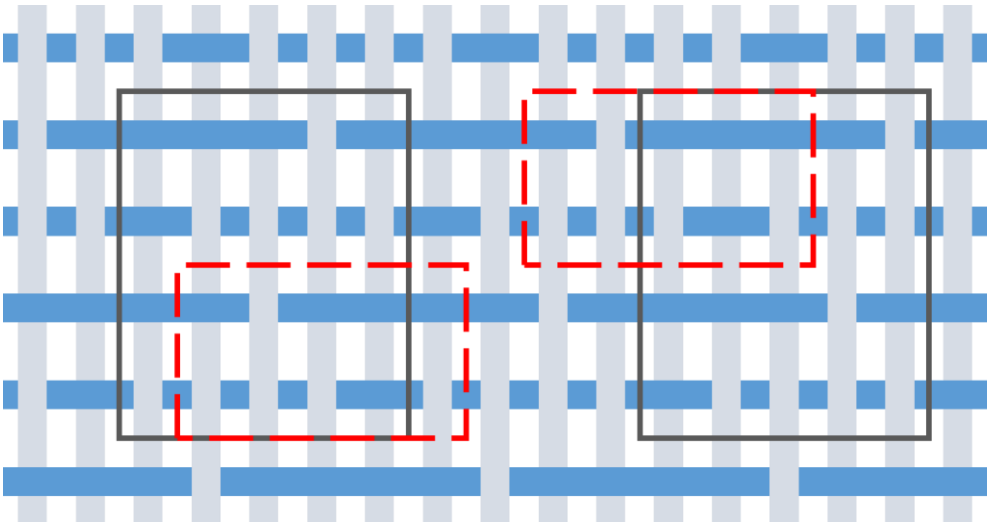
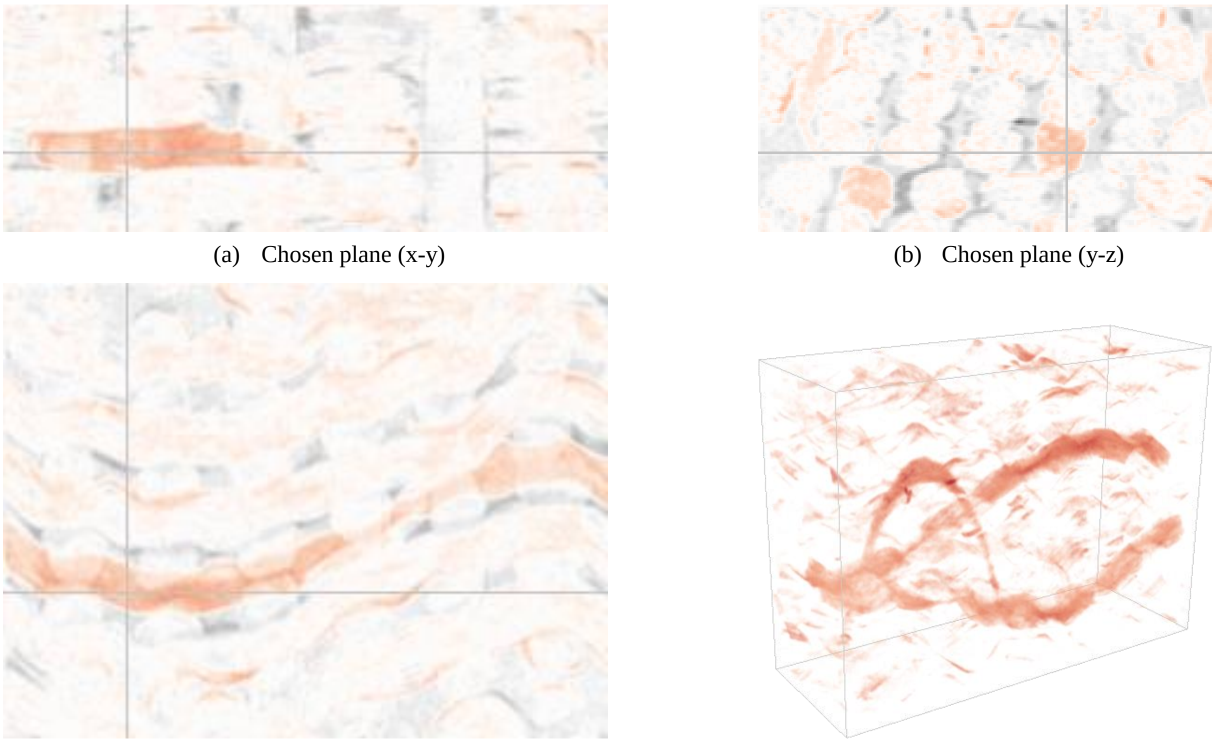


Figure 2: Sample textile showing the arrangement of warp and weft yarns in order to form unit cells.
It is also shown here some sample regions of interest Ω

Nevertheless, *large* topological differences, such as loops or missing yarns (weaving abnormalities), may exist between the studied volumes. For this particular case, the procedure is faced with the difficulty that no continuous displacement field will ever be able to bring the volumes into full coincidence as expected (i.e. as when no topological difference exists). This will result in an image of residuals η containing regions with large values. Such regions are of great interest because they naturally highlight any topological difference.

5 Results

The analysis is performed on a 3D woven composite part with a single weaving pattern appearing on two neighboring unit cells (sharing weft yarns as the sample regions of interest shown in figure 2) obtained through µCT using a regular procedure. The multiresolution isotropic approach is carried out on both unit cells, randomly choosing one of them as the "reference".
The isotropic kinematic basis is defined for all levels of the pyramid using a $6 \times 6 \times 10$ structured regular mesh composed of only H8 finite elements. The element size at the smallest scale is $2 \times 2 \times 2$ voxels, and uses the same scaling factor as in the pyramid. Additionally, the regularization length ξ is set to a constant value of twice the element size at the highest pyramid level.
A good indicator for the quality of the correlation procedure is the image of residuals, which tends to zero since that is precisely what is being sought. In fact, the obtained results show that around 90 % of the image of residuals contains values (in absolute value) lower than 5 % of the dynamic range of the input images. This can be interpreted as the algorithm correctly identifying the 3D kinematics that relates both studied unit cells. Yet the remaining "high" residual values are not scattered around the entire volume but clearly structured, as can be seen in figure 3. These can effortlessly be read as a prominent topological difference: missing yarns. Such results are in perfect agreement with the known positions of the intentionally placed weaving abnormalities. Furthermore, as figure 3 shows, the isotropic multiresolution approach is proven to be robust enough even in the presence of reconstruction artifacts such as ring artifacts (albeit scaled in the isotropic space).



(a) Chosen plane (x-y)

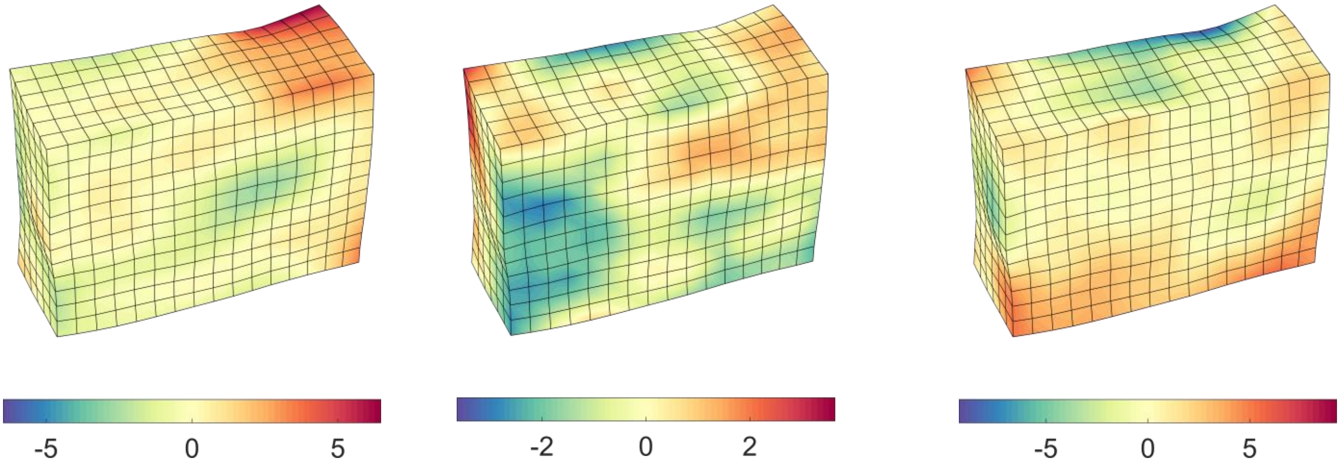
(b) Chosen plane (y-z)

(c) Chosen plane (x-z)

(d) Volumetric rendering

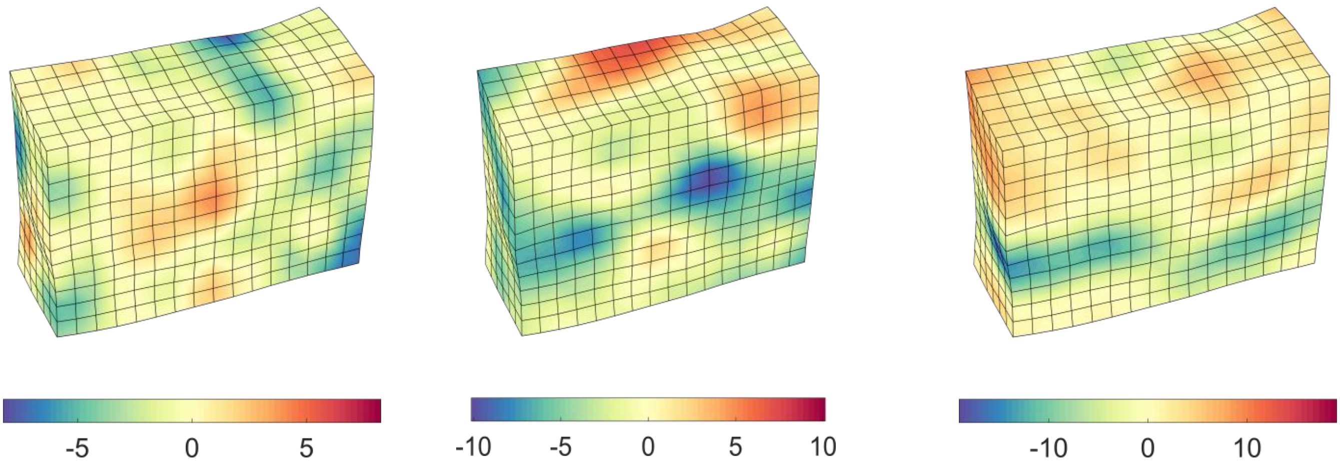
Figure 3: Visualization of the image of residuals η in 2D and 3D, missing yarns are identified.

As figure 4 shows, the obtained 3D displacement vector field $\boldsymbol{u}$ gives a detailed description of the geometrical transformation needed to "back-deform" a unit cell into another one defined as "reference". Furthermore, the displacement field can be used to compute another description of the deformation in terms of relative displacement that excludes rigid-body motion: the (logarithmic) strain tensor $\boldsymbol{\epsilon}$, shown in figure 5. Likewise, the strain $\boldsymbol{\epsilon}$ can be used to retrieve its trace $\mathrm{tr}(\boldsymbol{\epsilon})$, a useful descriptor accounting for volume changes; as well as the equivalent (von Mises) strain $\boldsymbol{\epsilon}_{eq}$, a descriptor for quantifying shear at constant volume (see figure 6). It should be noted that while the displacement field can be simply scaled into the anisotropic space, the strains need to be recalculated.



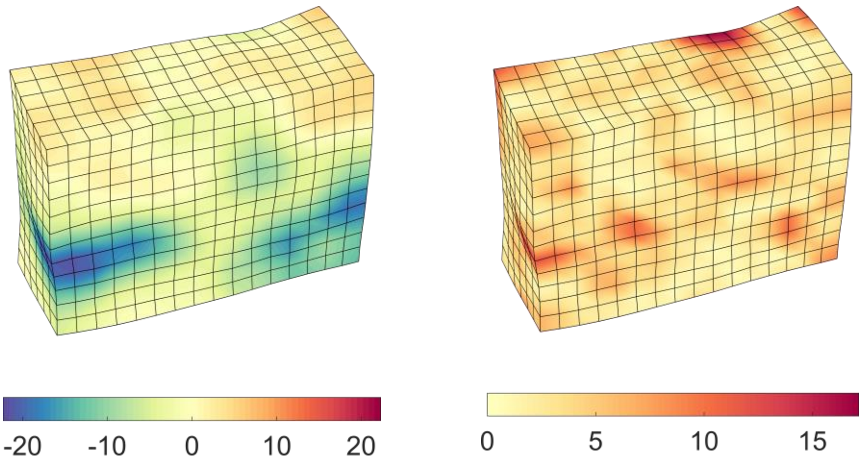
(a) $u_x(\boldsymbol{x})$ (b) $u_y(\boldsymbol{x})$ (c) $u_z(\boldsymbol{x})$

Figure 4: x, y and z components of the found displacement field $\boldsymbol{u}(\boldsymbol{x})$ in pixels



(a) $\epsilon_{xx}(\boldsymbol{x})$ (b) $\epsilon_{yy}(\boldsymbol{x})$ (c) $\epsilon_{zz}(\boldsymbol{x})$

Figure 5: Diagonal components of the computed strain tensor $\boldsymbol{\epsilon}(\boldsymbol{x})$ in %



(a) $\mathrm{tr}(\boldsymbol{\epsilon})$ (b) ϵ_{eq}

Figure 6: Volumetric and deviatoric strain fields in %

6 Conclusions

Digital Volume Correlation is employed as a tool to measure full 3D kinematics and to extract topological differences on a 3D woven composite part. The resulting continuous vector and tensor (strain) fields, in addition to the image of residuals, harmonize into a compelling quantitative (besides qualitative) descriptor for the processing of composite materials. Furthermore, the isotropic multiresolution approach is proven to be robust enough even in the presence of sizable topological differences, which pose a clear optimization challenge. These present a conflict between the minimization scheme (minimization of residuals) and the fact that high residuals are (as shown here) of great interest. Thus a tailored weighting scheme is currently being developed. Evidently this technique has many industrial applications. Starting from the material designing phase where the quantitative feedback of weaving deformations can be put in relation to certain weaving parameters, up to NDT applications where extreme deformations and large coherent residuals can be used for identifying weaving abnormalities. In any of such cases, a specific composite part could be selected as a *standard* and then be used as reference for the procedure as presented here.

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