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Seismic reconstruction using FWI with dual-sensors data.

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Romina Gaburro\textsuperscript{4} and Eva Sincich\textsuperscript{2}.

\textbf{GdR MecaWave, Fréjus, France}
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Overview

1. Introduction
2. Time-Harmonic Inverse Problem, FWI
3. Reconstruction procedure using dual-sensors data
4. Numerical experiments
   - Comparison of misfit functions
   - Changing the numerical acquisition with $J_G$
5. Conclusion
1 Introduction
Reconstruction of subsurface Earth properties from seismic campaign: collection of wave propagation data at the surface.
Seismic data

We work with back-scattered partial data from one-side illumination on large domain.
Seismic data

We work with back-scattered partial data from one-side illumination on large domain.
Seismic data

We work with back-scattered partial data from one-side illumination on large domain.
Seismic data

We work with **back-scattered partial data** from **one-side** illumination on large domain.
**Inverse problem**: from seismic traces to subsurface?

**nonlinear, ill-posed inverse problem.**
Plan

2 Time-Harmonic Inverse Problem, FWI
Time-harmonic wave equation

We consider propagation in acoustic media, given by the Euler’s equations, for the recovery of the medium parameters $\kappa$ and $\rho$:

\[
\begin{align*}
-\omega \rho \mathbf{v} &= -\nabla p, \\
-\omega p &= -\kappa \nabla \cdot \mathbf{v} + f.
\end{align*}
\]

$p$: scalar pressure field, \quad $\kappa$: bulk modulus, \\
$\mathbf{v}$: vectorial velocity field, \quad $\rho$: density, \\
$f$: source term, \quad $\omega$: angular frequency.
Time-harmonic wave equation

We consider propagation in acoustic media, given by the Euler’s equations, for the recovery of the medium parameters $\kappa$ and $\rho$:

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\begin{align*}
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-\imath \omega p &= -\kappa \nabla \cdot \mathbf{v} + f.
\end{align*}
\]

$p$: scalar pressure field, $\kappa$: bulk modulus,
$\mathbf{v}$: vectorial velocity field, $\rho$: density,
$f$: source term, $\omega$: angular frequency.

The system reduces to the Helmholtz equation when $\rho$ is constant,

\[
(-\omega^2 c^{-2} - \Delta) p = 0,
\]

with $c = \sqrt{\kappa \rho^{-1}}$. 

The inverse problem aims the recovery of the subsurface medium parameters from surface measurements of pressure and normal (vertical) velocity:

\[ \mathcal{F} : m = (\kappa, \rho) \rightarrow \{ \mathcal{F}_p ; \mathcal{F}_v \} = \{ p(x_1), p(x_2), \ldots, p(x_{n_{rcv}}); v_n(x_1), v_n(x_2), \ldots, v_n(x_{n_{rcv}}) \}. \]

D. Carlson, N. D. Whitmore et al.
Increased resolution of seismic data from a dual-sensor streamer cable – Imaging of primaries and multiples using a dual-sensor towed streamer
SEG, 2007 – 2010

CGG & Lundun Norway (2017–2018)
TopSeis acquisition (www.cgg.com/en/What-We-Do/Offshore/Products-and-Solutions/TopSeis)
Full Waveform Inversion (FWI)

FWI provides a **quantitative reconstruction** of the subsurface parameters by solving a minimization problem,

\[
\min_{m \in M} J(m) = \frac{1}{2} \| F(m) - d \|^2.
\]

- \(d\) are the observed data,
- \(F(m)\) represents the simulation using an initial model \(m\):

  - P. Lailly
    - The seismic inverse problem as a sequence of before stack migrations
    - Conference on Inverse Scattering: Theory and Application, SIAM, 1983
  - A. Tarantola
    - Inversion of seismic reflection data in the acoustic approximation
    - Geophysics, 1984
  - A. Tarantola
    - Inversion of travel times and seismic waveforms
    - Seismic tomography, 1987
FWI, iterative minimization

\[ \text{Observations} \rightarrow \text{Misfit functional } J \rightarrow \text{Initial model } m_0 \rightarrow k = 0 \rightarrow \text{Forward problem } F_\omega(m_k) \]
FWI, iterative minimization

Observations

Initial model $m_0$

Forward problem $F_\omega(m_k)$

Misfit functional $\mathcal{J}$

Optimization procedure

1. Gradient
2. Search direction $s_k$
3. Line search $\alpha_k$

Update model

$m_{k+1} = m_k + \alpha_k s_k$

Update $\omega$

$k = 0$

$k = k + 1$
FWI, iterative minimization

Observations → Initial model $m_0$ → Forward problem $F_\omega(m_k)$ → Misfit functional $J$ → Optimization procedure:

1. Gradient
2. Search direction $s_k$
3. Line search $\alpha_k$

$k = 0$
$k = k + 1$

Update model $m_{k+1} = m_k + \alpha_k s_k$

Numerical methods:

- Adjoint-method for the gradient computation,
- Forward problem resolution with Discontinuous Galerkin methods,
- Parallel computation, HPC, large-scale optimization,
- Rk: the code also works for elastic anisotropy and viscous media.
Plan

3 Reconstruction procedure using dual-sensors data
Minimization of the cost function

The appropriate misfit functional to minimize with pressure and vertical velocity measurements.

Compare the pressure and velocity fields separately:

\[
\mathcal{J}_{L2} = \sum_{\text{source}} \frac{1}{2} \| \mathcal{F}_p^{(s)} - d_p^{(s)} \|^2 + \frac{1}{2} \| \mathcal{F}_v^{(s)} - d_v^{(s)} \|^2.
\]
Minimization of the cost function

The appropriate misfit functional to minimize with pressure and vertical velocity measurements.

- Compare the pressure and velocity fields separately:

\[ J_{L2} = \sum_{\text{source}} \frac{1}{2} \| F_p^{(s)} - d_p^{(s)} \|^2 + \frac{1}{2} \| F_v^{(s)} - d_v^{(s)} \|^2. \]

- Compare the fields multiplication for all combinations:

\[ J_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \| d_v^{(s_1)T} F_p^{(s_2)} - d_p^{(s_1)T} F_v^{(s_2)} \|^2. \]
Minimization of the cost function

\[ J_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \| d_v^{(s_1)} T F_p^{(s_2)} - d_p^{(s_1)} T F_v^{(s_2)} \|_2^2. \]

From Euler’s equation, \( v_n(x_i) = -i(\omega \rho)^{-1} \partial_n p(x_i) \).
Minimization of the cost function

\[ J_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \| d_{v}^{(s_1)} T F_{p}^{(s_2)} - d_{p}^{(s_1)} T F_{v}^{(s_2)} \|^2. \]

From Euler’s equation, \( v_n(x_i) = -i(\omega \rho)^{-1} \partial_n p(x_i). \)

- **Cauchy data**: the cost function follows **Green’s identity**.
Minimization of the cost function

\[ J_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \| d_v^{(s_1)} T F_p^{(s_2)} - d_p^{(s_1)} T F_v^{(s_2)} \|_2^2. \]

From Euler’s equation, \( v_n(x_i) = -i(\omega \rho)^{-1} \partial_n p(x_i). \)

- **Cauchy data**: the cost function follows **Green’s identity**.
- **Reciprocity gap functional** in inverse scattering.

---

D. Colton and H. Haddar

An application of the reciprocity gap functional to inverse scattering theory

*Inverse Problems* 21 (1) (2005), 383398.

G. Alessandrini, M.V. de Hoop, R. Gaburro and E. Sincich

Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data

arXiv:1702.04222, 2017

G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich

Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization

preprint
Lipschitz type stability is obtained for the Helmholtz equation with partial Cauchy data.

\[ \| c_1 - c_2 \| \leq C \left( J_G(c_1, c_2) \right)^{1/2} \]
Stability results

Lipschitz type stability is obtained for the Helmholtz equation with partial Cauchy data.

$$\|c_1 - c_2\| \leq C (J_G(c_1, c_2))^{1/2}$$

- Using back-scattered data from one side in a domain with free surface and absorbing conditions,

- for piecewise linear parameters.

G. Alessandrini, M.V. de Hoop, R. Gaburro and E. Sincich
Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data
arXiv:1702.04222, 2017

G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich
Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization
preprint
It allows the non-collocation of numerical and observational sources:

\[ J_g = \frac{1}{2} \sum_{s_1} \sum_{s_2} \| d_v^{(s_1)} T F_p^{(s_2)} - d_p^{(s_1)} T F_v^{(s_2)} \|_2^2. \]

- \( s_1 \) is fixed by the observational setup,
- \( s_2 \) is chosen for the numerical comparisons.
Plan

4 Numerical experiments

- Comparison of misfit functions
- Changing the numerical acquisition with $J_g$
Experiment setup

3D velocity model $2.5 \times 1.5 \times 1.2\text{km}$ using dual-sensors data.
We work with time-domain data acquisition.
Experiment setup

We work with time-domain data (pressure and velocity).

Acquisition for the measures

- 160 sources,
- 100 m depth,
- point source,

For the reconstruction, we apply a Fourier transform of the time data.
Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of $J_{L2}$ or $J_G$, frequency from 3 to 15Hz.

$$J_{L2} = \sum_{\text{source}} \frac{1}{2} \| \mathcal{F}_p^{(s)} - d_p^{(s)} \|^2 + \frac{1}{2} \| \mathcal{F}_v^{(s)} - d_v^{(s)} \|^2.$$  

$$J_G = \frac{1}{2} \sum_{\text{source}} \sum_{\text{source}} \| d_v^{(s_1) T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1) T} \mathcal{F}_v^{(s_2)} \|^2.$$  

Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of $J_{L2}$ or $J_G$, frequency from 3 to 15Hz.

(a) True velocity

(b) Starting velocity
Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of $J_{L2}$ or $J_G$, frequency from 3 to 15Hz.

(a) Using $J_{L2}$

(b) Using $J_G$
Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of $J_{L2}$ or $J_{G}$, frequency from 3 to 15Hz.

(a) Using $J_{L2}$  
(b) Using $J_{G}$

But the major advantage of $J_{G}$ is the possibility to consider alternative acquisition setup.
Experiment with different obs. and sim. acquisition

\[ \min J_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \| d^{(s_1)T} F^{(s_2)}_p - d^{(s_1)T} F^{(s_2)}_v \|^2. \]

Acquisition for the measures \( s_1 \)

- 160 sources,
- 100 m depth,
- point source,
Experiment with different obs. and sim. acquisition

\[ \min J_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \| d_{v}(s_1)^T F_{p}(s_2) - d_{p}(s_1)^T F_{v}(s_2) \|^2. \]

Acquisition for the measures \( s_1 \)

- 160 sources,
- 100 m depth,
- point source,

Arbitrary numerical acquisition \( s_2 \)

- 5 sources,
- 80 m depth,
- multi-point sources,

- No need to known observational source wavelet.
- Differentiation impossible with least squares types misfit.
Experiment with different obs. and sim. acquisition

Data from frequency between 3 to 15 Hz, domain size $2.5 \times 1.5 \times 1.2$ km, **Simulation using 5 sources only**.

(a) True velocity  
(b) Starting velocity
Experiment with different obs. and sim. acquisition

Frequency from 3 to 15 Hz, $2.5 \times 1.5 \times 1.2$ km, Simulation using 5 sources only. -33% computational time.

(a) True velocity

(b) 15 Hz reconstruction
Plan

Conclusion
Seismic inverse problem using pressure and vertical velocity data:

- appropriate cost function to minimize,
- allow minimal information on the acquisition setup,
- other applications,
- **perspective:** design the most efficient numerical setup,
- Rk: possible for elastic media with measures of traction.
Conclusion

Seismic inverse problem using pressure and vertical velocity data:

- appropriate cost function to minimize,
- allow minimal information on the acquisition setup,
- other applications,
- perspective: design the most efficient numerical setup,
- Rk: possible for elastic media with measures of traction.

Quantitative reconstruction algorithm toolbox for time-harmonic wave,

- Discontinuous Galerkin discretization in HPC framework,
- large scale optimization scheme using back-scattered data,
- acoustic, elastic, anisotropy, 2D, 3D, attenuation.

P- and S-wavespeed reconstructions
Conclusion

Seismic inverse problem using pressure and vertical velocity data:

- appropriate cost function to minimize,
- allow minimal information on the acquisition setup,
- other applications,
- **perspective**: design the most efficient numerical setup,
- Rk: possible for elastic media with measures of traction.

Quantitative reconstruction algorithm toolbox for time-harmonic wave,

- **Discontinuous Galerkin** discretization in HPC framework,
- large scale optimization scheme using back-scattered data,
- acoustic, elastic, anisotropy, 2D, 3D, attenuation.

**Thank you**
Appendix
Stability of the Helmholtz Inverse Problem

\[ \|c_1^{-2} - c_2^{-2}\| \leq C(\|F(c_1^{-2}) - F(c_2^{-2})\|) \]

G. Alessandrini

Stable determination of conductivity by boundary measurement
Applicable Analysis 1988

N. Mandache

Exponential instability in an inverse problem for Schrödinger equation
Inverse Problems 2001
Stability of the Helmholtz Inverse Problem

\[ \| c_1^{-2} - c_2^{-2} \| \leq C (\| F(c_1^{-2}) - F(c_2^{-2}) \|) \]

- Stability associate data and model correspondence
- Reconstruction is based on the iterative minimization of the difference between observation and simulation using an initial model.

G. Alessandrini
Stable determination of conductivity by boundary measurement
Applicable Analysis 1988

N. Mandache
Exponential instability in an inverse problem for Schrödinger equation
Inverse Problems 2001
Stability of the Helmholtz Inverse Problem

\[ \| c_1^{-2} - c_2^{-2} \| \leq C \left( \| F(c_1^{-2}) - F(c_2^{-2}) \| \right) \]

- Stability associate data and model correspondence
- \( C(\delta) \leq C \left( \log(1 + \delta^{-1}) \right)^{-\alpha} \)

G. Alessandrini
Stable determination of conductivity by boundary measurement
Applicable Analysis 1988

N. Mandache
Exponential instability in an inverse problem for Schrödinger equation
Inverse Problems 2001
Conditional Lipschitz stability: assumptions

- $c(x)$ is bounded $B_1 \leq c^{-2}(x) \leq B_2$ in $\Omega$
- $c(x)$ has a **piecewise constant** representation of size $N$

$$c(x)^{-2} = \sum_{k=1}^{N} c_k \chi_k(x)$$

- $\Omega$ has Lipschitz boundary

$$\|c_1^{-2} - c_2^{-2}\|_{L^2(\Omega)} \leq C \|F(c_1^{-2}) - F(c_2^{-2})\| \quad (1)$$

---

G. Alessandrini and S. Vessella
Lipschitz stability for the inverse conductivity problem
Advances in Applied Mathematics 2005

E. Beretta, M. V. de Hoop, F. and O. Scherzer
Inverse boundary value problem for the Helmholtz equation: quantitative conditional Lipschitz stability estimates.
SIAM Journal of Mathematical Analysis 2016
The stability constant is bounded

\[
\frac{1}{4\omega^2} e^{K_1 N^{1/5}} \leq C \leq \frac{1}{\omega^2} e^{(K(1+\omega^2 B_2)N^{4/7})}
\]  

depends on the partitioning \( N \) and the frequency \( \omega \)

---

E. Beretta, M. V. de Hoop, F. and O. Scherzer
Inverse boundary value problem for the Helmholtz equation: quantitative conditional Lipschitz stability estimates 2016
In the case of partial Cauchy data ($p$ and $\partial_{\nu}p$), we have that, we can obtain a Lipschitz type stability:

$$\|c_1^{-2} - c_2^{-2}\| \leq C \left( J_G(c_1^{-2}, c_2^{-2}) \right)^{1/2}$$

Where $c_1^{-2}$ and $c_2^{-2}$ are piecewise linear.