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Ayman Chouayakh, Aurélien Bechler*, Isabel Amigo†, Loutfi Nuaymi†, Patrick Maillé†
*Orange Labs, Chatillon, France
† IMT Atlantique, Brest, Nantes and Rennes, France
Email:{ayman.chouayakh, aurelien.bechler}@orange.com
Email: {isabel.amigo, loutfi.nuaymi, patrick.maille}@imt-atlantique.fr

***Abstract*—Licensed Shared Access (LSA) is a new sharing approach that aims to optimize the use of 2.3-2.4 Ghz frequency band in order to support the deployment of 5G systems. Under LSA, Mobile networks Operators (MNOs) can share the 2.3-2.4 band with the incumbent of that band under some guarantees. Those guarantees are mentioned in a license attributed by the regulator. Using ascending auctions is a natural approach for attributing licenses. In this paper, we show how to implement the ascending version of Vickrey-Clarke-Groves (VCG) mechanism in the LSA context, since that implementation may introduce some computational complexity problems, we propose another ascending mechanism called C-LSA based on the clinching approach. To compare those two mechanisms, it will be much more easier to compare its equivalent (in terms of allocations and payments) one shot versions. Hence, we propose the equivalent version of C-LSA. Finally we compare those ascending mechanisms by comparing their one-shot versions. Simulations shows that C-LSA is viable candidate for LSA.**

I. Introduction

For fifth generation (5G) wireless networks, dealing with mobile data traffic is challenging, traffic volume being expected to explode. At the same time, some licensed frequency bands hold by governmental agencies are underutilized, leading to the emergence of the Licensed Shared Access (LSA) idea [1], [2]. LSA is a new concept, proposed by the Radio Spectrum Policy group (RSPG) in November 2011, which aims to optimize the use of spectrum: the incumbents or the owners of the 2.3-2.4 Ghz bandwidth can temporarily share their spectrum with Mobile Network Operators (MNOs). Contrary to the traditional concept of sharing in which secondary users (MNOs in the LSA context) have no guarantee for accessing to the incumbent's spectrum, under LSA the duration and conditions of sharing are precisely defined beforehand by the regulator via a license. Deploying an LSA system requires the introduction of two new architectural blocks called the LSA repository–basically a database containing information about the LSA band such as conditions of sharing and duration–and the LSA controller which controls the access to the LSA bandwidth [2]. Several trials have been carried out to show the applicability of the LSA concept[1].

Since the objective of the LSA is to optimize spectrum usage, spatial reusability (MNOs who do not interfere can use the same spectrum bands simultaneously) should be leveraged. Spectrum reusability has been addressed in spectrum markets in the last decade [3]–[6]; we will in particular consider a scenario in which multiple base stations of different operators compete for LSA spectrum at a defined period of time in a particular geographical area. In this scenario, a mechanism for attributing licenses needs to be adopted. A common approach is to design an auction mechanism, due to the need for information revealed by potential buyers. Designing an auction mechanism here raises two major challenges: it should take spectrum reusability into account, and should be truthful (strategy-proof) i.e., each player should be incentivized to be sincere regarding their willingness-to-pay for the good (LSA spectrum in our context) independently of the bids of other buyers. In general, players are expected to try to manipulate the mechanism in order to maximize their profit, which may hurt other players' (including the auctioneer's) interests. For this reason designing truthful mechanisms is very important.

Several mechanisms have been proposed to address this issue. In [3], [4], [7], authors have designed auction mechanisms for the case where there is only one spectrum block to allocate. LSAA [7] is the first auction mechanism which was proposed as candidate for the LSA context, it performs well in terms of social welfare assuming truth-telling by bidders, but sincere bidding is not an optimal strategy for bidders. In [8], we modified the payment rule of LSAA to make it truthful. In [6], we designed an auction mechanism for the case when bandwidth is infinitely divisible. But reality lies between those two extremes: spectrum can be split in several sub-bands or blocks with a predetermined size, and in [9] an auction mechanism for allocating K identical (interchangeable) resource blocks was proposed.

In this paper, we again suppose that the auctioned LSA spectrum is composed of K identical blocks, but contrary to the mentioned previous work which are sealed-bid one-shot auctions, we focus on "ascending auctions" where information is revealed by bidders during some convergence phase. Ascending auctions have been used with great success to auction spectrum and they are preferred over sealed auctions [10]. Compared with one-shot auctions, ascending auctions have several advantages: they preserve the privacy of the winning bidder(s) because the winner(s) do(es) not reveal his/their valuation(s). Also, they give bidders the opportunity to adjust their valuations over the convergence phase. This benefit of price discovery is ignored in one-shot auctions, which as-

[1] https://www.cept.org/ecc/topics/lsa-implementation

sume that each bidder perfectly knows his valuation. Another advantage is the transparency because each bidder sees the evolution of the auction. There are several types of ascending auction mechanisms, in this document we first focus on two mechanisms that achieve the outcome of the Vickrey-Clarke-Groves (VCG) auction [11]–[13], known to be jointly truthful and efficient (the total value extracted from the resource is maximal). We start by presenting the model of Mishra and Parkes [14], who introduce a new concept called "Universal Competitive Equilibrium" (UCE). Their mechanism can be applied for general valuations (items may be different) and yields the same outcome as the VCG mechanism. The other mechanism has been developed by Ausubel [15], based on the "clinching approach" which we detail in Section IV-B, can be applied when items are identical. We investigate those two mechanisms under the LSA concept. The rest of the paper is organized as follows: in Section II we present those two mechanisms. Section III introduces our LSA model, and Section IV present our first contributions, namely adaptations of the UCE (U-LSA) and the clinching mechanisms (C-LSA) to the LSA context, i.e., encompassing spectrum reusability. We then show in Section V that each of those mechanisms has a one-shot auction equivalent, which helps us establish some key properties of the studied mechanisms. Finally, the performance of those schemes are compared through simulations in Section VI, and we conclude and give some directions for future work in Section VII.

The contributions of this paper can be summarized as follows:

1) We adapt the UCE auction to the LSA context (taking into account spectrum reusability), and highlight some computational complexity issues with the resulting mechanism (U-LSA), even when auctioned items (spectrum blocks) are identical.
2) To deal with computation complexity of UCE, we propose to adapt the clinching approach by proposing a new ascending mechanism which we call C-LSA.
3) We propose a new one-shot mechanism that is equivalent to C-LSA, and use it to establish some properties of C-LSA in terms of truthfulness and revenue guarantees.
4) We compare U-LSA with C-LSA through simulations, by comparing their equivalent one-shot versions, and show that C-LSA yields in most cases much larger revenues (with a multiplicative factor between 5 and 10) for a very limited cost in terms of resource usage efficiency (around 4%).

Hence we think our proposed mechanism C-LSA is a viable candidate for LSA spectrum allocation and pricing applied in a wireless 5G network.

II. Related work: two efficient ascending mechanisms

In this section, we summarize some desirable properties and present two ascending auction mechanisms from the literature known to be equivalent to VCG under different scenarios: the first mechanism is proposed by Mishra and Parkes and can be applied for general valuations (items may be different), and the second applies when items are identical and valuation of a player for an extra item decreases with the number of items obtained. Note that both mechanisms work only under the following restrictions which are standard in the literature.

1) Private valuations: Each player i is the only one knowing his valuation $v_i(S) \geq 0$ for each bundle of items S.
2) Free disposal: for each player i and every bundles S and T, $S \subset T \Rightarrow v_i(S) \leq v_i(T)$.
3) Quasi-linear utility: The utility of a buyer or bidder i is $v_i(S) - p_i(S)$ where p_i is the price paid by buyer i when obtaining the bundle S .
4) Zero seller valuations: The seller values the items at zero. His utility is his revenue which is the total payment of buyers.

We moreover assume (without much loss of generality, since one can select the monetary unit) that all valuation values are integers.

A. Desirable properties of an auction mechanism

Here we summarize the most frequently used desirable properties of an auction mechanism [16], [17]:

1) Social welfare maximization (also referred to as efficiency): Suppose there are N players, social welfare SW is defined as the sum of all utilities including the regulator (whose utility is his revenue)
$$\mathrm{SW} = \sum_{i=1}^{N}(v_i(S) - p_i(S)) + \sum_{i=1}^{N} p_i(S) = \sum_{i=1}^{N} v_i(S)$$
2) Truthfulness: this property means that bidders' best strategy is to behave sincerely, i.e., lying about one's preferences is not beneficial. The strongest version is when truth-telling is a dominant strategy, but it can also be a (weaker) ex-post Nash equilibrium strategy : When truthful bidding is an ex-post equilibrium, each player knows that bidding truthfully is a best strategy if all other players also bid truthfully and without knowing the other players' valuations [18].
3) Maximize the revenue of the seller which is the sum of payments of all buyers.

B. Mishra and Parkes' UCE mechanism

Consider N players and a set of different items $\boldsymbol{I} = \{1, ..., K\}$, and denote by $\Omega = \{S \subseteq \boldsymbol{I}\}$ the set of all bundles of items. Mishra and Parkes [14] define an ascending auction–which we will call UCE–as a price path that starts from round 0 with vector price P^0 and ends at some round T with vector price P^T, and develop an ascending auction that is equivalent to the VCG mechanism by introducing the concept of Universel Competitive Equilibrium (UCE). The price vector P^t at each round t is of dimension $N2^K$ (each player i faces a price vector P_i^t of 2^K elements, one for each bundle), and is therefore non-anonymous (each player sees a different price for the same bundle). Before presenting the auction, we introduce some necessary notations and definitions.

1) Notations and definitions for UCE:

a) Feasible allocation: An allocation is a vector of bundles on buyers, the set of feasible allocations is denoted by $\mathbb{X}$.

b) Demand set: The demand set of a player i, d_i^t at round t is defined as the set of bundles that maximize his profit at price vector P_i^t, i.e., mathematically,

$$d_i(P) := \arg\max_{S\in\Omega} \left(v_i(S) - P_i(S)\right).$$

Note that $d_i(P)$ can contain several elements if the player has several utility-maximizing bundles. Each player reports his demand set at each round; if a buyer demands the empty bundle, then this player is called inactive, and must receive a zero utility from any obtained bundle, i.e., $P_i(S) \geq V_i(S)$ $\forall S \in \Omega$.

c) Supply set: The supply set is the set of allocations that maximize the payoff of the seller, which is the sum of payments of all players at price P:

$$L(P) := \arg\max_{X\in\mathbb{X}} \sum P_i(X_i)$$

Note that those are not the real paid prices: the price vector aims at eliciting preference revelation, but each player will have a discount at the end of the auction. The final payment by each player is indeed defined in [14] as:

$$p_i = P_i(S) - \underbrace{\left(\Pi(P) - \Pi(P_{-i})\right)}_{\text{discount}}, \tag{1}$$

where $\Pi(P)$ is the sum of prices (based on P) of players for the final allocation, and $\Pi(P_{-i})$ is the sum of payments of all players when i is absent (i.e., for a new revenue-maximizing allocation ignoring i).

Definition 1. *Competitive Equilibrium (CE): a price P and an allocation X are a competitive equilibrium (CE) if $X \in L(P)$ and $X_i \in d_i(P_i)$ for every buyer i, i.e., for this price the allocation both maximizes revenue and satisfies each buyer.*

Definition 2. *Universal Competitive Equilibrium (UCE): a price P is a UCE if it is a CE and the projection of P on every marginal economy (that is, the same situation but removing one player: in total there are N marginal economies) is a CE. This means that we can always satisfy all buyers while maximizing the revenue of the regulator after excluding any individual player.*

Achieving a universal competitive equilibrium price is very important because as it was proven in [14], Vickrey payments can be computed from P if and only if P is a UCE price vector.

We illustrate these notions using an example with two items (A and B) and three players having the following valuations:

$$\begin{pmatrix} v_1(A) & v_1(B) & v_1(A,B) \\ v_2(A) & v_2(B) & v_2(A,B) \\ v_3(A) & v_3(B) & v_3(A,B) \end{pmatrix} = \begin{pmatrix} 4 & 0 & 4 \\ 0 & 5 & 5 \\ 0 & 2 & 4 \end{pmatrix}$$

We suppose that $P_1(\emptyset) = P_2(\emptyset) = P_3(\emptyset) = 0$, consider the price vector $P = \begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 2 \\ 0 & 2 & 4 \end{pmatrix}$. The player demands are then $d_1(P) = \{A, \{A, B\}\}$, $d_2(P) = \{B, \{A, B\}\}$ and $d_3(P) = \{\emptyset, A, B, \{A, B\}\}$. In this example (P, X) –where X is the allocations that assigns item A to player one and item B to player two– is a CE equilibrium for the main economy because the seller can maximize his revenue and satisfy the players. However, after excluding player one, P is not anymore a competitive equilibrium in the resulting marginal economy because maximizing revenue implies allocating both items to player three, in which case player two is not satisfied.

On the other hand, the price $P = \begin{pmatrix} 2 & 0 & 4 \\ 0 & 4 & 4 \\ 0 & 2 & 4 \end{pmatrix}$ is a universal competitive equilibrium: in the main economy composed by all players, the seller can maximize his revenue (6) while satisfying all buyers by allocating item A for player one and item B for player two. In addition:

- this still holds in the marginal economy where player three is removed;
- in the economy composed by player two and three, revenue is maximized by allocating both items (or only item B) to player two, both players having no better option;
- similarly in the economy composed by player one and three, the seller can allocate both items to player one.

Now applying (1), we get the paid prices $p_1 = 2-(6-4) = 0$, $p_2 = 4 - (6 - 4) = 2$, and $p_3 = 0$.

2) The UCE auction steps: We now explain how the UCE auction can be implemented in practice.

The auction starts with all prices set to 0 in P^0. At each round t, the seller asks player's demands for the price vector, and checks whether a Universal Competitive Equilibrium is reached. If it is not the case, a subset of active players (i.e., not having $\emptyset$ in their demand set) is selected and all the prices of their demand sets are increased by one unit in the next price P^{t+1}. How to choose this subset opens some trade-offs, larger subsets speeding up the convergence while possibly increasing the communication overhead and the revealed valuations of players. An example of choosing that subset is given in Section IV. We can summarize the auction steps as follows:

1) At every round t, each buyer reports his demand set for the price vector P^t. Players should respect two activity rules:
   - Round Monotonicity: for every buyer $d_i(P^t) \subset d_i(P^{t+1})$
   - Bundle Monotonicity: if $S \subset T$ and $S \in d_i(P^t)$ then $T \in d_i(P^t)$.

   Note that these rules are satisfiable because valuations are integers and prices only increase by one unit (or zero) between t and $t+1$ for bundles in $d_i(P^t)$, and never increase if $\emptyset \in d_i(P^t)$.
2) The seller computes the supply. If the situation is not an UCE, the auctioneer chooses a set of players who will see a price increase at each demanded bundle.
3) The auction ends when a UCE price vector P is reached; then a CE allocation is chosen, i.e., revenue is maximized and every buyer gets a bundle from his demand set.

4) Each buyer is charged an amount p_i computed from the final price P, applying (1) for the chosen allocation.

In the next subsection, we present the clinching auction, which is another way to reach an efficient allocation, in the specific case of identical items and decreasing marginal valuations. Contrary to the previous mechanism in which players report their demand set, in this auction, since items are identical, each player just reports the number of blocks he wants. Also, the payment of each player is computed dynamically during the auction.

C. The clinching auction

The clinching auction is an ascendant auction for K homogeneous goods, where bidders have decreasing marginal valuations: the willingness-to-pay for an extra item decreases with the number of items already obtained. At each round t, the auctioneer declares a price p^t and bidders respond by asking for a quantity (at each round the demanded quantity can not be higher than the demanded quantity in the previous round) at that price, the price increasing (in general we can increment by $\epsilon > 0$ but here with integer valuations we take $p^{t+1} = p^t + 1$) until demand is no greater than supply K. Bidders' payments are computed during the auction: an active bidder clinches (obtains) an item at price p if the demand of the other players at that price is less than the supply. The seller computes two quantities namely *cumulative clinch* and *current clinch*, defined as follows.The cumulative clinch Cl_i^t of player i at round t is defined as:

$$\mathrm{Cl}_i^t = \max\{0, K - \sum_{j \neq i} d_j^t\}, \tag{2}$$

with d_j^t the demand of player j at round t. The increment of the obtained blocks is called the current clinch at round t of player i, and denoted by cl_i^t:

$$\mathrm{cl}_i^t = \mathrm{Cl}_i^t - \mathrm{Cl}_i^{t-1}. \tag{3}$$

When the auction ends, each bidder i obtains a quantity equal to his cumulative clinch Cl_i, and his payment p_i is:

$$p_i = \sum_{t=0}^{T} p^t \mathrm{cl}_i^t. \tag{4}$$

An illustrative example is provided in Table I, with three items, and three players with respective marginal valuations $\{6, 4, 0\}$, $\{5, 3, 2\}$, $\{2, 1, 0\}$. (Please note that we will suppose that players are not willing to pay a price per block equal to the valuation of that block, as an example if player one gets one block for a price $p^t = 6$ then his utility is zero, hence we will suppose that for $p^t = 6$ player one will not demand any block i.e., his demand is zero.)

For $p = 2$, the sum of demands of player two and three is equal to 2, hence, $\mathrm{cl}_1^2 = 1 - 0 = 1$, player one clinches his first block at price 2. Similarly, player two clinches his block at the same price. At $p = 3$, $\mathrm{cl}_1^3 = 2 - 1 = 1$, thus player one clinches his second block. Finally the auction concludes at price $p = 3$ $(d_1 + d_2 + d_3 = 3)$, player one obtains two blocks and pays $2 + 3 = 5$ and player two obtains one block and pays 2.

Round	0	1	2	3
Price	0	1	2	3
Total demand	7	6	4	$3 = K$
d_1	2	2	2	2
Cl_1^t	0	0	1	2
cl_1^t	0	0	1	1
p_1	0	0	2	2+3
d_2	3	3	2	1
Cl_2^t	0	0	1	1
cl_2^t	0	0	1	0
p_2	0	0	2	2
d_3	2	1	0	0
Cl_3^t	0	0	0	0
cl_3^t	0	0	0	0
p_3	0	0	0	0

TABLE I: An example of clinching auction for $K = 3$ items.

III. SYSTEM AND BIDDER MODEL

In this section, we present the LSA system model and we detail preferences of base stations and the regulator.

A. Grouping operators before the auction

We consider N base stations of different operators in competitions over K identical blocks. We intend to leverage spacial reusability: two base stations can use the same bandwidth simultaneously if they do not interfere with each other. Following the approach of [3], [4], [6], [7], [19], the competition between the N base stations is transformed into a competition between M groups in such a way that two base stations in the same group h (the set of base stations in that group is denoted by g_h) do not interfere. While the group formation has a non-negligible impact, in this paper (as in [3], [4], [6], [7] that also rely on groups) we assume that the groups are formed by the auctioneer, and advertised to bidders before the auction takes place. We indeed focus here on how to allocate the resource among groups, based on the submitted bids.

B. Preferences of base stations

We assume that each base station i has a private valuation v_i vector of size K, each element $v_{i,k}$ representing the willingness-to-pay for one extra resource block. As in [20], we suppose that the value of an extra block, for a base station, decreases with the number of blocks already obtained. This corresponds to a discretization of concave valuation functions for spectrum [20], as illustrated in Figure 1. Finally, we adopt a quasi-linear utility model: if a base station i obtains n_i blocks and pays p_i, its utility then is

$$u_i = \sum_{n=1}^{n_i} v_{i,n} - p_i,$$

In particular, a base station obtaining no block gets a utility equal to zero. We denote by V_{i,n_i} the valuation of player i for n_i blocks.

$$V_{i,n_i} = \sum_{n=1}^{n_i} v_{i,n} \tag{5}$$

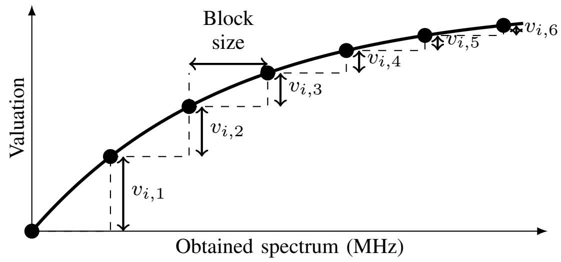

Fig. 1: An example of a concave valuation function of obtained spectrum, and the corresponding block valuations $v_{i,n}$ for player i.

IV. Proposed ascending auction mechanisms for LSA

In this section we show how to adapt the UCE and clinching auction schemes to the LSA context.

A. Adapting the UCE auction to the LSA context

To implement the UCE auction mechanism, we need to take into account a set of feasible allocations different from the original design since the allocation will be made for groups instead of individual bidders while bids are individual.

1) LSA-UCE rules: We denote by $\mathbb{X}^g$ the set of feasible allocations, that assign to each player i a bundle X_i^g such that $X_i^g = X_j^g$ if players i and j are in the same group and $X_i^g \cap X_j^g = \emptyset$ otherwise. Note that all the demonstrations of [14] are still valid when replacing $\mathbb{X}$ with $\mathbb{X}^g$ so the truthfulness and social welfare maximization properties can be easily proved. The auction steps are exactly the same as those presented in Section II-B, except that the allocation is made for groups i.e., players of the same group obtain the same bundle. We illustrate in the following how we can adapt Mishra and Parkes model in the LSA context.

Note that the price vector for each player is composed by K components instead of 2^K (since items are identical), the first component representing the price paid for all blocks and the K^{th} components represent the price paid for only one block.

An example is provided in Table II. At each round, following a proposition in [14], we take a minimum set of buyers who cannot be jointly satisfied until a CE, i.e., until Round 7. Then we pick one of the active players (those having strictly positive utilities) until a UCE is reached, at Round 9: in the marginal economies where one player of Group 1 is removed, revenue is maximized by giving one block to each player, and if player four is removed each player can get two blocks. Finally, Group 1 obtains two blocks since this maximizes revenue, and the payments yield: $p_1 = 3-(9-7) = 1$, $p_2 = 3-(9-7) = 1$, $p_3 = 3-(9-8) = 2$, and $p_4 = 0-(9-9) = 0$.

2) Computational complexity problems with U-LSA: Even if the original problem of computing an optimal allocation is not NP-hard when we know the players' valuations, implementing UCE may involve having to solve NP-hard problems, which prevents its use in practice.

Round		**Group 1**			**Group 2**
		buyer 1	buyer 2	buyer 3	buyer 4
	$V_{i,2}, V_{i,1}$	$5, 3$	$6, 4$	$6, 3$	$7, 6$
0	Price	$0, 0$	$0, 0$	$0, 0$	$0, 0$
	Utility	$(5), 3$	$(6), 4$	$(6), 3$	$(7), 6$
1	Price	$1, 0$	$0, 0$	$0, 0$	$1, 0$
	Utility	(4),3	(6),4	$(6), 3$	$(6), (6)$
2	Price	$1, 0$	$0, 0$	$1, 0$	$2, 1$
	Utility	$(4), 3$	(6),4	$(5), 3$	$(5), (5)$
3	Price	$2, 0$	$0, 0$	$1, 0$	$3, 2$
	Utility	$(3), (3)$	$(6), 4$	$(5), 3$	$(4), (4)$
4	Price	$2, 0$	$1, 0$	$1, 0$	$4, 3$
	Surplus	$(3), (3)$	$(5), 4$	$(5), 3$	$(3), (3)$
5	Price	$2, 0$	$1, 0$	$2, 0$	$5, 4$
	Utility	$(3), (3)$	$(5), 4$	$(4), 3$	$(2), (2)$
6	Price	$2, 0$	$2, 0$	$2, 0$	$6, 5$
	Utility	$(3), (3)$	$(4), (4)$	$(4), 3$	$(1), (1)$
7	Price	$2, 0$	$2, 0$	$3, 0$	$7, 6$
	Utility	$(3), (3)$	$(4), (4)$	$(3), (3)$	$(0), (0)$
8	Price	$2, 0$	$3, 1$	$3, 0$	$7, 6$
	Utility	$(3), (3)$	$(3), (3)$	$(3), (3)$	$(0), (0)$
9	Price	$3, 1$	$3, 1$	$3, 0$	$7, 6$
	Utility	$(2), (2)$	$(3), (3)$	$(3), (3)$	$(0), (0)$

TABLE II: Example of an UCE auction in the LSA context for two resource blocks and two groups. At each round, bidder utility in parentheses indicate the demand sets (as example, in the round 0 player one demands two blocks because he maximizes his utility if he obtains two blocks), and grayed cells indicate the bidders whose prices (of the demand set) will be raised by one unit (as example, in the round 0 player one demands two blocks and player four demands two blocks, we choose them as minimum set, thus prices of their demanded set will increase in the next round.

Proposition 1. *Even when blocks are identical, an implementation of UCE can lead to the regulator having to solve NP-hard problems.*

Proof. We show that the step of finding a revenue-maximizing allocation can correspond to solving a knapsack optimization problem with N items, item i ($i = 1, ..., N$) having weight w_i and value $\tilde{v}_i$ and maximum allowed weight W. Note that we assume $W < \sum_i w_i$ otherwise the problem is trivial.
We consider an instance of the previous problem and let us reduce it to our problem:

- The maximum allowed weight W is the number of blocks K to allocate.
- Each item of weight w_i and value $\tilde{v}_i$ corresponds to a group which contains only one player with vector of valuations v_i such that $v_{i,n} = \tilde{v}_i \mathbb{1}_{n \le w_i}$, this means that the valuation of player i for n_i blocks is $n_i \tilde{v}_i$ if $n_i \le w_i$ and $w_i \tilde{v}_i$ if $n_i \ge$w$_i$.

Then, as long as its price is below $\tilde{v}_i$, player i will keep asking for being allocated w_i blocks or more, and the corresponding prices will be of the form "p_i for w_i blocks or more, and 0 otherwise". Since the algorithm does not specify whose (unsatisfied) bidder prices will be raised, it can happen that the prices of each bidder i reach $\tilde{v}_i$. As an example suppose that $W = 6$ and let us consider an item of weight $w_i = 3$ and valuation $\tilde{v}_i = 5$, this object corresponds to a player with $v_i = (5, 5, 5, 0, 0, 0)$ thus $V_i = (15, 15, 15, 15, 10, 5)$, the price starts with $(0, 0, 0, 0, 0, 0)$, player i keep asking for $3, 4, 5$ or 6

blocks, thus the price can achieves $(5, 5, 5, 5, 0, 0)$. Since the algorithm does not specify whose (unsatisfied) bidder prices will be raised, it can happen that the price p_i of each bidder i reaches $\tilde{v}_i$, and finding an allocation maximizing revenue is equivalent to solving the knapsack problem. [1] □

Thus, we propose in the following mechanism with a simpler allocation based on the concept of clinching.

B. Adapting the clinching mechanism to the LSA context: C-LSA

In this section we study the possibility of implementing the clinching approach in the LSA context, through a mechanism that we call C-LSA.

1) C-LSA rules: As in the initial clinching auction, the auctioneer broadcasts a per-block price Pstarting with $P = 0$ (to simplify notation we write P instead of P^t), each group h responds with its demand $D_h(P)$, that is, a number of blocks the group is willing to buy at that price, we describe later on how to compute such demand. The auctioneer keeps increasing P by one unit until the sum of demands of all groups is equal or below K. To perform clinching (decide on blocks allocation), we use the same model as before but adapted for groups: the cumulative clinch Cl_h^t of group h is then defined as:

$$\mathrm{Cl}_h(P) := \max\{0, K - \sum_{j \neq h} D_j(P)\}. \tag{6}$$

As in the original scheme, the current clinch at time t for group h is the increment of Cl_h.

$$\mathrm{cl}_h(P) = \mathrm{Cl}_h(P) - \mathrm{Cl}_h(P-1). \tag{7}$$

The payment of each group h is:

$$P_h = \sum_{t=0}^{T} P^t \mathrm{cl}_h(P). \tag{8}$$

At each round, if group h obtains a block at price P^t then he pays P^t. An overview of the clinching approach for groups is presented in Fig.2. Please note that this is the price paid by the group, the price paid by each player belonging to such group and the computation of each group's demand is defined in the following.

To apply (6) the demand $D_h(P)$ of each group h at price P needs to be defined. We define $D_h(P)$ as the maximum number of blocks that a subset of players in group h are willing to buy if they equally share the unit price P. Mathematically,

$$\begin{aligned} D_h(P) = \max\{n : \exists\ \omega \subset g_h \text{ and } r\ \in \mathbb{R} \text{ s.t.} \\ \forall\ i\ \in \omega\ d_i(r) = n \text{ and } r|\omega| = P\}. \end{aligned}$$

Thus, we have to know demands of players in order to compute groups demand.

[1] From Isabel: pas assez clair pour moi

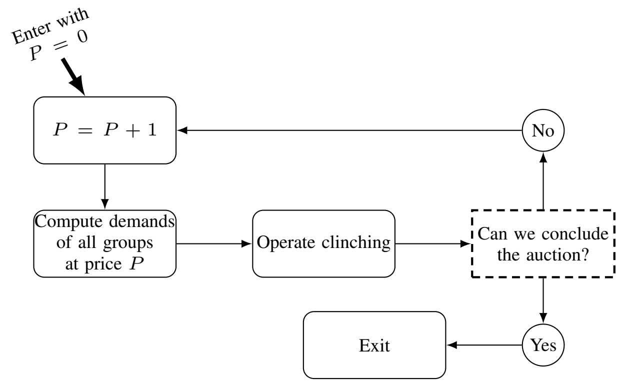

Fig. 2: Overview of the clinching approach for groups

2) Deriving group demands in C-LSA: The demand $D_h(P)$ can be obtained by requesting demands from individual group members. We propose to introduce a price p_h per group h, and ask each player i to reveal his demand $d_i(p_h)$ for that price. As shown in Fig. 3 We keep increasing p_h, $(p_h = p_h + 1)$ until we can compute $D_h(P)$ i.e.,find the maximum number of block n, the subset w and the price r. If each player reports his demand truthfully, then $d_i(p_h) = \max\{n \text{ s.t } v_i(n) > p_h\}$, we have supposed that player i is not interested on paying a price equals to his valuation for a block. Note that we can derive the vector of valuations of player i from his demand: if $d_i(p_h) = n$ and $d_i(p_h + 1) = n - 1$ then $v_i(n) = p_h + 1$.

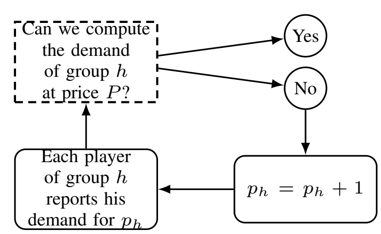

Fig. 3: Relation between P and p_h

The following simple example illustrates the process of collecting individual demands and deriving group demands.

Example 1. Consider $K = 4$ blocks, and one group (say, group 1) with 3 players having the following valuations $v_1 = \{9, 7, 6, 5\}$, $v_2 = \{7, 7, 5, 2\}$, and $v_3 = \{6, 3, 3, 2\}$.

To compute $D_1(P)$ for $P = 1$, we start with $p_1 = 1$ and ask players their demand at p_1. All players are willing to buy 4 blocks at that price, hence we know that $D_1(P) = 4$ as long as $P \leq 3$, each player paying a unit price $r = P/3 \leq 1$.

For $P = 4$ we need to ask individual demands at $p_1 = 2$. Truthful answers give $d_1(2) = 4$ and $d_2(2) = d_3(2) = 3$, so we know that $D_1(P) = 4$ as long as $r = P/3 < 2$, i.e., when $P \leq 5$.

For $P = 6$, we know from the responses for $p_1 = 2$ that $D_1(P) \geq 3$, since all three players are interested to buy 3 blocks at a unit price $6/3 = 2$. But possibly $D_1(P) = 4$, if player one is willing to buy 4 blocks at a unit price 6. So we increase p_1 and ask players their demand until either 6 is reached or $d_1(p_1) < 4$. The latter occurs first, for $p_1 = 5$, which leads to the conclusion that $D_1(6) = 3$ (each player paying 2).

Following that process, we derive the group demands $D_1(P) = 3$ for $P = 7, 8, 9$, $D_1(P) = 2$ for $10 \leq P \leq 13$, $D_1(P) = 1$ for $14 \leq P \leq 17$, and $D_1(P) = 0$ for $P \geq 18$

C. The working of C-LSA

We now put together C-LSA rules and computation of demands to build the complete C-LSA process. Its working is detailed in Fig. 4 and Algorithm 1: at each price P and for each group h, we keep increasing p_h until being able to compute $D_h(P)$, then we operate clinching (perform allocation); and we increment P until we can conclude the auction i.e., the sum of demands of all groups is below or equal to the capacity K.

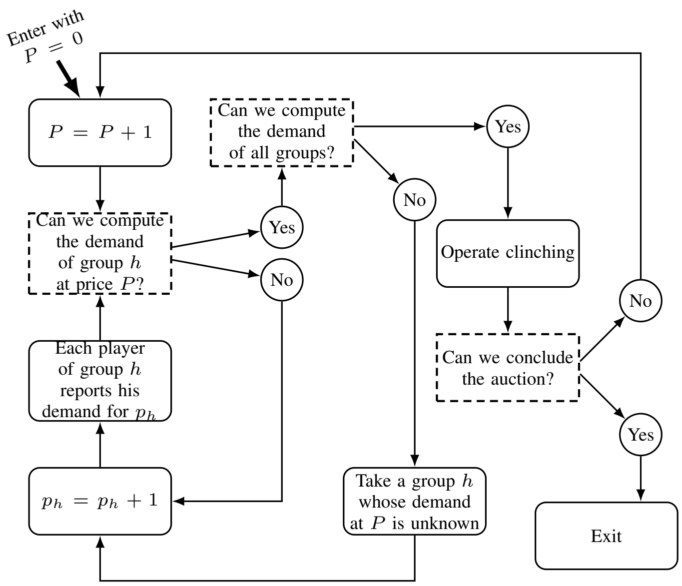

Fig. 4: C-LSA: applying the clinching approach in the LSA context

Through the following example we illustrate the complete mechanism.

Example 2. Consider three groups and $K = 4$ blocks, the first group is composed by three players, the second by two and the third by one player, with the following valuation vectors:

- in the first group: $\{9, 7, 6, 5\}$, $\{7, 7, 5, 2\}$, $\{6, 3, 3, 2\}$.
- in the second group: $\{7, 6, 4, 3\}$ $\{9, 8, 5, 2\}$
- in the third group: $\{13, 10, 3, 1\}$

At each P, we compute the demand of all groups (as we did before in the previous example), until total demand gets equal to or below K. The details for each round are shown in Table III. We conclude that the auction stops at $P = 12$ (sum of demands equals to 4):

1) The first and the second group clinch their first block at $P = 10$. Each player of the first group obtains that block and pays $\frac{10}{3}$. Each player of the second group pays $\frac{10}{2}$.
2) At $P = 12$ the first group clinches its second block and the third group clinches its first block. Player one and two of the first group pay 6 each one for their second block (player three will not obtain that block because he is not willing to pay 4 for a second block). The player of the third group pays 12 for his first block.

Price	1	2	3	4	5	6	7	8	9	**10**	11	**12**
D_1	4	4	4	4	4	3	3	3	3	2	2	2
D_2	4	4	4	3	3	3	3	2	2	2	2	1
D_2	3	3	2	2	2	2	2	2	2	1	1	1

TABLE III: Demand and price evolution for Example 2.

Algorithm 1 C-LSA allocation and pricing

Set $P = 0$
Set $p_h = 0$ for each group h
Set $\mathrm{Cl}_h^0 = 0$ for each group h
Set $D_h(P) = K$ for each group h
while $\sum_{h=1}^{M} D_h(P) > K$ **do**
  $P = P + 1$
  get-demand-at-P=False
  while get-demand-at-P==False **do**
    if demands of all groups can be computed at price P **then**
      get-demand-at-P=True
      Perform clinching
    else demand of some group h cannot be computed
      $p_h = p_h + 1$
      Ask bidders in group h their demand at p_h
    end if
  end while
end while

Note that our proposed clinching approach is not equivalent to VCG, as the clinching approach presented in Section II was. Indeed, suppose we have two groups and one block, the first group counting two players with equal value 5 and the second group with one player with value 2. Then implementing VCG means that each player of the first group obtains that block and pays zero. Having an equivalent clinching mechanism would imply that group one clinches that block when $P = 0$, which is impossible because when $P = 0$ the demand of the second group is one.

V. Equivalent one-shot auctions for U-LSA and C-LSA

We now introduce a new One-Shot auction mechanism, which we will call OS-LSA, and which will turn out to be equivalent to C-LSA. The equivalent one shot version will then allow as to probe diffrent properties of C-LSA.

A. VCG and UCE

As was mentioned before, UCE is an ascending implementation of the VCG mechanism. VCG allocates blocks in a way that maximizes social welfare and charges each player i with the loss of declared welfare his presence incurs on others. If we take the example provided in Table II , when player 1 is absent SW=13 (each group obtains one block) and when he is present the social welfare without counting him is 12 (player one and two obtain two blocks) thus: $p_1^{\mathrm{VCG}} = 13 - 12 = 1$. The other VCG payment are $p_2^{\mathrm{VCG}} = 12 - 11 = 1$, $p_3^{\mathrm{VCG}} = 13 - 11 = 2$, and $p_4^{\mathrm{VCG}} = 17 - 17 = 0$. We obtain the same payments with UCE. In the following, we illustrate the equivalence between VCG and UCE. To implement a mechanism which is equivalent to VCG means (Fig. 5):

1) Achieving an efficient allocation. For UCE we reach an efficient allocation by reaching a competitive equilibrium (CE).
2) Preserving truthful telling. We preserve truthful telling for UCE by applying two activity rules presented in Section IV.
3) Charging each player his Vickrey payment. For UCE, we can compute Vickrey payment of each player after reaching an universal competitive equilibrium (UCE)

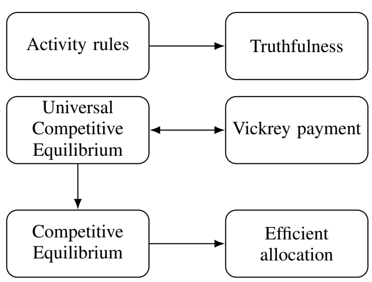

Fig. 5: Analogy between VCG and UCE

B. OS-LSA: a one-shot equivalent to C-LSA

Mirroring the relationship between one-shot and ascending auctions, we now introduce a new one-shot mechanism, OS-LSA. We first describe the mechanism's rules, and then show its equivalence with C-LSA.

1) Bidding, allocation and pricing rules: In OS-LSA, we require each base station, which belongs to one and only one group, to submit a bid vector b_i composed of non-increasing components, the n-th element $b_{i,n}$ ($n \geq 1$) representing the declared willingness-to-pay of base station i for an n-th block, if it already obtained $n-1$ blocks. After collecting bids, and for each group h (with set of players g_h), we sort the bids of its players for their n-th block in a non-increasing order. We then construct the group-bid vector $B_h = \{B_{h,1}.., B_{h,K}\}$ as follows:

$$B_{h,n} = \max_{i \in g_h} \left(b_{i,n} \mathrm{rank}(b_{i,n}) \right). \tag{9}$$

Here $B_{h,n}$ represents the bid of group h for its n-th block, which can be interpreted as the maximum amount that a subset of players of group h can agree to equitably share to obtain an n-th block.

Considering allocation, intuitively, since groups are independent (there is no player in common), given the group bids, in order to maximize social welfare, blocks should be allocated to the highest K bids among the $M \times K$ bids.

Finally, the payment rule is inspired by VCG. We first denote by C_h the (nondecreasing) vector of competing bids facing group h, i.e., C_h is composed by the highest K bids of all groups but group h, sorted in an ascendant order. From our allocation rule, the number of blocks that a group h wins is the number of bids in C_h it defeats. We propose that a group h obtaining j blocks pays $\sum_{n=1}^{j} C_{h,n}$. More specifically, we propose that for the n-th block obtained by group h, a subset of its players pays $C_{h,n}$ equitably, and we choose as set the largest subset of players in the group such that the price paid by each for each obtained block is not greater than his declared valuation for that block. Therefore, we define:

$$j_n = \max \left\{ \mathrm{rank}(b_{i,n}), i \in g_h \text{ and } \mathrm{rank}(b_{i,n}) b_{i,n} \geq C_{h,n} \right\};$$

where $\mathrm{rank}(b_{i,n})$ is the rank of $b_{i,n}$ among bids of players in group h (to which i belongs) for their n^{th} block. Our proposed payment rule is then that each player with rank below or equal to j_n pays $C_{h,n}/j_n$, players with rank above j_n will not get that block and will not pay for it.

Let us now run our allocation and pricing rules on a simple example, for illustration and clarification purposes.

Example 3. We take the same configuration as for example 2. Following (9), for computing groupbid vector for the first group,B_1, we have:

- $B_{1,1} = \max\{9 \times 1,\ 7 \times 2,\ 6 \times 3\} = 18$
- $B_{1,2} = \max\{7 \times 1,\ 7 \times 2,\ 3 \times 3\} = 14$
- $B_{1,3} = \max\{6 \times 1,\ 5 \times 2,\ 3 \times 3\} = 10$
- $B_{1,4} = \max\{5 \times 1,\ 2 \times 3\} = 6$

Thus, $B_1 = \{18, 14, 10, 6\}$. Similarly $B_2 = \{14, 12, 8, 4\}$ and $B_3 = \{13, 10, 3, 1\}$. Also, the sets of highest bids from competing groups (C_h) are, following our definition, $C_1 = \{10, 12, 13, 14\}$, $C_2 = \{10, 13, 14, 18\}$ and $C_3 = \{12, 14, 14, 18\}$. Ranking all groupbid in a descending order, we obtain that the first group is allocated two blocks, priced as follows.

- For the first block, the group pays 10. Since each player is willing to pay at least $\frac{10}{3}$ for his first block, then all players get that block and each one pays $\frac{10}{3}$ for it.
- For the second block of group 1, group one pays 12, player 3 will not pay 4 for this block, only player 1 and 2 get the block and each one pays 6.

The second group obtains one block and pays 10, with each player paying 5. Finally, the third group obtains a block and pays 12.

2) Equivalence with C-LSA: We establish in the following the equivalence between the one shot auction OS-LSA and the ascending auction C-LSA.

Proposition 2. *Any instance of C-LSA can be mapped from its history to an instance of OS-LSA that generates the same outcome via an appropriate mapping of demands into a bids vector. The opposite direction is also true i.e., any OS-LSA outcome can be obtained in C-LSA via an appropriate mapping of bids vector into demands.*

Proof. In fact, there is an equivalence between demands and bids: proposing a bid $b_{i,n}$ is equivalent to say that $d_i(b_{i,n}) =$

$n-1$ and $d_i(b_{i,n}-1) = n-1$. Let us fix a C-LSA sequence that starts from a round 0 and ends at round T, the demand of each player during the C-LSA auction can be mapped to a bids vector as follows: we build loosing bids from the history as explained before i.e., if $d_i(p_h) = n$ and $d_i(p_h+1) = n-1$ then we set $b_{i,n} = p_h + 1$. For winning bids, please note that it is not always possible to build them from history as before, since an auction might finish with a non zero demand. However, we can fix each uncompleted component of the bids vector of each player from his payment, if a player gets n block and pays p_h for his n^{th} block then we fix each winning bids $b_{i,n}$ such that $b_{i,n}$ is higher than p_h. Please note that this way an equivalent mechanism is obtained since prices are computed only based on losing bids.

The opposite direction mapping (from OS-LSA to C-LSA) is also intuitive: player i just has to act in C-LSA with respect to the proposed bid vector in OS-LSA as his valuations i.e., report $d_i(p_h)$ in accordance to the reported OS-LSA bid b_i. □

In the following proposition, we provide the connection between C-LSA and OS-LSA in terms of truthfulness.

Proposition 3. *If sincere bidding in OS-LSA is a dominant strategy (each player can maximize his utility by proposing $b_i = v_i$ independently of bids of other players) then sincere bidding in the C-LSA ascending auction is an ex-post Nash Equilibrium.*

Proof. Let us fix a player i when C-LSA is applied, suppose that all other players act truthfully (demands are derived from valuations), suppose that by bidding truthfully player i obtains a utility u_1 and by using any other strategy he obtains a utility u_2, we have to show that $u_1 \geq u_2$. From Proposition 2, player i can obtain that same utility u_1 by bidding truthfully in OS-LSA and can obtain u_2 by proposing other bids vector in OS-LSA. Since bidding truthfully is a dominant strategy in OS-LSA, u_1 must be higher than u_2. □

3) Truthfulness of C-LSA and OS-LSA: We prove that the proposed mechanism is truthful. To establish that result, we will establish two intermediate lemmas.

Lemma 1. *Payments for blocks can only increase i.e., if player i pays p_n for his n-th block then he pays $p_{n+1} \geq p_n$ for his $(n+1)$-th block.*

Proof. Consider a player who obtains an n-th block and is charged a price p_n for it. We denote by q_n the number of players in the same group, who are willing to pay at least p_n to obtain an n-th block (we therefore have $q_n p_n = C_{h,n}$). Since $C_{h,n+1} \geq C_{h,n}$ and bids decrease with the number of obtained blocks, we have $q_{n+1} \leq q_n$. Thus $p_{n+1} = \frac{C_{h,n+1}}{q_{n+1}} \geq p_n = \frac{C_{h,n}}{q_n}$. □

In the following, we call a component $b_{i,n}$ of a bid vector b_i of a player i a *winning component* if $b_{i,n} \geq p_n$ and a *losing component* if $b_{i,n} < p_n$.

Lemma 2. *If a player gets n blocks, then his winning components of his bid are exactly his first n components.*

Proof. Assume that there is a situation in which $b_{i,n}$ is a losing bid and $b_{i,n+1}$ is a winning bid: $b_{i,n}$ being a losing bid means that $b_{i,n} < p_n$ with p_n the price paid by the other bidders in the group for that block, and $b_{i,n+1}$ being a winning bid means that $b_{i,n+1} \geq p_{n+1}$, hence from the Lemma 1 $b_{i,n+1} \geq p_n$ and then $b_{i,n+1} > b_{i,n}$, a contradiction □

In the following proposition, we prove that truthful bidding is a dominant strategy for OS-LSA.

Proposition 4. *Truthful bidding in the proposed mechanism is a dominant strategy.*

Proof. Suppose that by bidding truthfully, player i (who belongs to group h) gets n blocks. For his first n bids, player i cannot do better than proposing his true valuations: lowering the corresponding bids could make him lose blocks that are charged below his valuation for them, and increasing those bids would have no impact because the payments are independent of his bids. If player i wants an $(n+1)-$ th block then he has to propose a bid $b_{i,n+1} \geq v_{i,n+1}$ such that $B_{h,n+1} \geq C_{h,n+1}$, however this leads to a lower utility: because $v_{i,n+1}\mathrm{rank}(v_{i,n+1}) \leq C_{h,n+1}$, where $\mathrm{rank}(v_{i,n+1})$ is just $\mathrm{rank}(b_{i,n+1})$ when $b_{i,n+1} = v_{i,n+1}$, and by proposing $b_{i,n+1} \geq v_{i,n+1}$ (which ensures $B_{h,n+1} \geq C_{h,n+1}$), player i pays at least $\frac{C_{h,n+1}}{\mathrm{rank}(v_{i,n+1})}$ which is higher than $v_{i,n+1}$. □

4) Revenue guarantee of C-LSA and OS-LSA: In the following, we establish some revenue guarantee for OS-LSA. We have $B_1, .., B_M$ group bids vector. Each element is composed by K components. In total we have KM components. We sort all those components in a non increasing order to form a vector bids R of size KM, $R = \{R_1, .., R_{KM}\}$. Let us first introduce the following lemma.

Lemma 3. *if a group h wins n blocks then he pays the first n bids in $\{R_{K+1}, ..., R_{2k}\}$ proposed by other groups.*

Proof. C_h is composed by the highest K bids of other groups which are a set in $\{R_1, ..., R_{2k}\}$. $\{C_h(n+1), .., C_h(K)\}$ are in $\{B_1 .., B_K\}$ because they are a winning bids (group h can not defeat those bids). The first n components of C_h (which group h will pay) are in $\{R_{K+1}, ..., R_{2k}\}$ because they are defeated bids and by definition since C_h is composed by the highest K bids of other groups, the first n components of C_h must be the first n bids in $\{R_{K+1}, ..., R_{2k}\}$ proposed by other groups. □

In the following proposition, we establish some revenue guarantee for OS-LSA and C-LSA.

Proposition 5. *The revenue of OS-LSA is in* $[\sum_{i=K+1}^{2K} R_i, K \times R_{K+1}]$

Proof. Clearly for each group, the facing vector can not have a component lower than R_{2K} hence the revenue from each block is higher than R_{2K}, also the first K components $\{R_1, ..., R_k\}$ are the winning bids thus the maximum revenue from each block is R_{K+1}.

- Upper bound: The best case in terms of revenue is when all blocks are allocated to K different groups (here we suppose that M higher than K) and R_{K+1} is a bid from another group who does not get any block, in this situation the revenue is $K \times R_{K+1}$

• Lower bound: Suppose that the revenue could be lower than $\sum_{i=K+1}^{2K} R_i$, this means that it exist at least R_j which will be not paid and at least a component B_i lower than R_j which will be paid at least twice by a group h and another group h'. We can distinguish two cases:

1) R_j is not a bid of group h: in this situation and using lemma 3, since group h pays R_i then he must pay R_j.
2) R_j is a bid of group h': in this situation and using lemma 3, since group h' pays R_i then he must pay R_j.

Thus, R_j must be paid by some group. Hence the revenue can not be lower than $\sum_{i=K+1}^{2K} R_i$ □

VI. Results and Analysis

In the following we compare UCE and C-LSA mechanisms in terms of social welfare and revenue by comparing OS-LSA and VCG, their equivalent one-shot versions that are easier to simulate. We denote by:

- Rev_1: The average revenue of VCG. For each instance, the VCG revenue is defined as

$$\sum_{h=1}^{M}\sum_{i=1}^{|g_h|}\sum_{j=1}^{n_h}[C_{h,j}^{\mathrm{VCG}} - (B_{h,j}^{\mathrm{VCG}} - b_{i,j})]^+$$

where n_h is the number of blocks obtained by g_h, $B_{h,j}^{\mathrm{VCG}} = \sum_{i=1}^{|g_h|} b_{i,j}$ and C_h^{VCG} is the vector of competing bids facing group h when VCG is applied.
- Rev_2: The average revenue of OS-LSA. The revenue of OS-LSA is defined as:

$$\sum_{h=1}^{M}\sum_{j=1}^{n_h} C_{h,j} \tag{10}$$

- SW_N: The average normalized social welfare, the normalized social welfare is defined as $\frac{\text{social welfare of OS}-\text{LSA}}{\text{social welfare of VCG}}$

A. Simulation settings

For our simulation we go through the following steps:

1) Fix the number of blocks K and the number of groups M.
2) The number of players is chosen randomly from the discrete uniform distribution $[1\ ;\ 30]$
3) For each player i we create the bid vector b_i which is composed by K elements: the first bid is drawn from the uniform distribution over the interval $[0, 100]$ and the n-th element $(n > 1)$ is drawn from the uniform distribution $[0, b_{i,n-1}]$

For each number of blocks and number of groups, the average revenue and social welfare are computed over 10 000 draws. A draw means that we generate the number of player for each group then we generates vector bid of each player.

K	1	2	4	8	50	100
$\mathrm{SW_N}$	0.86	0.878	0.91	0.939	0.992	0.9997
Rev_1	179.5	220.34	230.42	263.16	74.9	3.82
Rev_2	726.75	1275.3	1970.93	2448.5	580.16	26.84

TABLE IV: Average normalized social welfare as a function of the number of blocks for $M = 10$

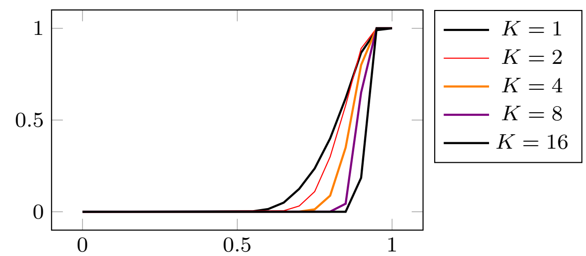

Fig. 6: Cumulative density function of $\mathrm{SW_N}$ as a function of the number of blocks for $M = 10$

M	2	5	10	20	40
$\mathrm{SW_N}$	0.944	0.885	0.86	0.847	0.839
Rev_1	30.29	105.2	187.31	280.8	372.72
Rev_1	308.87	817.23	724.96	821.63	898.9

TABLE V: Average normalized social welfare as a function of the number of groups for $K = 1$

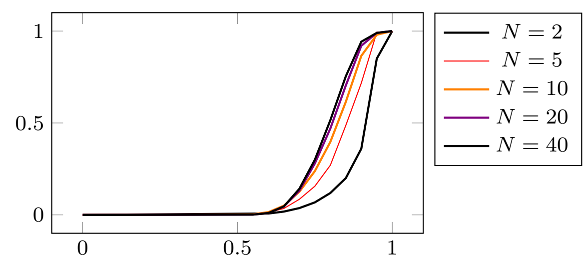

Fig. 7: Cumulative density function of $\mathrm{SW_N}$ as a function of the number of groups for $K = 1$

B. Simulation results

In terms of Social welfare, Table IV shows that the clinching approach converges to the optimal social welfare as we keep increasing the number of blocks which can be justified as follows: we can see from Fig 1 that if we keep moving on to the right side by adding blocks, the value of an extra block is very small. i.e., adding a block has a very low impact on social welfare. In terms of revenue, we can see that the revenue achieves its maximum for a value of $K \in [8, 16]$. However, if we keep increasing K, the revenue converges to zero, this is can be explained from Proposition 5, as we increase the number of blocks B_K converges to zero, which justifies the low revenue.

VII. Conclusion

In this paper, we have adapted UCE auction to the LSA context, since the allocatilon my be an NP-hard problem, we have designed another ascending mechanism, based on the clinching approach, with a simple allocation. We have proposed its equivalent one-shot auction and we have derived some revenue guarantee. We have proven that truthfull telling is a dominant strategy for OS-LSA and an ex-post Nash Equilibrium for C-LSA.We have easily evaluate through simulations the performance of the proposed mechanism with respect to UCE in terms of social welfare and revenue, thanks to their one-shot equivalent mechanisms. Simlation results suggests that the revenue generated by the proposed mechanism is much larger than the UCE revenue with a multiplicative factor between 5 and 10 for a very limited cost in terms of resource usage efficiency (around 4%). Hence we think our proposed mechanism C-LSA is a viable candidate for LSA spectrum allocation and pricing.

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