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Experimental prediction of the vibration response of panels under a turbulent boundary layer excitation from sensitivity functions

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Abstract

This study aims at validating an experimental method for characterizing the vibration behavior of panels excited by a turbulent boundary layer excitation as a possible alternative to standard means like wind-tunnels or *in situ* tests. The approach takes advantage of an explicit separation of the excitation contribution from the dynamic behavior of the panel. Based on the measurement of deterministic transfer functions on the panel, called ‘sensitivity functions’, which are then combined with either measurements or a model of the wall-pressure fluctuations induced by the turbulent boundary layer excitation, the vibration response under such an excitation can be retrieved. For validation purposes, the wall-pressure fluctuations of the turbulent flow generated in an anechoic wind tunnel are measured with a flush-mounted microphone array. The decay rates and the convection velocity which mainly characterize the excitation are extracted from these measurements. The plate velocity response to this excitation is estimated following the proposed method using the measured sensitivity functions and the model of Mellen fed with experimentally estimated decay rates and convection velocity. A comparison between a directly measured vibration autospectrum under the actual flow and the one predicted following the suggested method shows satisfactory agreement.

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1 I. INTRODUCTION

2 The experimental characterization of panels vibration under a turbulent boundary layer
3 (TBL) excitation is of great interest for the transport industry and for researchers studying
4 flow-induced vibration topics. Panel-like structures are mainly tested in wind tunnels or via
5 *in situ* measurements. These experimental methods are hard to control, costly and subjected
6 to variability between different laboratories and/or measuring techniques. Over the past
7 decades, studies have been carried out to experimentally synthesize the pressure field induced
8 by a TBL on a panel surface using an array of acoustic sources [1–3] and ultimately providing
9 improved methods to characterize the vibration behavior of a panel under this excitation.
10 However, these synthesis methods require a large number of sources (approximately 4 per
11 smallest wavelength) to reproduce the small correlation lengths of the surface pressure field
12 induced by the TBL, especially for subsonic velocities of the convected TBL. As frequency
13 increases, the number of reproduction sources thus becomes very large and then prohibitive.
14 A synthesis of the TBL excitation focussed on a subdomain of the simulation surface [4]
15 helps reaching higher frequencies while ensuring correct reproduction of the TBL excitation,
16 but limits the observation area to a fraction of the actual panel. Also, some of the proposed
17 methods [5, 6] are not able to accurately reproduce the TBL-induced wall-pressure field
18 outside the acoustic wavenumber domain, where the most energetic components of a subsonic
19 TBL are yet located and should be taken into account.

20 In this context, this study investigates an alternative approach to experimentally predict
21 the vibration response of panels under a TBL excitation by separating the contributions of
22 the forcing wall-pressure excitation from the vibration behavior of the panel. Indeed, the
23 mathematical formulation in the wavenumber domain of a panel vibration response when
24 submitted to random excitations allows estimating the system response at any point on
25 the structure from wall-pressure cross-spectral density (CSD) functions (characterizing the
26 excitation) and from so-called ‘sensitivity functions’. The latter are defined as the panel
27 vibration response to wall-pressure acoustic plane waves and characterize the intrinsic vi-
28 bration behavior of the panel. Since the contributions of the excitation and those from the
29 structural behavior are separated, the method can be fed with numerical and/or experimen-
30 tal data either for the excitation or for the sensitivity functions. This allows performing fast
31 parametric studies by changing the properties of the panel or those of the excitation.

32 A method [7] has recently been proposed for estimating the sensitivity functions exper-
33 imentally without having to excite the panel by sets of surface plane waves which is hard
34 to realize in practice. This alternative approach is based on the reciprocity principle, which
35 states that the sensitivity functions at any point on the structure are equivalent to the panel
36 velocity response expressed in the wavenumber domain when the system is excited by a nor-
37 mal effort at the point of interest. Following this, the experimental process for estimating the
38 sensitivity functions consists in exciting the panel with a transverse force at the point where
39 the panel vibration response is to be determined. The spatial vibratory response of the panel
40 to this force is measured with a scanning laser vibrometer. In a subsequent post-processing
41 phase, a discrete 2-D wavenumber transform of the measured vibratory field normalized
42 by the input force is performed to deduce the sensitivity functions. Finally, by combining
43 the wall-pressure CSD function of the considered random excitation and the previously es-
44 timated sensitivity functions, the response of the panel excited by the random excitation
45 can be deduced at the point of interest. This approach has been successfully applied in the
46 case of a panel excited by a diffuse acoustic field (DAF) [7]. In this particular case, the
47 wall-pressure field (WPF) is described in the wavenumber domain by components restricted
48 to the acoustic domain and the sensitivity functions of the panel have to be evaluated only
49 for wavenumbers of magnitude smaller than the acoustic wavenumber.

50 In principle, the method proposed in [7] for the case of a diffuse acoustic field excitation
51 can be applied for a panel excited by a spatially homogeneous stationary turbulent boundary
52 layer. However, from the authors' knowledge, this has never been assessed experimentally.
53 The work presented in this paper consists in applying and experimentally validating this
54 approach for such an excitation by comparison with direct vibration measurements in an
55 anechoic wind tunnel. This constitutes the main novelty of the paper. Even if the study
56 is limited to the vibration response of a panel to a turbulent boundary layer excitation,
57 its acoustic response (radiated pressure, acoustic intensity) could be further obtained by
58 exciting the panel with a monopole and a dipole source, as pointed out in [7]. In contrast
59 with the DAF, the WPF of a subsonic TBL excitation exhibits components located outside
60 the acoustic domain. The results of the method would therefore be sensitive to the accuracy
61 of the measured sensitivity functions for wavenumbers larger than the acoustic wavenumber.
62 A particular attention is therefore paid to the estimation of these sensitivity functions for a
63 set of wavenumbers adapted for dealing with TBL excitations. Moreover, unlike the DAF

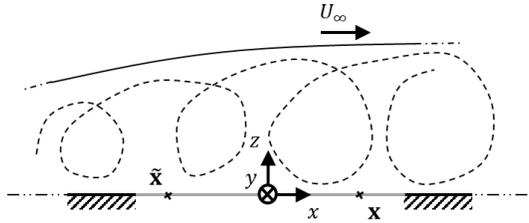
64 for which the theoretical CSD functions are well defined [8], various models of the TBL
65 excitation [9, 10] exist but none of them is able to accurately describe the WPF induced by
66 a TBL excitation on a large wavenumber range. These models are mostly semi-empirical and
67 can thus be adjusted through parameters like decay rates and convective wavenumber. As
68 there is no clear consensus in the literature regarding universal values for these parameters
69 and/or the model to be used, the WPF of the excitation considered in this study has been
70 measured and used to fit the model of Mellen [11].

71 The paper is organized as follows: the mathematical formulation of the vibration problem
72 is presented in Sec. II A, and the sensitivity functions involved in the problem are defined in
73 Sec. II B based on the reciprocity principle. The proposed methodology for characterizing
74 the panel vibration response under a TBL is summarized in Sec. II C. The characterization
75 of the WPF under the experimental TBL excitation is presented in Sec. III. In Sec. IV the
76 procedure is validated for an aluminum panel excited by a turbulent flow, based on measured
77 sensitivity functions and an adjusted model of the WPF. The measured sensitivity functions
78 are presented in Sec. IV B. Finally, the complete method is implemented to predict the
79 vibration response of the panel to the experimental TBL, which are compared to direct
80 measurements performed in an anechoic wind tunnel in Sec. IV C.

81 **II. PROPOSED METHOD FOR ESTIMATING THE VIBRATION RESPONSE** 82 **OF PANELS UNDER A TURBULENT BOUNDARY LAYER EXCITATIONS**

83 Let us consider a baffled panel of surface Σ_p with arbitrary boundary conditions. As illus-
84 trated in Fig. 1, a fully developed TBL with a flow velocity U_∞ outside the boundary layer
85 is supposed to excite the panel on one of its sides. This excitation is considered stationary
86 in time and spatially homogeneous. The plate and the boundary layer are supposed to be
87 weakly coupled; in other words, the vibration of the plate does not interfere with the WPF
88 and the forcing term is thus not modified by the panel vibration response. This assumption
89 is generally admitted when the panel displacements are much smaller than the characteristic
90 length scales of the flow [12]. The TBL excitation is then characterized by the WPF (*i.e.*,
91 lateral force) induced on a smooth rigid surface. We also define $\mathbf{x} = (x, y)$ an observation
92 point and $\tilde{\mathbf{x}} = (\tilde{x}, \tilde{y})$ an excitation point (where the surface pressure fluctuation induced by
93 the TBL is prescribed). Both points are defined in a Cartesian coordinate system (x, y, z)

94 with the origin at the center of the panel, as shown in Fig. 1, and are located on the panel
 95 surface $z = 0$.



96
 FIG. 1. Illustration of a baffled panel (gray line) excited by a TBL and coordinate system.

97 To characterize the vibration response of the panel under this excitation, the one-sided
 98 normal vibration velocity spectrum $v(\mathbf{x}, f)$ at point \mathbf{x} is considered, where f is the frequency
 99 and is considered positive. As the excitation is random, this quantity is derived from the
 100 normal velocity auto-spectral density (ASD) function $G_{vv}(\mathbf{x}, f)$. An approach for evaluating
 101 this quantity based on deterministic transfer functions and making use of the reciprocity
 102 principle has been thoroughly presented in [7] and is briefly summarized in Secs. II A and
 103 II B.

104 A. Mathematical formulation of the vibration response

105 The one-sided frequency ASD function of the velocity $G_{vv}(\mathbf{x}, f)$ at point \mathbf{x} can be ex-
 106 pressed as the following inverse space-wavenumber Fourier transform:

$$G_{vv}(\mathbf{x}, f) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} |H_v(\mathbf{x}, \mathbf{k}, f)|^2 G_{p_b p_b}(\mathbf{k}, f) d\mathbf{k}, \quad (1)$$

107 where

$$H_v(\mathbf{x}, \mathbf{k}, f) = \iint_{\Sigma_p} H_{v/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, f) e^{-j\mathbf{k}\tilde{\mathbf{x}}} d\tilde{\mathbf{x}}, \quad (2)$$

108 where $\mathbf{k} = (k_x, k_y)$ is the wavevector defined in the plane (x, y) . The function $G_{p_b p_b}(\mathbf{k}, f)$
 109 corresponds to the CSD function of the WPF on the excitation side (for instance a TBL
 110 excitation). The $H_v(\mathbf{x}, \mathbf{k}, f)$ function is called the sensitivity function [13] and characterize
 111 the vibration behavior of the panel. The term $H_{v/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, f)$ corresponds to the transfer
 112 function between the panel velocity v at point \mathbf{x} and a normal point force F_n applied at
 113 point $\tilde{\mathbf{x}}$. According to Eq. (2), the sensitivity function can be interpreted as the vibration

114 response of the panel at point \mathbf{x} due to a plane wave excitation with a wavevector $-\mathbf{k}$ (as
 115 illustrated in Fig. 2(a)).

116 Eq. (1) shows that the panel has a filtering effect on the excitation in the wavenumber
 117 space [12], which somehow limits the integration to a finite wavenumber domain $\Omega_{\mathbf{k}}$ while
 118 ensuring a correct estimation of the integral (see Sec. IV A). This filtering effect allows
 119 approximating the integral in Eq. (1) by performing a sum over an appropriately defined
 120 finite set of wavevectors $\mathbf{k} \in \Omega_{\mathbf{k}}$ (using the rectangular integration rule). The one-sided
 121 frequency ASD function of the velocity at point \mathbf{x} is thereby estimated with

$$G_{vv}(\mathbf{x}, f) \approx \frac{1}{4\pi^2} \sum_{\mathbf{k} \in \Omega_{\mathbf{k}}} |H_v(\mathbf{x}, \mathbf{k}, f)|^2 G_{p_b p_b}(\mathbf{k}, f) \delta \mathbf{k}, \quad (3)$$

122 where $\delta \mathbf{k}$ represents the wavenumber resolution. To evaluate this quantity, the sensitivity
 123 functions H_v for wavenumbers belonging to $\Omega_{\mathbf{k}}$ have to be determined.

124 B. Sensitivity functions based on the reciprocity principle

125 In its most general form, the reciprocity principle states that the response of a system
 126 is invariant with respect to the exchange of excitation and observation points [14]. For the
 127 particular case of a normal force applied at point $\tilde{\mathbf{x}}$ and normal velocity observed at point
 128 \mathbf{x} , the reciprocity relationship can be translated following the previous notations into [15]

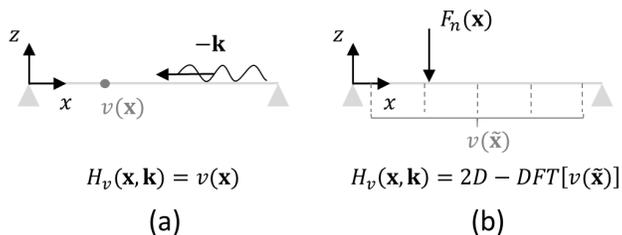
$$H_{v/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, f) = H_{v/F_n}(\tilde{\mathbf{x}}, \mathbf{x}, f), \quad (4)$$

129 Introducing Eq. (4) in Eq. (2) one obtains

$$H_v(\mathbf{x}, \mathbf{k}, f) = \iint_{\Sigma_p} H_{v/F_n}(\tilde{\mathbf{x}}, \mathbf{x}, f) e^{-j\mathbf{k}\tilde{\mathbf{x}}} d\tilde{\mathbf{x}}. \quad (5)$$

130 The right hand side of Eq. (5) can be interpreted as the space-wavenumber transform of
 131 $H_{v/F_n}(\tilde{\mathbf{x}}, \mathbf{x}, f)$ with respect to the space variable $\tilde{\mathbf{x}}$. The points $\tilde{\mathbf{x}}$ become observation points
 132 on the panel surface Σ_p , which means that the space-wavenumber transform is performed
 133 over the vibration velocity field of the panel. To sum up, the sensitivity function $H_v(\mathbf{x}, \mathbf{k}, f)$
 134 may be obtained by exciting the panel with a normal effort F_n at point \mathbf{x} and by calculating
 135 the space-wavenumber transform of the transfer function between the panel velocity at the

136 observation points and the applied effort (as illustrated in Fig. 2(b)).



137

138 FIG. 2. Determination of the sensitivity functions H_v : (a) based on the direct interpretation, (b)
139 using the reciprocity principle.

140 In practice, the vibration field has to be measured on a regular grid of points denoted
141 $\Gamma_{\tilde{\mathbf{x}}}$, using a scanning laser vibrometer for example. The space-wavenumber transform is
142 therefore approximated by a 2D discrete Fourier transform (2D-DFT). In order to avoid
143 aliasing effects, the spatial resolution $\delta\tilde{\mathbf{x}}$ over $\Gamma_{\tilde{\mathbf{x}}}$ should be determined so that the spatial
144 variations of the vibration field can be correctly represented by the grid of points. For a
145 homogeneous isotropic thin panel, $\delta\tilde{\mathbf{x}}$ should be less than or equal to a quarter of the natural
146 flexural wavelength of the panel λ_f at the highest frequency of interest (as considered in
147 [7]). According to [16], having four points per smallest flexural wavelength allows a non-
148 biased estimation of the vibration field up to a frequency corresponding to two times this
149 flexural wavenumber. It ensures correct estimation of the sensitivity functions close the
150 flexural wavenumber (where vibratory levels are the largest) and avoids potential aliasing
151 effects. For a more complex panel, a preliminary study should be carried out to define this
152 parameter (for instance, by using a numerical model of the panel or by using a trial and
153 error procedure).

154 C. Description of the proposed methodology

155 A methodology for experimentally estimating the vibration response of a panel excited by
156 a TBL is derived from Eq. (3) and the sensitivity functions determined using the previously
157 described reciprocity principle. The methodology for evaluating the velocity ASD function
158 G_{vv} at a given point \mathbf{x} of the panel ($z = 0$) can be summarized as follows:

- 159 - Excite the panel with a normal mechanical force at point \mathbf{x} and measure the normal
160 velocity response of the panel at points $\tilde{\mathbf{x}} \in \Gamma_{\tilde{\mathbf{x}}}$ to determine $H_{v/F_n}(\tilde{\mathbf{x}}, \mathbf{x}, f)$,

- 161 - Perform a 2D-DFT of the panel velocity response $H_{v/F_n}(\tilde{\mathbf{x}}, \mathbf{x}, f)$ (with respect to $\tilde{\mathbf{x}}$)
162 to obtain the sensitivity functions $H_v(\mathbf{x}, \mathbf{k}, f)$ at point \mathbf{x} for $\mathbf{k} \in \Omega_{\mathbf{k}}$,
- 163 - Use Eq. (3) and an estimation of the CSD functions of the wall-pressure fluctuations
164 $G_{p_b p_b}(\mathbf{k}, f)$ to estimate the velocity ASD function G_{vv} at point \mathbf{x} under the considered
165 TBL excitation.

166 In the following, the vibration behavior of a thin isotropic homogeneous plate will be
167 investigated. The proposed method is however valid for any panel having a linear mechanical
168 behavior and, isotropic and homogeneous conditions are thus not mandatory.

169 III. CHARACTERIZATION OF THE EXCITATION

170 In addition to the knowledge of the vibration behavior of the panel through the sensitivity
171 functions, solving Eq. (3) requires that CSD functions of the blocked wall-pressure of the
172 excitation are known. Over the past few years, numerous studies have shown that the
173 coherent power of the wall-pressure fluctuations induced by a TBL decays exponentially
174 with the increasing separation distances along flow and transverse directions [17]. It has
175 also been shown that the phase of the cross-spectrum is directly related to the convection
176 wavenumber $k_c = \omega/U_c$, where ω is the angular frequency and U_c is the convection speed
177 (usually defined as a constant fraction of the free flow velocity U_∞). These dependencies are
178 included in most of the semi-empirical models [9, 10] aiming at predicting the CSD functions
179 of the wall-pressure fluctuations.

180 In order to validate the proposed methodology in comparison to actual measurements
181 of the panel response in an anechoic wind tunnel, the wall-pressure fluctuations induced
182 by a subsonic turbulent flow generated in a low-speed anechoic wind tunnel have been
183 measured on the considered frequency range in this study ([170, 2000 Hz], see Sec. IV).
184 These measurements will then be used to fit the model of Mellen in Sec. III B.

185 A. Spiral-shaped surface microphone array

186 The wall-pressure fluctuations have been measured at two flow velocities: $U_\infty = 20 \text{ m.s}^{-1}$
187 and $U_\infty = 40 \text{ m.s}^{-1}$ using the spiral-shaped rotating microphone array introduced by Robin
188 *et al.* [18].

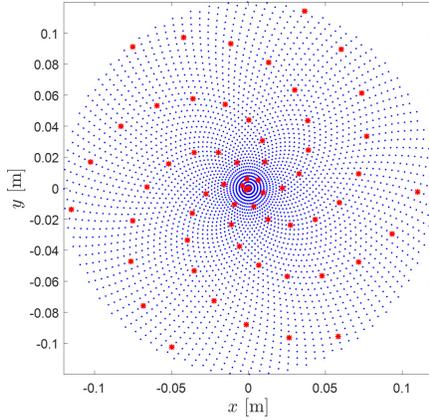
189 The array is composed of 61 microphones of three different types in order to reach the
190 desired microphone density and to tackle congestion issues at the center of the array. A
191 deported B&K 4182 probe and three flush-mounted Knowles FG-23629-P16 microphones
192 are used at the center of the array (as shown in Fig. 3(b)). The remaining part of the array
193 consists of 57 quarter inch pinhole-mounted B&K 4957 microphones (for further details refer
194 to [18]).

195 The pattern over which the 61 microphones are positioned (red markers in Fig. 3(a))
196 has been designed so that each microphone has a different radial position with a radial
197 separation of 2 mm. Measurements following 180 consecutive rotations allow reaching an
198 angular resolution of 2° and thereby reconstructing a high density microphone array at a
199 post-processing step (as illustrated in Fig. 3(a)). Under the assumption of a stationary
200 and homogeneous turbulent WPF, the translation or rotation of an array with at least one
201 reference sensor has proven to be a solution to obtain the needed compromise between a
202 small sensor diameter (to avoid spatial averaging) and small sensor spacing (to gain high
203 spatial resolution) [18] [19]. Since the central microphone position is the only invariant one,
204 it is used as a reference in the calculations of wall-pressure CSD functions.

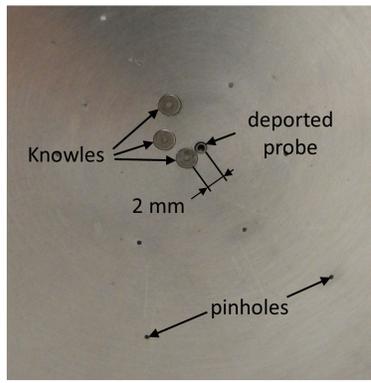
205 An amplitude calibration of the microphone array was performed by placing a Larson
206 Davis CAL200 calibrator over each microphone separately and by exciting it with a sine wave
207 of 94 dB SPL at 1000 Hz. Since the B&K 4182 probe is deported from the microphone array
208 surface by approximately 4 cm, an additional phase calibration was necessary. To do so, the
209 calibrator was positioned over the three microphones at the center of the array (the B&K
210 4182 probe and the two closest Knowles microphones). A simultaneously triggered acoustic
211 pressure signal at 1000 Hz was extracted for each microphone. The time offset between the
212 signal measured by the deported probe and the signal measured by the two adjacent Knowles
213 microphones (both signals were identical) was compensated in all resulting measurements.
214 Once the signals are expressed in the frequency domain, as a time offset has been applied,
215 the phase calibration was effective regardless of the frequency.

216 **B. Measurement of the wall-pressure fluctuations and adjusted model of Mellen**

217 The considered TBL-like excitation is reproduced in a low-speed anechoic wind tunnel.
218 The installation consists in a closed-loop wind tunnel powered by two rotating fans. The



(a)



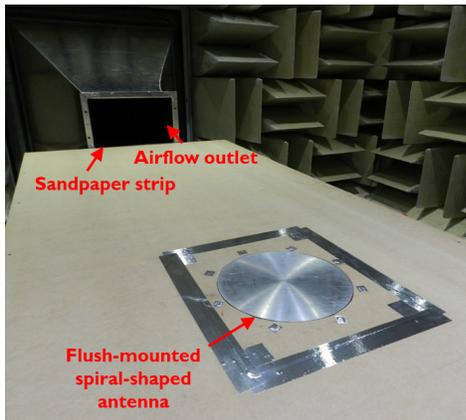
(b)

FIG. 3. (Color online) (a) Microphone positions (red markers) and illustration of a reconstructed grid (blue dots). (b) Close-up view of the mounted sensors and pinholes on measurement side.

219 air flow is directed to the anechoic chamber through a vent pipe containing a screen to
 220 homogenize the flow. A convergent is mounted at the end of the vent pipe inside the
 221 anechoic chamber which allows reaching higher flow velocities (up to $\text{Mach} \leq 0.12$). An
 222 $1.22 \times 2.44 \text{ m}^2$ plywood panel of 0.019 m thickness was mounted in the anechoic wind tunnel
 223 at the end of the convergent and to help the TBL develop, a sandpaper strip was glued
 224 at the end of the convergent (as shown in Fig. 4). The spiral-shaped array was flush-mounted
 225 1.8 m away from the convergent and 30 seconds acquisitions were performed with a sampling
 226 frequency of 8192 Hz. Time signals of the wall-pressure fluctuations were extracted for all
 227 microphones and all 180 rotations.

228 The reconstructed microphone measurement grid theoretically allows reaching a maxi-
 229 mum wavenumber of 1570 m^{-1} , which is sufficient to capture the convective contributions
 230 over the whole frequency range and for both considered flow velocities (at 2000 Hz and for

231 $U_\infty = 20 \text{ m.s}^{-1}$, the convective wavenumber k_c equals 838 m^{-1} with $U_c = 0.75 \times U_\infty$).



232

233 FIG. 4. (Color online) Measurement of the wall-pressure fluctuations using the spiral-shaped
234 surface microphone array flush-mounted in the wind tunnel.

235 The spatial CSD functions of the wall-pressure fluctuations $G_{p_b p_b}(\xi_x, \xi_y, f)$ were estimated
236 as a function of the spatial separations (ξ_x, ξ_y) in both x and y directions using the “cpsd”
237 MATLAB command with a fixed reference point at the center of the array ($x = 0, y = 0$).
238 For the remainder of this paper, the “cpsd” MATLAB command was defined with a Hanning
239 window applied to time signals and with a 50% overlap. Two approaches were investigated
240 to post-process the spatial CSD functions. The first consists in performing a 2D-DFT on
241 $G_{p_b p_b}(\xi_x, \xi_y, f)$ in order to directly estimate the blocked wall-pressure CSD functions in the
242 wavenumber domain [19]. However, the finite dimensions of the microphone array result
243 in unrealistic predictions in the low-wavenumber domain. As the low-wavenumber domain
244 mainly dictates the response of the panel to a TBL excitation (see Sec. IV A), it has to be
245 accurately estimated. Deconvolution methods can be used to compensate for this windowing
246 effect, but they require significant and prohibitive computation time [20].

247 With this in mind and for computation time to be reasonable, the measurements were
248 fitted to the model of Mellen [11] in the spatial domain

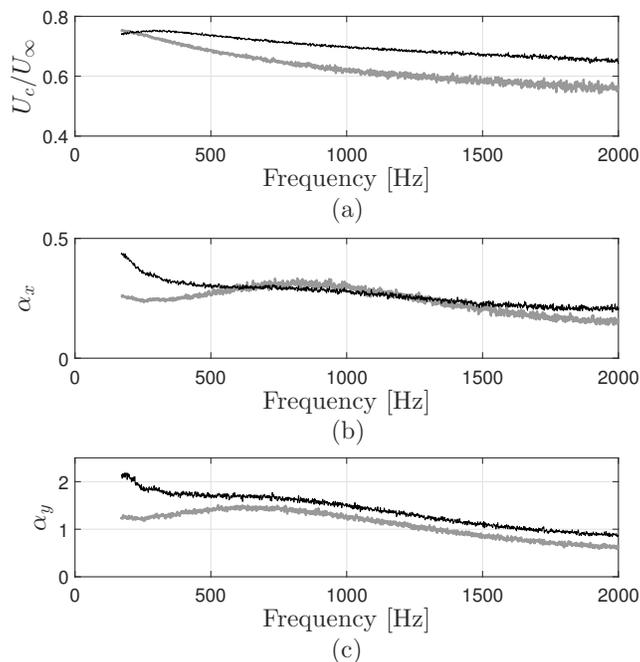
$$G_{p_b p_b}(\xi_x, \xi_y, f) = G_{p_b p_b}(f) e^{-\sqrt{(\alpha_x k_c \xi_x)^2 + (\alpha_y k_c \xi_y)^2}} e^{j k_c \xi_x}, \quad (6)$$

249 where $G_{p_b p_b}(f)$ is the measured blocked wall-pressure ASD function, (α_x, α_y) are the expo-
250 nential decay rates along x and y directions and k_c is the convective wavenumber. Performing
251 a space-wavenumber transform of Eq. (6) yields an expression of blocked wall-pressure CSD
252 functions in the wavenumber domain [11]

$$G_{p_b p_b}(\mathbf{k}, f) = G_{p_b p_b}(f) \frac{2\pi (\alpha_x \alpha_y k_c^2)^2}{[(\alpha_x \alpha_y k_c^2)^2 + (\alpha_x k_c k_y)^2 + (\alpha_y k_c)^2 (k_c - k_x)^2]^{3/2}}. \quad (7)$$

253 The model of Mellen has been chosen because, like the Corcos model, it can easily be
 254 adjusted by estimating α_x , α_y and k_c which is directly related to the convection speed U_c .
 255 Also, the convective peak in the model of Mellen expressed in the wavenumber domain has
 256 an oval shape which is in better accordance with the measurements, as opposed to the model
 257 of Corcos which has a diamond-like shape [9].

258 In order to estimate the decay rates (α_x, α_y) and convective wavenumber k_c , the consid-
 259 ered model has been fitted to measurements by solving Eq. (6) following the least squares
 260 method using the “lsqcurvefit” MATLAB command. First the decay rates have been esti-
 261 mated by solving Eq. (6) while considering the modulus of all the terms in this equation.
 262 In a second step, Eq. (6) has been solved by implementing the previously determined de-
 263 cay rates and while considering all terms as complex values to extract U_c . The extracted
 264 parameters are presented in Fig. 5 for the two considered flow velocities.



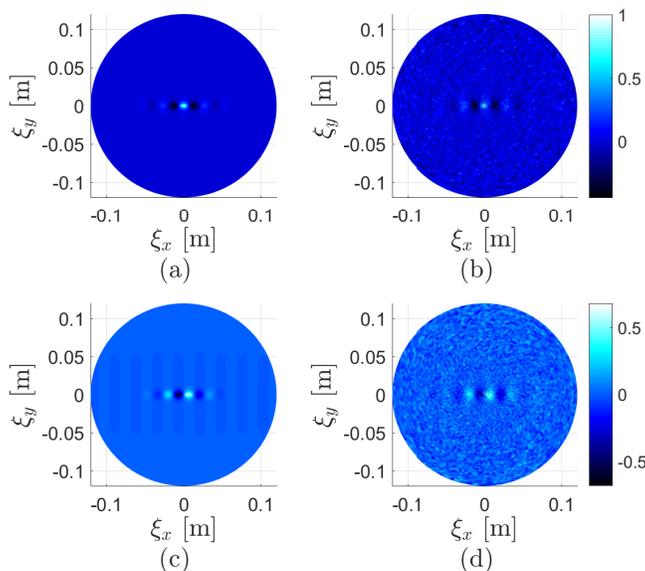
265

266 FIG. 5. TBL parameters extracted from measurements at $U_\infty = 20 \text{ m.s}^{-1}$ (bold gray line) and at
 267 $U_\infty = 40 \text{ m.s}^{-1}$ (light black line) based on the model of Mellen. (a) Convection speed normalized
 268 by the flow velocity. (b) Streamwise exponential decay rate α_x . (c) Spanwise exponential decay
 269 rate α_y .

270 The convection speed normalized by the flow velocity is presented in Fig. 5(a). As already
 271 observed in the literature [19, 22, 23], the convection speed normalized by the flow velocity
 272 decreases with the increasing frequency and has values in the range [0.55 – 0.75].

273 The exponential decay rates estimated along x and y directions are shown in Figs. 5(b)
 274 and 5(c) respectively. For the two considered flow velocities, the values of decay rates are
 275 generally larger than those commonly found in the literature for both Corcos and Mellen
 276 models (*i.e.*, $\alpha_x = 0.116$ and $\alpha_y = 0.7$ [23]). Like for the convection speed, these estimated
 277 decay rates are implemented in the model of Mellen as functions of the frequency.

278 Finally, the real and imaginary parts of the spatial CSD functions of the blocked wall-
 279 pressure obtained by fitting the model of Mellen are compared to those derived from mea-
 280 surements with the microphone array in Figs. 6 and 7 for flow velocities of $U_\infty = 20 \text{ m.s}^{-1}$
 281 and $U_\infty = 40 \text{ m.s}^{-1}$, respectively and at a frequency of 500 Hz.



282

283 FIG. 6. (Color online) Spatial CSD function of the blocked wall-pressure normalized by the auto-
 284 spectrum at the center of the array $G_{p_b p_b}(\xi_x, \xi_y, f) / G_{p_b p_b}(f)$ at 500 Hz and at a flow velocity
 285 $U_\infty = 20 \text{ m.s}^{-1}$. (a) Mellen model, real part. (b) Direct measurement, real part. (c) Mellen model,
 286 imaginary part. (d) Direct measurement, imaginary part.

287 Numerically, it has been observed that to correctly estimate the CSD functions of the
 288 WPF beneath a TBL, a large number of realizations has to be considered [24]. From an
 289 experimental point of view, this can be directly related to the time of acquisition required
 290 to average the random process. Since the time required to measure the TBL fluctuations
 291 at successive rotated positions of the antenna was already significant (30 seconds per each

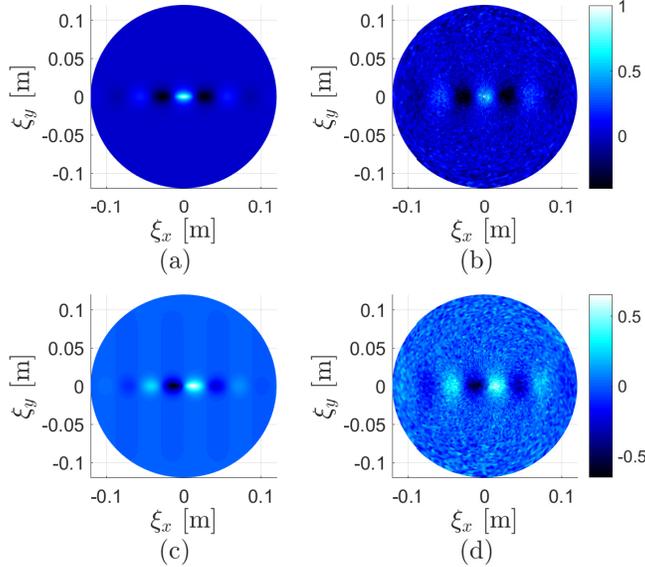


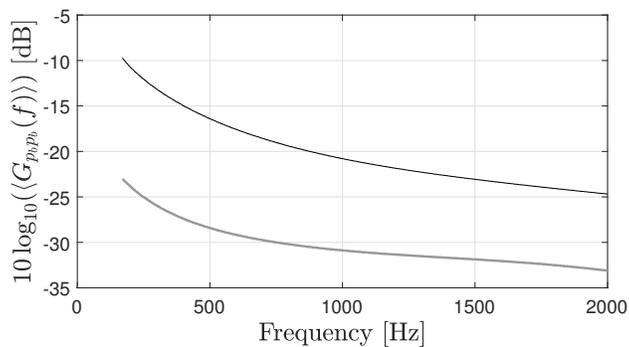
FIG. 7. (Color online) Spatial CSD function of the blocked wall-pressure normalized by the auto-spectrum at the center of the array $G_{p_b p_b}(\xi_x, \xi_y, f) / G_{p_b p_b}(f)$ at 500 Hz and at a flow velocity $U_\infty = 40 \text{ m}\cdot\text{s}^{-1}$. (a) Mellen model, real part. (b) Direct measurement, real part. (c) Mellen model, imaginary part. (d) Direct measurement, imaginary part.

consecutive 180 rotations), no attempt was made to increase the acquisition time. However, using a fitted model tends to eliminate measurement noise while ensuring a satisfactory estimation of the wall-pressure fluctuations.

The adjusted model of Mellen is in good agreement with the measurement at both considered flow velocities, which suggests that the proposed method to fit measurements to a model is accurate. It also shows that the extracted parameters can be implemented in the model of Mellen expressed in the wavenumber domain (see Eq. (7)) to apply the proposed methodology for predicting the vibration response of the plate. However, the model of Mellen also needs to define the wall-pressure ASD function $G_{p_b p_b}(f)$ which is the object of the next paragraph.

Under the assumption of a spatially homogeneous TBL, the auto-spectrum of the wall-pressure should be invariant with the observation point. In reality, our measurement results indicate that the auto-spectrum varies (essentially in the streamwise direction). In the proposed approach the wall-pressure auto-spectrum should be estimated at the point of interest \mathbf{x} where the plate response is to be obtained. This information could not be retrieved from this experiment because response point \mathbf{x} considered in Sec. IV was slightly outside and downstream the area covered by the microphone array. The auto-spectra at each micro-

309 phones have been estimated using the “cpsd” MATLAB command and their mean value
 310 was finally considered for $G_{p_b p_b}(f)$ in Eq. (3) in the remainder of this work. It is therefore
 311 assumed that the spatially averaged ASD function of the blocked wall-pressure $\langle G_{p_b p_b}(f) \rangle$
 312 provides an acceptable estimation of the auto-spectrum at all points sufficiently close to the
 313 spiral-shaped array. The spatially averaged ASD function of the blocked wall-pressure is
 314 presented in Fig. 8 at both considered flow velocities. The trends of the two curves are quite
 315 identical. The increase of flow speed results in a nearly constant shift of the auto-spectrum
 316 level. The results presented in Figs. 5 and 8 fully describe the parameters used in the model
 317 of Mellen which will be considered in Sec. IV C.



318

319 FIG. 8. Spatially averaged ASD function of the blocked wall-pressure $\langle G_{p_b p_b}(f) \rangle$ (dB, ref.
 320 $1 \text{ Pa}^2 \cdot \text{Hz}^{-1}$) at a flow velocity $U_\infty = 20 \text{ m.s}^{-1}$ (bold gray line) and $U_\infty = 40 \text{ m.s}^{-1}$ (light black
 321 line).

322 IV. VALIDATION OF THE PROPOSED METHODOLOGY

323 A. Test case description

324 For experimental validation purposes an academic test case was considered, which consists
 325 in a baffled rectangular thin aluminum plate with simply supported boundary conditions on
 326 all edges. This baffled plate is submitted to a subsonic TBL air flow on one side. The plate’s
 327 geometrical and mechanical properties are detailed in Table I. Simply-supported boundary
 328 conditions have been chosen because they lead to a simple analytical solution of the plate
 329 sensitivity functions. The experimental fabrication method proposed by Robin et al. [21]
 330 has been used to setup panels with representative simply supported boundary conditions.

331 The considered frequency range is [170, 2000] Hz and the modal structural loss factors
 332 η_{mn} have been experimentally estimated using the -3 dB bandwidth method at all resonance
 333 peaks in the considered frequency range. They are taken into account in the numerical
 334 simulations and given in Table II with their corresponding resonance frequencies for the
 335 first eight resonance peaks.

336 In this work, the methodology described in Sec. II C was implemented at a given, arbitrary
 337 position \mathbf{x} of coordinates ($x = 0.18$ m, $y = 0.09$ m, $z = 0$ m) on the plate. Therefore, the
 338 quantities of interest are the sensitivity functions H_v at point \mathbf{x} and the ASD function of
 339 the vibration velocity $G_{vv}(\mathbf{x}, f)$ at point \mathbf{x} when the plate is excited by the TBL.

340 Like in the previous section, two flow velocities are considered: $U_\infty = 20$ m.s⁻¹ and
 341 $U_\infty = 40$ m.s⁻¹. The frequency range is well above the aerodynamic coincidence frequency
 342 f_c defined as the frequency at which the flexural wavenumber k_f equals the convective
 343 wavenumber k_c . These two wavenumbers are defined by:

$$k_f = \sqrt{\omega} \sqrt[4]{\frac{\rho h}{D}} \quad (8)$$

344 where $D = \frac{Eh^3}{12(1-\nu^2)}$ is the flexural rigidity and,

$$k_c = \frac{\omega}{U_c}, \quad (9)$$

345 where U_c is the convection speed which has been extracted from measurements of the wall-
 346 pressure fluctuations in Sec. III B. The aerodynamic coincidence frequency can thus be
 347 expressed as follows

$$f_c = \frac{U_c^2}{2\pi} \sqrt{\frac{\rho h}{D}}. \quad (10)$$

348 In the considered case, $f_c = 7.5$ Hz at $U_\infty = 20$ m.s⁻¹ and $f_c = 30$ Hz at $U_\infty = 40$ m.s⁻¹. It is
 349 clear from Eq. (1) that the plate filters out the excitation and this filtering effect gets effective
 350 above f_c , in which case $k_c > k_f$. To illustrate this effect, the theoretical squared absolute
 351 value of the sensitivity functions have been plotted in Fig. 9(a) at point \mathbf{x} , as a function of
 352 k_x ($k_y = 0$) and as a function of the frequency (see [7] for details on the numerical model).
 353 The strongly decreasing magnitude of $|H_v(\mathbf{x}, k_x, f)|^2$ above the flexural wavenumber can be
 354 noticed. Similarly, the CSD function of the wall-pressure fluctuations according to the fitted
 355 model of Mellen is plotted in Fig. 9(b) while considering the previously obtained decay rates

356 and convection speed at a flow velocity of $U_\infty = 40 \text{ m.s}^{-1}$. Strong contributions around
 357 the convective wavenumber k_c can be noticed. Fig. 9(c) shows the product of the squared
 358 absolute value of the sensitivity functions and the wall-pressure CSD functions (which is
 359 directly involved in the ASD function of the plate velocity), normalized by the maximum
 360 value at each frequency. Components are considered filtered out in the (k_x, k_y) space when
 361 their magnitude is smaller than the maximum value at the corresponding frequency minus
 362 10 dB. In this particular case, the convective wavenumber components centered on k_c are
 363 entirely filtered out on the whole considered frequency range and the vibration response
 364 of the plate to the experimental TBL is mainly driven by the region inside and close to
 365 the circle of radius k_f , the flexural wavenumber. It should be noted that in this work the
 366 excitation is a low-speed subsonic TBL. The higher the flow velocity, the lower the slope of
 367 the curve $k_c(f)$ and therefore, the less the plate filtering effect is effective.

368 In order to fully characterize the filtering effect of the plate, this study has been performed
 369 in both x and y directions and for both considered flow velocities. It allowed retrieving the
 370 maximum wavenumbers that need to be considered in Eq. (3) while ensuring a correct
 371 estimation of the response of the plate to the experimental TBL. Based on this numerical
 372 study and for the considered test case, the wavenumber domain $\Omega_{\mathbf{k}}$ over which Eq. (3) is
 373 calculated is limited to wavenumbers $|k_x| \leq 55 \text{ m}^{-1}$ and $|k_y| \leq 55 \text{ m}^{-1}$. These limits are
 374 slightly above the flexural wavenumber at 2000 Hz ($k_f \approx 51 \text{ m}^{-1}$) which can be explained
 375 by the fact that significant contributions remain slightly above the flexural wavenumber (see
 376 Fig. 9(c)).

377 This study on the filtering effect of the plate shows that with an *a priori* knowledge on the
 378 panel, the limits of the wavenumber domain involved in Eqs. (1) and (3) can be minimized
 379 while ensuring a correct estimation of the response of the plate to a TBL excitation. From
 380 a practical point of view, it allows optimizing the grid of points over which the vibration
 381 response of the structure to a normal effort is to be estimated in order to determine the
 382 sensitivity functions. For a more complex panel with unknown properties, it might be
 383 necessary to extend the wavenumber domain as much as possible (define a refined mesh to
 384 determine the sensitivity functions) to minimize errors linked to truncation effects in the
 385 wavenumber domain.

386 To apply the methodology described in Sec. II C, the panel velocity field has to be mea-
 387 sured on a grid of points $\Gamma_{\tilde{\mathbf{x}}}$. A uniform mesh of 37×27 points was considered in directions

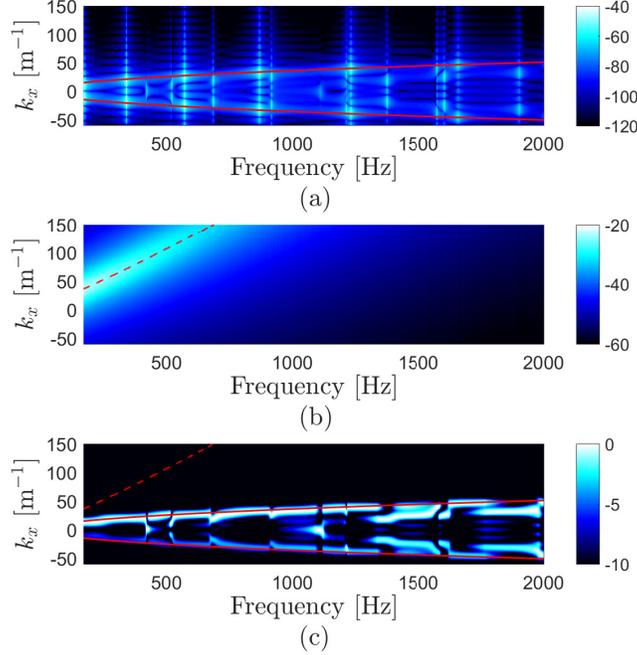


FIG. 9. (Color online) Illustration of the filtering effect of the plate. (a) Theoretical squared absolute value of the sensitivity functions $|H_v(\mathbf{x}, k_x, 0, f)|^2$ (dB, ref. $1 \text{ m.s}^{-1}.\text{Hz}^{-1}$). (b) Model of Mellen $G_{p_b p_b}(k_x, 0, f)$, $U_\infty = 40 \text{ m.s}^{-1}$ (dB, ref. $1 \text{ Pa}^2.\text{Hz}^{-1}$). (c) Product $|H_v(\mathbf{x}, k_x, 0, f)|^2 \times G_{p_b p_b}(k_x, 0, f)$ (dB, ref. $1 \text{ Pa}^2 \text{ m}^2.\text{s}^{-2}.\text{Hz}^{-1}$) normalized by the maximum value at each frequency. Continuous line: flexural wavenumber k_f according to Eq. (8). Dashed line: convective wavenumber k_c according to Eq. (9).

388 x and y respectively and a gap of 1 cm along the edges was left for practical reasons. This
 389 leads to spatial separations of $\delta_x \simeq 1.3 \text{ cm}$ and $\delta_y \simeq 1.6 \text{ cm}$. These separations are well
 390 above the criterion of 4 points per flexural wavelength for all frequencies of interest. The
 391 density of points has voluntarily been set to reach wavenumbers higher than the flexural
 392 wavenumber in order to experimentally validate the filtering effect of the plate.

393 Based on this grid of points, the highest wavenumbers k_x^{max} and k_y^{max} that can be resolved
 394 in directions x and y , respectively, are given by

$$k_x^{max} = \frac{\pi}{\delta_x} \simeq 246 \text{ m}^{-1} ; k_y^{max} = \frac{\pi}{\delta_y} \simeq 204 \text{ m}^{-1}. \quad (11)$$

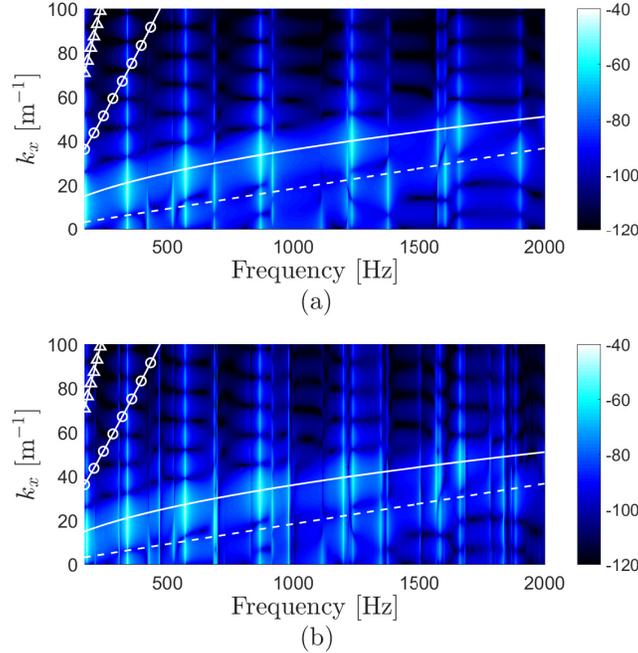
395 The chosen discretization prevents significant spatial aliasing and thereby ensures a cor-
 396 rect estimation of the sensitivity functions on $\Omega_{\mathbf{k}}$. The wavenumber resolutions δk_x and δk_y
 397 in directions x and y respectively, are given by

$$\delta k_x = \frac{2\pi}{L_x} \simeq 13 \text{ m}^{-1} ; \delta k_y = \frac{2\pi}{L_y} \simeq 15 \text{ m}^{-1}. \quad (12)$$

398 These wavenumber resolutions are relatively large because of the small dimensions of the
 399 panel. In order to improve the wavenumber resolution, zero-padding was used to obtain a
 400 wavenumber resolution of 1 m^{-1} along k_x and k_y .

401 B. Experimental sensitivity functions

402 The accuracy of the reciprocity approach for evaluating the panel sensitivity functions
 403 has been assessed in [7] for wavenumbers restricted to the acoustic wavenumber circle (of
 404 radius defined by $k_0 = \omega/c_0$, with c_0 the speed of sound). In this work, the sensitivity
 405 functions have to be determined on a larger wavenumber domain (inside and close to the
 406 flexural wavenumber circle of radius k_f).



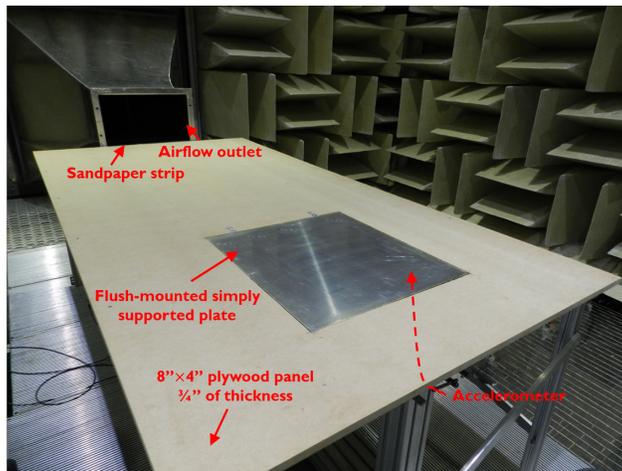
407
 408 FIG. 10. (Color online) Squared absolute value of the sensitivity functions $|H_v(\mathbf{x}, \mathbf{k}, f)|^2$ (dB,
 409 ref. $1 \text{ m}^2 \cdot \text{s}^{-2} \cdot \text{Hz}^{-1}$) along $k_x \geq 0$ for $k_y = 0$: (a) numerical result, (b) experimental result.
 410 The superimposed lines represent: k_f according to Eq. (8) (continuous line); k_0 (dashed line); k_c
 411 according to Eq. (9) for $U_\infty = 20 \text{ m.s}^{-1}$ (line with triangle markers); k_c according to Eq. (9) for
 412 $U_\infty = 40 \text{ m.s}^{-1}$ (line with circle markers).

413 The sensitivity functions of the panel have been estimated from measurements based on
 414 the methodology described in Sec. IIC. A normal effort was applied at point \mathbf{x} using a
 415 TMS SmartShaker K2007E01 with integrated amplifier, which was fed with a swept sine
 416 over the considered frequency range and the force was measured using an impedance head
 417 PCB288D01. An adapter was used between the impedance head and the plate reducing
 418 the area of mechanical coupling to approximately a 5 mm diameter circle. The vibratory
 419 response of the panel was measured on the grid of 37×27 points with a single point scanning
 420 laser vibrometer (PSV-300 Polytec) and a time Fourier transform was directly performed in
 421 the post-processing software with ten linear averages. A frequency resolution of 0.625 Hz
 422 was chosen and fixed for the remainder of this paper. The numerically obtained sensitivity
 423 functions in Fig. 10(a) (see [7] for details on the numerical model) are compared to the
 424 experimental sensitivity functions in Fig. 10(b). In both cases, the sensitivity functions are
 425 presented along $k_x \geq 0$ and for $k_y = 0$ as a function of the frequency. The panel vibration
 426 modes are noticeable below or close to the dispersion curve of the flexural motion, Eq. (8).
 427 Since the plate is considered isotropic, its vibration behavior is similar in both direction
 428 which explains why results are only presented in the flow-direction, where the excitation
 429 has its most energetic components. A representation along k_y would have shown similar
 430 results with differences linked to the dimensions of the plate resulting in different modal
 431 wavenumbers in direction y . Results presented in Fig. 10 show that the experimentally
 432 obtained sensitivity functions are in good agreement with numerical simulations on the
 433 whole frequency range. Slight differences between experimental and numerical results can
 434 be observed mainly for the upper part of the frequency range. They can be attributed to
 435 the boundary conditions of the actual plate, which are close to simply supported boundary
 436 conditions but not absolutely perfect.

437 **C. Application of the proposed methodology and comparison with direct mea-** 438 **surements in the wind tunnel**

439 The vibration response of the plate estimated from the proposed approach (Eq. (3)) is
 440 finally compared to direct measurements in the wind tunnel. Measurements in the wind
 441 tunnel were performed with a plate identical to the one used for sensitivity functions mea-
 442 surement (Sec. IV B) and described in Table I (similar dimensions, material and boundary

443 conditions). The plate was flush-mounted at the location of the previously mounted spiral-
 444 shaped array (as shown in Fig. 11). Aside from replacing the spiral-shaped array with the
 445 plate, the experimental conditions remained unchanged in order to keep the turbulent flow
 446 excitation identical to the one characterized in Sec. III.



447

448 FIG. 11. (Color online) Experimental setup used to measure the vibration response of the plate to
 449 the TBL excitation generated in the wind tunnel.

450 On the one hand, the vibration velocity of the plate was measured at point \mathbf{x} using a
 451 PCB 353B18 accelerometer, the acceleration signal over time was extracted from the post-
 452 processing software with a sampling frequency of 8192 Hz and the vibration velocity ASD
 453 function G_{vv} was estimated using the “cpsd” MATLAB command. On the other hand, the
 454 vibration velocity ASD function was estimated by applying the proposed methodology using
 455 Eq. (3), the measured sensitivity functions $H_v(\mathbf{x}, \mathbf{k}, f)$ and the fitted Mellen model of the
 456 WPF CSD function $G_{p_b p_b}(\mathbf{k}, f)$.

457 The plate velocity ASD function measured in the anechoic wind tunnel room at point \mathbf{x}
 458 is compared to the result obtained with the proposed method in Figs. 12 and 13 for flow
 459 velocities of 20 m.s^{-1} and 40 m.s^{-1} , respectively. One can notice that all modes are excited
 460 by the reproduced TBL. According to the formulation presented in Eq. (1), where both
 461 functions in the integral are positive or null, any type of mode (even or odd) can be excited
 462 by any random excitation.

463 For both considered flow velocities, slight shifts of the resonance peaks in the high fre-
 464 quency range are noticed. They can be explained by the fact that an adapter was used
 465 between the impedance head and the plate for the measurement of the sensitivity functions,

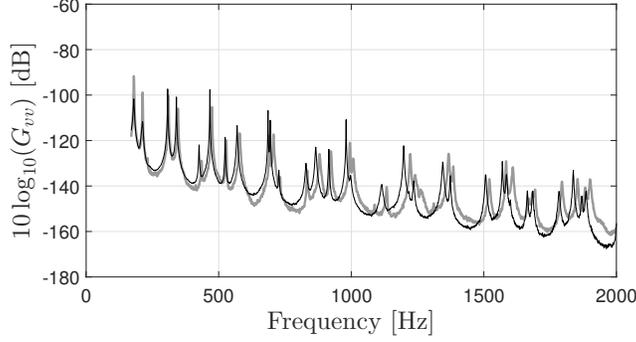


FIG. 12. Velocity ASD functions G_{vv} (dB, ref. $1 \text{ m}^2 \cdot \text{s}^{-2} \cdot \text{Hz}^{-1}$) at flow velocity $U_\infty = 20 \text{ m} \cdot \text{s}^{-1}$: proposed approach (light black line) vs. wind tunnel measurements (bold gray line).

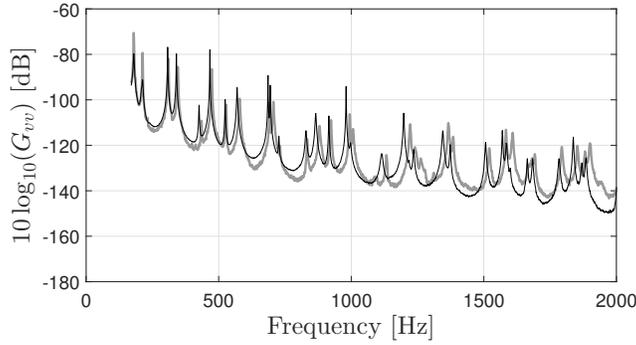


FIG. 13. Velocity ASD functions G_{vv} (dB, ref. $1 \text{ m}^2 \cdot \text{s}^{-2} \cdot \text{Hz}^{-1}$) at flow velocity $U_\infty = 40 \text{ m} \cdot \text{s}^{-1}$: proposed approach (light black line) vs. wind tunnel measurements (bold gray line).

466 adding a mass to the system. It can also be explained by the fact that the sensitivity
 467 functions were not measured on the same plate as the plate installed in the wind tunnel to
 468 directly measure the response. Despite all efforts made to have two identical panels, slight
 469 differences in dimensions, material properties and boundary conditions were unavoidable.
 470 The structural damping is implicitly taken into account through the measurement of the
 471 sensitivity functions in the proposed approach and through the direct measurement of the
 472 vibration response in the wind tunnel. The differences in peak values, as well as off reso-
 473 nance values, can again be linked to the fact that the two plates had differences in structural
 474 damping values. For instance, the damping loss factors estimated at the first two peaks for
 475 the plate mounted in the wind tunnel ($\eta_{21} = 0.55 \%$ and $\eta_{12} = 0.56 \%$) are lower than those
 476 estimated on the plate used to measure the sensitivity functions (given in Table II). For
 477 the following six peaks, an opposite trend has been observed which is in line with obtained

478 results and, thereby, explains the amplitude differences at the resonance peaks.

479 The differences can also be attributed to the fact that the auto-spectrum of the blocked
480 wall-pressure has not been estimated directly at point \mathbf{x} but by averaging the auto-spectra
481 over the area covered by the spiral-shaped array. This leads to slight errors in terms of
482 overall trend and thereby also contributes to the under-estimation by the proposed method,
483 particularly at high frequencies. Despite the differences at the first two peaks of resonance,
484 which again are linked to discrepancies in the structural loss factors of both plates, the
485 predicted and measured data are in good agreement, which shows that for the considered
486 test case, the velocity response of the plate can be fairly well estimated experimentally by
487 applying the proposed methodology.

488 V. CONCLUSION

489 In this paper, an alternative methodology for characterizing the vibration response of
490 panels to a turbulent boundary layer excitation was proposed. This approach is based on
491 the concept that the panel response at a point \mathbf{x} on the panel to a random pressure field
492 depends on two quantities in the wavenumber domain. First, the wall-pressure cross-spectral
493 density function, which characterizes the excitation. Second, so-called ‘sensitivity functions’
494 determined at point \mathbf{x} and which characterize the dynamic behavior of the panel. Those sen-
495 sitivity functions can be determined using the reciprocity principle, which states that they
496 are equivalent to the panel velocity frequency response function when the panel is excited
497 by a normal force at the point of interest \mathbf{x} , expressed in the wavenumber domain. The
498 sensitivity functions can be estimated easily by experiments based on this reciprocal inter-
499 pretation. The method has been validated experimentally for a simply supported aluminum
500 plate. The confrontation to direct measurements in an anechoic wind tunnel has shown that
501 a fairly good estimate of the vibration response can be obtained by applying the proposed
502 methodology. This indicates that the excitation, as well as the panel behavior, have been
503 correctly characterized.

504 The main limitations of the proposed approach rely on the assumptions of the mathe-
505 matical formulation of the problem: the system should be linear (*i.e.*, elastic material, small
506 deformations) and time invariant. It is also assumed that the wall-pressure field exciting the
507 panel corresponds to that of the turbulent flow in rigid conditions. Models provided in the

508 literature can be used but should be adjusted to actual measurements in order to correctly
509 represent the considered excitation (it is however not mandatory to apply the method).

510 From a practical point of view, the acquisition time for the reciprocal approach is signif-
511 icantly longer than for a direct measurement in the wind tunnel because a scanning laser
512 vibrometer was used. However, it could be well reduced with the recently developed full-
513 field vibration measuring techniques (such as digital image correlation or deflectometry [25]).
514 Once the excitation is characterized, the overall cost for a measurement in a given facility
515 as well as variability between measurements in different ones can be greatly reduced using
516 the proposed approach.

517 The strongest asset of the proposed methodology is that it allows performing an *ex situ*
518 characterization of a panel under turbulent boundary layer excitation. The characteriza-
519 tion of the excitation and/or the panel can be experimental, but might as well come from
520 numerical models. This approach is therefore well suited for parametric studies. Once the
521 excitation is defined, the response of panels with various mechanical properties under the
522 considered excitation can easily be deduced.

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594 Fig. 1. Illustration of a baffled panel (gray line) excited by a TBL and coordinate
595 system.

596 Fig. 2. Determination of the sensitivity functions H_v : (a) based on the direct inter-
597 pretation, (b) using the reciprocity principle.

598 Fig. 3. (a) Microphone positions (red markers) and illustration of a reconstructed grid
599 (blue dots). (b) Close-up view of the mounted sensors and pinholes on measurement
600 side.

601 Fig. 4. Measurement of the wall-pressure fluctuations using the spiral-shaped surface
602 microphone array flush-mounted in the wind tunnel.

603 Fig. 5. TBL parameters extracted from measurements at $U_\infty = 20 \text{ m.s}^{-1}$ (bold gray
604 line) and at $U_\infty = 40 \text{ m.s}^{-1}$ (light black line) based on the model of Mellen. (a)
605 Convection speed normalized by the flow velocity. (b) Streamwise exponential decay
606 rate α_x . (c) Spanwise exponential decay rate α_y .

607 Fig. 6. Spatial CSD function of the blocked wall-pressure normalized by the auto-
608 spectrum at the center of the array $G_{p_b p_b}(\xi_x, \xi_y, f) / G_{p_b p_b}(f)$ at 500 Hz and at a flow
609 velocity $U_\infty = 20 \text{ m.s}^{-1}$. (a) Mellen model, real part. (b) Direct measurement, real
610 part. (c) Mellen model, imaginary part. (d) Direct measurement, imaginary part.

611 Fig. 7. Spatial CSD function of the blocked wall-pressure normalized by the auto-
612 spectrum at the center of the array $G_{p_b p_b}(\xi_x, \xi_y, f) / G_{p_b p_b}(f)$ at 500 Hz and at a flow
613 velocity $U_\infty = 40 \text{ m.s}^{-1}$. (a) Mellen model, real part. (b) Direct measurement, real
614 part. (c) Mellen model, imaginary part. (d) Direct measurement, imaginary part.

615 Fig. 8. Spatially averaged ASD function of the blocked wall-pressure $\langle G_{p_b p_b}(f) \rangle$ (dB,
616 ref. $4 \times 10^{-10} \text{ Pa}^2 \cdot \text{Hz}^{-1}$) at a flow velocity $U_\infty = 20 \text{ m.s}^{-1}$ (bold gray line) and
617 $U_\infty = 40 \text{ m.s}^{-1}$ (light black line).

618 Fig. 9. Illustration of the filtering effect of the plate. (a) Theoretical squared abso-
619 lute value of the sensitivity functions $|H_v(\mathbf{x}, k_x, 0, f)|^2$ (dB, ref. $1 \text{ m.s}^{-1} \cdot \text{Hz}^{-1}$). (b)
620 Model of Mellen $G_{p_b p_b}(k_x, 0, f)$, $U_\infty = 40 \text{ m.s}^{-1}$ (dB, ref. $1 \text{ Pa}^2 \cdot \text{Hz}^{-1}$). (c) Product

621 $|H_v(\mathbf{x}, k_x, 0, f)|^2 \times G_{p_b p_b}(k_x, 0, f)$ (dB, ref. $1 \text{ Pa}^2 \text{ m}^2 \cdot \text{s}^{-2} \cdot \text{Hz}^{-1}$) normalized by the max-
622 imum value at each frequency. Continuous line: flexural wavenumber k_f according to
623 Eq. (8). Dashed line: convective wavenumber k_c according to Eq. (9).

624 Fig. 10. Squared absolute value of the sensitivity functions $|H_v(\mathbf{x}, \mathbf{k}, f)|^2$ (dB, ref.
625 $1 \text{ m}^2 \cdot \text{s}^{-2} \cdot \text{Hz}^{-1}$) along $k_x \geq 0$ for $k_y = 0$: (a) numerical result, (b) experimental
626 result. The superimposed lines represent: k_f according to Eq. (8) (continuous line); k_0
627 (dashed line); k_c according to Eq. (9) for $U_\infty = 20 \text{ m} \cdot \text{s}^{-1}$ (line with triangle markers);
628 k_c according to Eq. (9) for $U_\infty = 40 \text{ m} \cdot \text{s}^{-1}$ (line with circle markers).

629 Fig. 11. Experimental setup used to measure the vibration response of the plate to
630 the TBL excitation generated in the wind tunnel.

631 Fig. 12. Velocity ASD functions G_{vv} (dB, ref. $1 \text{ m}^2 \cdot \text{s}^{-2} \cdot \text{Hz}^{-1}$) at flow velocity $U_\infty =$
632 $20 \text{ m} \cdot \text{s}^{-1}$: proposed approach (light black line) vs. wind tunnel measurements (bold
633 gray line).

634 Fig. 13. Velocity ASD functions G_{vv} (dB, ref. $1 \text{ m}^2 \cdot \text{s}^{-2} \cdot \text{Hz}^{-1}$) at flow velocity $U_\infty =$
635 $40 \text{ m} \cdot \text{s}^{-1}$: proposed approach (light black line) vs. wind tunnel measurements (bold
636 gray line).

TABLE I. Mechanical properties of the simply supported aluminum plate.

Parameter (Symbol), Unit	Value
Young's modulus (E), GPa	68.9
Poisson's ratio (ν)	0.3
Mass density (ρ), kg/m ³	2740
Length (L_x), mm	480
Width (L_y), mm	420
Thickness (h), mm	3.17

TABLE II. Modal properties of the simply supported aluminum plate.

(m, n)	Resonance frequency, f_{mn} [Hz]	Modal structural loss factor, η_{mn} [%]
(2, 1)	180	1.71
(1, 2)	213.1	2.05
(2, 2)	307.5	0.28
(3, 1)	340.6	0.3
(1, 3)	426.3	0.5
(3, 2)	466.9	0.11
(2, 3)	524.4	0.29
(4, 1)	569.4	0.41