Sugeno Utility Functionals for Monotonic Classification
Decision Rules
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Sugeno Utility Functionals for Monotonic Classification
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Monotonic Classification & Decision Rules

Feature space: \( X = X_1 \times \cdots \times X_n \), where \( X_i \) is a totally ordered set. Each object is represented by a tuple \( x = (x_1, \ldots, x_n) \in X \).

Labels: Each object has a label \( l(x) \) from a totally ordered set \( L \).

The relation between descriptions and labels is assumed to be order-preserving
\[
a_1 \leq b_1, \ldots, a_n \leq b_n \Rightarrow l(a_1, \ldots, a_n) \leq l(b_1, \ldots, b_n).
\]

Aim: to predict the label of objects from their descriptions, with a non-decreasing function \( f : X \rightarrow L \).

Example: These rules express the function at the left.
\[
x_1 \geq \frac{3}{7} \Rightarrow l(x) \geq \blacksquare
\]
\[
x_1 \geq \frac{4}{7}, x_2 \geq \frac{3}{7} \Rightarrow l(x) \geq \blacksquare
\]

Sugeno Utility Functionals (SUF)
A capacity \( \mu : 2^{\{1, \ldots, n\}} \rightarrow L \) is a set function verifying
- \( \mu(\emptyset) = 0 \) and \( \mu(\{1, \ldots, n\}) = 1 \)
- \( I \subseteq J \Rightarrow \mu(I) \leq \mu(J) \).

The Sugeno integral \( S_\mu \) defined by \( \mu \) is the aggregation function
\[
\max_{I \subseteq \{1, \ldots, n\}} \min_{i \in I} (\mu(I), \min x_i).
\]

Let \( \varphi = (\varphi_1, \ldots, \varphi_n) \), where each mapping \( \varphi_i : X_i \rightarrow L \) verifies
- \( \varphi_i(0) = 0 \) and \( \varphi_i(1) = 1 \)
- \( a_i \leq b_i \Rightarrow \varphi_i(a_i) \leq \varphi_i(b_i) \).

A SUF is a combination of a Sugeno integral and mappings \( \varphi_1, \ldots, \varphi_n \) of the form
\[
S_\mu(\varphi_1(x_1), \ldots, \varphi_n(x_n)).
\]

A single SUF is less expressiv than decision rules. A maximum of several SUFs can represent any set of decision rules.

Application
Maxima of SUFs enable a non-parametric method [1] for monotonic classification.

Principle: To fit the data with a max-SUF using the smallest possible number of SUFs.

The max-SUF can then be translated back into rules.


References
Try the method on your data: https://github.com/QGBrabant/SUF4OC
