Sugeno Utility Functionals for Monotonic Classification & Decision Rules
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To cite this version:
Quentin Brabant, Miguel Couceiro. Sugeno Utility Functionals for Monotonic Classification & Decision Rules. ISWS 2018 - International Semantic Web Research Summer School, Jul 2018, Bertinoro, Italy. hal-01906052

HAL Id: hal-01906052
https://hal.archives-ouvertes.fr/hal-01906052
Submitted on 26 Oct 2018

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Monotonic Classification & Decision Rules

Feature space: $X = X_1 \times \cdots \times X_n$, where $X_i$ is a totally ordered set. Each object is represented by a tuple $x = (x_1, \ldots, x_n) \in X$.

Labels: Each object has a label $l(x)$ from a totally ordered set $L$. The relation between descriptions and labels is assumed to be order-preserving $a_1 \leq b_1, \ldots, a_n \leq b_n \Rightarrow l(a_1, \ldots, a_n) \leq l(b_1, \ldots, b_n)$.

Aim: to predict the label of objects from their descriptions, with a non-decreasing function $f : X \rightarrow L$.

Example: These rules express the function at the left.

\[ x_1 \geq \frac{3}{7} \Rightarrow l(x) \geq \]
\[ x_1 \geq \frac{4}{7}, \ x_2 \geq \frac{3}{7} \Rightarrow l(x) \geq \]

Sugeno Utility Functionals (SUF)

A capacity $\mu : 2^{\{1, \ldots, n\}} \rightarrow L$ is a set function verifying
- $\mu(\emptyset) = 0$ and $\mu(\{1, \ldots, n\}) = 1$
- $I \subseteq J \Rightarrow \mu(I) \leq \mu(J)$.

The Sugeno integral $S_\mu$ defined by $\mu$ is the aggregation function

$$\max_{I \subseteq \{1, \ldots, n\}} \min_{i \in I} (\mu(I), \min_{i \in I} x_i).$$

Let $\varphi = (\varphi_1, \ldots, \varphi_n)$, where each mapping $\varphi_i : X_i \rightarrow L$ verifies
- $\varphi_i(0) = 0$ and $\varphi_i(1) = 1$
- $a_i \leq b_i \Rightarrow \varphi_i(a_i) \leq \varphi_i(b_i)$.

A SUF is a combination of a Sugeno integral and mappings $\varphi_1, \ldots, \varphi_n$ of the form $S_\mu(\varphi_1(x_1), \ldots, \varphi_n(x_n))$.

A single SUF is less expressive than decision rules. A maximum of several SUFs can represent any set of decision rules.

Application

Maxima of SUFs enable a non-parametric method [1] for monotonic classification.

Principle: To fit the data with a max-SUF using the smallest possible number of SUFs.

The max-SUF can then be translated back into rules.


References

Try the method on your data: https://github.com/QGBrabant/SUF4OC
