Sugeno Utility Functionals for Monotonic Classification & Decision Rules
Quentin Brabant, Miguel Couceiro

To cite this version:
Quentin Brabant, Miguel Couceiro. Sugeno Utility Functionals for Monotonic Classification & Decision Rules. ISWS 2018 - International Semantic Web Research Summer School, Jul 2018, Bertinoro, Italy. hal-01906052

HAL Id: hal-01906052
https://hal.archives-ouvertes.fr/hal-01906052
Submitted on 26 Oct 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Monotonic Classification & Decision Rules

Feature space: $X = X_1 \times \cdots \times X_n$, where $X_i$ is a totally ordered set. Each object is represented by a tuple $x = (x_1, \ldots, x_n) \in X$.

Labels: Each object has a label $l(x)$ from a totally ordered set $L$.

The relation between descriptions and labels is assumed to be order-preserving

$$a_1 \leq b_1, \ldots, a_n \leq b_n \Rightarrow l(a_1, \ldots, a_n) \leq l(b_1, \ldots, b_n).$$

Aim: to predict the label of objects from their descriptions, with a non-decreasing function $f : X \to L$.

Example: These rules express the function at the left.

$$x_1 \geq 3/7 \Rightarrow l(x) \geq \square$$

$$x_1 \geq 4/7, x_2 \geq 3/7 \Rightarrow l(x) \geq \square$$

Sugeno Utility Functionals (SUF)

A capacity $\mu : 2^{\{1, \ldots, n\}} \to L$ is a set function verifying

- $\mu(\emptyset) = 0$ and $\mu(\{1, \ldots, n\}) = 1$
- $I \subseteq J \Rightarrow \mu(I) \leq \mu(J)$.

The Sugeno integral $S_\mu$ defined by $\mu$ is the aggregation function

$$\max_{I \subseteq \{1, \ldots, n\}} \min\{\mu(I), \min_{i \in I} x_i\}.$$

Let $\varphi = (\varphi_1, \ldots, \varphi_n)$, where each mapping $\varphi_i : X_i \to L$ verifies

- $\varphi_i(0) = 0$ and $\varphi_i(1) = 1$
- $a_i \leq b_i \Rightarrow \varphi_i(a_i) \leq \varphi_i(b_i)$.

A SUF is a combination of a Sugeno integral and mappings $\varphi_1, \ldots, \varphi_n$ of the form

$$S_\mu(\varphi_1(x_1), \ldots, \varphi_n(x_n)).$$

A single SUF is less expressive than decision rules. A maximum of several SUFs can represent any set of decision rules.

Application

Maxima of SUFs enable a non-parametric method [1] for monotonic classification.

Principle: To fit the data with a max-SUF using the smallest possible number of SUFs.

The max-SUF can then be translated back into rules.


References

Try the method on your data: https://github.com/QGBrabant/SUF4OC
