Sugeno Utility Functionals for Monotonic Classification & Decision Rules
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Monotonic Classification & Decision Rules

**Feature space:** $X = X_1 \times \cdots \times X_n$, where $X_i$ is a totally ordered set. Each object is represented by a tuple $x = (x_1, \ldots, x_n) \in X$.

**Labels:** Each object has a label $l(x)$ from a totally ordered set $L$.

The relation between descriptions and labels is assumed to be order-preserving

$$a_1 \leq b_1, \ldots, a_n \leq b_n \Rightarrow l(a_1, \ldots, a_n) \leq l(b_1, \ldots, b_n).$$

**Aim:** to predict the label of objects from their descriptions, with a non-decreasing function $f : X \to L$.

The function can be specified by decision rules of the form:

$$\forall i \in A, \; x_i \geq a_i \Rightarrow l(x) \geq \delta,$$

where $A \subseteq \{1, \ldots, n\}$. Sets of such rules can describe any non-decreasing function from $X$ to $L$.

**Example:** These rules express the function at the left.

$$x_1 \geq 3/7 \Rightarrow l(x) \geq \Box$$

$$x_1 \geq 4/7, \; x_2 \geq 3/7 \Rightarrow l(x) \geq \Box$$

Sugeno Utility Functionals (SUF)

A capacity $\mu : 2^{\{1, \ldots, n\}} \to L$ is a set function verifying

- $\mu(\emptyset) = 0$ and $\mu(\{1, \ldots, n\}) = 1$
- $I \subseteq J \Rightarrow \mu(I) \leq \mu(J)$.

The Sugeno integral $S_\mu$ defined by $\mu$ is the aggregation function

$$\max_{I \subseteq \{1, \ldots, n\}} \min(I, \min_{i \in I} x_i).$$

Let $\varphi = (\varphi_1, \ldots, \varphi_n)$, where each mapping $\varphi_i : X_i \to L$ verifies

- $\varphi_i(0) = 0$ and $\varphi_i(1) = 1$
- $a_i \leq b_i \Rightarrow \varphi_i(a_i) \leq \varphi_i(b_i)$.

A SUF is a combination of a Sugeno integral and mappings $\varphi_1, \ldots, \varphi_n$ of the form

$$S_\mu(\varphi_1(x_1), \ldots, \varphi_n(x_n)).$$

A single SUF is less expressive than decision rules. A maximum of several SUFs can represent any set of decision rules.

Application

Maxima of SUFs enable a non-parametric method [1] for monotonic classification.

**Principle:** To fit the data with a max-SUF using the smallest possible number of SUFs.

The max-SUF can then be translated back into rules.

**Result:** The method is competitive (in terms of accuracy) with state of the art methods [2] for learning decision rules.

References

Try the method on your data: https://github.com/QGBrabant/SUF4OC
