Numerical Scheme for a Stratigraphic Model with Erosion Constraint and Nonlinear Gravity Flux
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Objectives

- Simulate the evolution of sedimentary basins over large time and space scales:
  - dimension of domains: 10 ~ 100 km,
  - simulations times: 0.1 ~ 100 My.
- Take into account sedimentary processes:
  - gravity- and water-driven transport,
  - sediment accumulation and erosion.
- Improve classical numerical schemes [2, 3, 4] to better describe physical processes.

Simplified model

**Sediment transport** The sediment flux $F$, depending on the sediment height $h$, is nonlinearly proportional to the local slope $\nabla h$:  
$$ F = -K(h)|\nabla h|^{p-2} \nabla h = -|\nabla h|^{p-2} \nabla \psi(h), $$

where the diffusion coefficient $K$ depends on maritime and continental domains, and $p > 2$ to ensure finite propagation speed.

**Maximum erosion rate constraint** The actual sediment flux is limited by the sediment availability, constrained by a maximum erosion rate $E > 0$. The diffusive flux $F$ is multiplied by a factor $\lambda$ so as to guarantee  
$$ \partial_t h + E \leq 0, \quad \lambda \geq 0, \quad (\partial_t h + E)(1 - \lambda) = 0. $$

The complete system then reads

$$ \partial_t h + \nabla \cdot (\lambda F) = 0, $$

$$ \min \left(1 - \lambda, E - \nabla \cdot (\lambda F)\right) = 0. $$

References


Application

**Physical data**

- Dimensions: 180 x 180 km
- Diffusion coefficients:
  - continental: 500 km²/My
  - maritime: 10 km²/My
- Constraint: $E = 0.04$ km/My
- Sea level: $H = 0$ km
- Input fluxes: 2 sources
- Simulation time: $T = 1$ My

**Numerical parameters**

- Discretization: 361 x 361 cells
- Exponent value: $p = 2.5$
- Maximum time step: $10^{-3}$ My
- Solver: BiCGStab with ILU(0)

<table>
<thead>
<tr>
<th>Without constraint</th>
<th>With constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accepted time steps</td>
<td>1016</td>
</tr>
<tr>
<td>Refused time steps</td>
<td>0</td>
</tr>
<tr>
<td>Mean Newton iterations per accepted time step</td>
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</tr>
<tr>
<td>Mean solver iterations per Newton iteration</td>
<td>1.99</td>
</tr>
<tr>
<td>Computing time (s)</td>
<td>783</td>
</tr>
</tbody>
</table>

Gradient norm reconstruction

Following [1], the term $|\nabla h|^2$ is approximated on each cell dual by the formula

$$ B_{i,j}^{n+1/2,j+1/2} = \frac{1}{2} \left( h_{i,j}^{n+1/2,j+1/2} - h_{i,j}^{n+1/2,j-1/2} \right)^2 + \frac{1}{2} \left( h_{i+1,j+1/2}^{n+1/2,j+1/2} - h_{i+1,j-1/2}^{n+1/2,j-1/2} \right)^2 + \frac{1}{2} \left( h_{i,j+1}^{n+1/2,j+1/2} - h_{i,j}^{n+1/2,j-1/2} \right)^2 + \frac{1}{2} \left( h_{i+1,j+1}^{n+1/2,j+1/2} - h_{i+1,j}^{n+1/2,j-1/2} \right)^2. $$

It is coercive in the sense $B_{i,j}^{n+1/2,j+1/2} = 0$ if and only if $h_{i,j}^{n+1} = h_{i,j}^{n} = h_{i+1,j}^{n+1} = h_{i+1,j}^{n}$. The normal flux $\lambda F$ is upwinded according to the sign of the flux:

$$ (\lambda F)_{i,j}^{n+1/2} = \lambda_{i,j}^{n+1/2} (F_{i,j}^{n+1/2})^+ - \lambda_{i,j+1}^{n+1/2} (F_{i,j+1/2}^{n+1/2})^-, $$

with $u^+ = \max(u, 0)$ and $u^- = -\min(u, 0)$. The normal flux $\mathbf{F} \cdot \mathbf{n}$ is discretized using the approximation of $|\nabla h|^2$ previously introduced:

$$ F_{i,j}^{n+1/2} = \frac{1}{2} \left[ (B_{i,j+1/2,j+1/2}^{n+1/2})^{1/2} + (B_{i+1,j+1/2,j+1/2}^{n+1/2})^{1/2} \right] \psi(h_{i,j}^{n+1/2} - h_{i,j+1}^{n+1/2}) - \psi(h_{i+1,j}^{n+1/2} - h_{i,j+1}^{n+1/2}) \Delta x. $$

Complementarity equation

$$ \lambda_{i,j}^{n+1} = \min \left( 1, \frac{\Delta x \Delta y E_{i,j} + \Delta y \lambda F_{i,j}^{n+1/2} + \Delta x \lambda E_{i+1,j}^{n+1/2}}{\Delta y (F_{i,j}^{n+1}) + \Delta x (F_{i+1,j}^{n+1/2})} \right), $$

where $h_{i,j}^{n+1}$ represents the outgoing flux from the cell $i,j$, and $F_{i,j}^{n+1/2}$ the limited incoming flux.