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d-WAVE SUPERCONDUCTIVITY IN THE ANDERSON LATTICEM LAVAGNA^{*,†}, A J MILLIS^{%,†} and P A Lee⁺*Department of Physics, MIT, Cambridge, MA 02139, USA*

We show, within a slave-boson $1/N$ expansion, that the $U = \infty$ lattice Anderson model is unstable to d-wave superconductivity. The pairing mechanism is slave-boson exchange between heavy quasiparticles. The model is extended to include the Coulomb pseudopotential. We discuss the pressure dependence of T_c in light of our theory.

The heavy fermion superconductors have been of considerable and continuing interest to condensed matter physicists since their discovery seven years ago. It is now believed that the superconducting state in the heavy fermion materials is “unconventional” in two senses. One sense has to do with the form of the gap function, which appears to vanish along points or lines on the Fermi surface [1], and implies that heavy fermion superconductivity involves electrons paired in a state of non-zero angular momentum. The other, perhaps more fundamental, unconventionality is that heavy fermion superconductivity is unlikely to be due to the conventional electron-phonon interaction [2].

In this paper, we show that the $U = \infty$ Anderson model is unstable to d-wave superconductivity in the same limit in which it describes heavy fermion behavior. We use the slave boson large degeneracy (N) formulation, originally devised for the f -impurity [3–5] and recently extended to the lattice [6–8], in which the Hamiltonian can be written within the constraint $Q_i = n_i^+ + n_i^- = 1$

$$H_F = \sum_{km} \epsilon_k c_{km}^\dagger c_{km} + E_0 f_{km}^\dagger f_{km} + \frac{V}{\sqrt{N}} \times \sum_{kqm} c_{km}^\dagger f_{k+qm} b_q^\dagger + \text{h.c.} \quad (1)$$

All energies are measured from the chemical potential, μ . We are interested in the Kondo limit,

in which $-E_0 > \rho_0 V^2$. ρ_0 is the c-electron density of states of width $2W$. To enforce the constraint, one introduces a Lagrange multiplier λ . The model possesses a $1/N$ expansion around the mean-field theory. The fluctuations may be studied in either the Cartesian or the radial gauge. To avoid the complications of infrared divergences appearing in the Cartesian gauge [5], we choose here the radial-gauge $b_i^+ = r_i e^{i\theta_i}$ and $f_i = f_i' e^{i\theta_i}$. The main results of the paper eq. (7) could be obtained in the Cartesian gauge, also.

We define in this way two Bose-fields r and λ (θ being incorporated into λ), and consider their fluctuations around their mean-field values r_0 and $\lambda_0 = \epsilon_F - E_0$

$$\epsilon_F = W e^{-|E_0|/\rho_0 V^2}, \quad (2a)$$

$$V^2 r_0^2 = \sigma_0^2 = q_0 \epsilon_F n_F / \rho_0, \quad (2b)$$

with

$$n_F = (1 + \epsilon_F / \rho_0 V^2)^{-1}, \quad (2c)$$

ϵ_F is the Kondo temperature, q_0 is defined as Q_i/N , but to obtain a sensible $1/N$ expansion, we must regard q_0 as a parameter independent of the explicit N , and only set $q_0 = 1/N$ at the end of the calculation. The mean-field theory describes electrons moving in a bandstructure presenting a gap around ϵ_F . The elementary excitations are heavy quasiparticles of mass enhancement $m^*/m = \rho/\rho_0 = (\sigma_0/\epsilon_F)^2 = q_0/\rho_0 \epsilon_F \gg 1$.

Corrections to the mean-field theory are due to electron-boson interactions of order $\leq 1/N$. In this paper we show that, even though weak, this electron-boson interaction leads to d-wave superconductivity. We proceed by investigating

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divergences in the scattering amplitude Γ for two quasiparticles of opposite momenta near the fermi surface

The quasiparticle scattering amplitude is conventionally decomposed into spin singlet and triplet channels, this is meaningless in our spin N model. An alternative procedure is to decompose the scattering amplitude into orbital angular momentum channels. Since $\mathbf{k}$ and $\mathbf{k}'$ lie on the fermi surface (see fig. 1), Γ is a function only of the angle φ between them $|\mathbf{q}| = |\mathbf{k}' - \mathbf{k}| = 2k_F \sin \varphi/2$. The interaction in the l th orbital angular momentum channel is

$$\Gamma_l = (2l+1)/2 \int d\varphi \sin \varphi P_l(\cos \varphi) \Gamma(\varphi) \quad (3)$$

Γ can be expanded in terms of the two-particle irreducible (2PI) vertex, Γ_0 (fig. 1). To leading order in $1/N$, Γ_0 consists of two parts: a dressed boson propagator and the matrix elements coupling this to the fermion operators. Because Γ_0 is order $1/N$, we need only consider contributions to Γ in which the smallness of Γ_0 is compensated by a logarithmic divergence arising from a particle-particle propagator. We find that logarithmic divergences only occur when both intermediate lines are in the lower band and the frequency ω and momentum p are such that $|\omega|, |E_{p1}| < \epsilon_f$. In this region we may, to logarithmic accuracy, replace the full interaction by its static limit, $\Gamma_0(q, \omega = 0)$, and cut off the energy integrals at $\epsilon = \epsilon_F$.

After decomposing Γ_0 as in eq. (3) we may immediately write

$$\Gamma_l = \Gamma_{0l}/(1 + \rho\Gamma_{0l} \ln \epsilon_F/T) \quad (4)$$

The criterion for l -wave superconductivity is $\Gamma_{0l} < 0$.

We have computed $\Gamma_0(q, \omega = 0)$, we find (if

Fig. 1 Expansion of Γ in terms of Γ_0 . The solid lines with arrows are non-interaction Green's functions, p_i and ω_i vary over all possible values. The wiggly line is the dressed boson propagator discussed in refs. [6,7]

$$\alpha = q_0/\mu\rho_0)$$

$$\rho\Gamma_{0s} = 1/N - 0.2\alpha/N, \quad \rho\Gamma_{0p} = 0.1\alpha/N,$$

$$\rho\Gamma_{0d} = -0.07\alpha/N \quad (5)$$

$\Gamma_{0s,p} > 0$, but $\Gamma_{0d} < 0$, the Γ_{0l} are very small for $l > 2$. We therefore expect the boson mediated interaction to give d-wave ($l = 2$) superconductivity.

We now extend the Anderson model to include Coulomb interactions involving the c electrons. Two additional terms are possible: we write them $U_{cc}n_c(r)n_c(r')$ and $U_{fc}n_f(r)n_c(r')$. The f - f interaction is already included in the Anderson model. We are interested in the short range part of the interaction, U_{cc} and U_{fc} are therefore phenomenological fermi liquid parameters expected to be weak $N\rho_0 U < 1$. The U_{fc} term is negligible [8]. A straightforward extension of the usual derivation of the Coulomb pseudopotential [9] leads to a new effective interactions for $l = 2$

$$\lambda_{\text{eff}} = -\rho\Gamma_{0d} - (m/m^*)\mu^*/N \quad (6)$$

Here $\mu^* = \mu/(1 + \mu L n(W/\sigma_0))$ is the $l = 2$ projection of $N\rho_0 U_{cc}$, rescaled via the Anderson-Morel scaling which operates from W to σ_0 , where the electrons lose their c character and the scaling breaks down. The factor (m/m^*) comes from the combination of the small weight of the c electrons at the Fermi surface and the density of states enhancement.

Eq. (6) immediately yields

$$T_c = \epsilon_f \exp(-1/\lambda_{\text{eff}}) \quad (7)$$

We comment: (i) although the attractive part of our interaction is of order $q_0/N \sim 1/N^2$, our calculation, which included only $1/N$ contributions to Γ_0 , is consistent. Any two boson contributions to Γ_0 would give a term in order $q_0/N^2 = 1/N^3$ for $\Gamma_{0l \neq 0}$. (ii) The attractive pairing interaction is specific to the lattice: in the l -impurity version of our model, one finds the s -channel interaction is repulsive and all other interaction amplitudes are zero. (iii) Because the Coulomb term is multiplied by m/m^* , it is negligible in the Kondo limit, $\eta = -E_0/\rho_0 V^2 \gg 1$, and superconductivity is possible despite the weakness of the pairing interaction. Using eqs. (3), (6), (7) one can show that as

η decreases from ∞ , T_c must increase, pass through a maximum at some $\eta = \eta_{\max}$ and then decrease (iv) It seems possible to vary η experimentally by applying pressure, P . In Ce and U compounds, $d\eta/dP < 0$ [1]. In CeCu_2Si_2 $T_c(P)$ has the qualitative behavior expected if $\eta(P=0) > \eta_{\max}$ [10,11], In UPt_3 [12] and UBe_{13} [13], $T_c(P)$ monotonically decreases with increasing P , as expected if $\eta(P=0) < \eta_{\max}$.

In conclusion, we have shown that in the Anderson lattice there is a non-phonon mechanism which leads to a weak, d-wave superconducting instability, at least within our $1/N$ model. The superconducting instability is specific to the heavy fermion state, and is due to slave boson exchange.

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