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***d*-Wave Superconductivity in the Large-Degeneracy Limit of the Anderson Lattice**

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We show, within a Kondo-boson $1/N$ expansion, that the infinite- U lattice Anderson model is unstable to d -wave superconductivity in the limit in which the model describes heavy-fermion behavior. The pairing mechanism is Kondo-boson exchange between heavy quasiparticles found in mean-field theory. The model is extended to include the Coulomb pseudopotential.

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The heavy-fermion superconductors have been of considerable and continuing interest to condensed-matter physicists since their discovery seven years ago.¹ It is now believed that the superconducting state in the heavy-fermion materials is "unconventional" in two senses. One sense has to do with the form of the gap function, which appears to vanish along points or lines on the Fermi surface,² unlike the gap in conventional superconductors, which is essentially isotropic over the Fermi surface. One infers from this that heavy-fermion superconductivity involves electrons paired in a state of nonzero angular momentum, unlike conventional superconductors, in which the electrons form s -wave pairs. The other, perhaps more fundamental, unconventionality is that the pairing interaction that causes heavy-fermion superconductivity is unlikely to be the conventional electron-phonon interaction that leads to superconductivity in ordinary metals.³

In this paper we start from the $U = \infty$ lattice Anderson

model, which is believed³ to contain the essential physics of the heavy-fermion materials and which contains only electron-electron interactions. We show within a "slave boson" large-degeneracy expansion, originally devised for the one-impurity Anderson model^{4,5} and recently extended to a lattice,^{6,7} that the model is unstable with respect to d -wave superconductivity.

The $U = \infty$ Anderson model describes an essentially structureless band of conduction (c) electrons hybridizing with a localized set of f electrons and subject to the constraint that the number of f electrons on any lattice site i , $n_f^i \leq 1$. To obtain a tractable theory we have assumed both c and f electrons have "spin N ." In the slave-boson technique, one introduces a new Bose field, $b_i^\dagger$, which creates a hole on the f orbital on site i , replaces the constraint $n_f^i \leq 1$ by

$$Q_i = n_f^i + n_b^i = 1, \quad (1a)$$

and writes the Hamiltonian

$$H = \sum_{k,m} \epsilon_k c_{km}^\dagger c_{km} + E_0 f_{km}^\dagger f_{mk} + \left[\frac{V}{\sqrt{N}} \sum_{k,q,m} c_{km}^\dagger f_{k+q,m} b_q^\dagger + \text{H.c.} \right]. \quad (1b)$$

All energies are measured from the chemical potential, μ . We are interested in the Kondo limit, in which $\eta = -E_0/\rho_0 V^2 > 1$. ρ_0 is the c -electron density of states. The model given by Eqs. 1(a) and 1(b) is equivalent to the original Anderson model, but may be studied with use of conventional field-theoretical techniques. We note that the f operator in Eq. (1) does not have a simple physical meaning; the creation operator for a physical f state is given⁴ by $(f^\dagger b)$.

One studies the low-temperature properties of the system (1) by adding to Eq. 1(b) a Lagrange multiplier field λ to enforce the constraint (1a), and then decomposing b and λ into mean-field and fluctuating components. If one retains only the mean-field parts of b and λ , the model may easily be solved. This mean-field theory is exact as $N \rightarrow \infty$.^{4,5} Corrections to the mean-

field theory are due to interactions between the fermions and the fluctuating parts of the Bose operators; these corrections may be expanded in powers of $1/N$. The problem may be formulated in either the Cartesian or the radial gauge. In the former we write $b = \sqrt{N}(a + A)$ and consider A as the fluctuating field. In the Cartesian formulation, infrared divergences appear at intermediate stages of many calculations and must be carefully canceled from physical results.^{6,8} It is often more convenient to use the radial gauge,^{5,7} in which one rewrites the problem in terms of new Bose fields r_i and θ_i defined via $b_i^\dagger = r_i e^{i\theta_i}$ and a new Fermi operator $f'_i = f_i e^{i\theta_i}$. Then, following Ref. 5, we may (after some algebraic manipulation) rewrite the problem in functional integral form with a Lagrangean given, as a sum of

fermion, boson, interaction, and constraint terms, by $L(t) = L_F + L_B + L_I + L_C$, where

$$L_F = \sum_{k,m} (c_{km}^\dagger f_{km}^\dagger) \begin{pmatrix} \partial_\tau + \varepsilon_k & \sigma_0 \\ \sigma_0 & \partial_\tau + \varepsilon_f \end{pmatrix} \begin{pmatrix} c_{km} \\ f_{km}' \end{pmatrix}, \quad (2a)$$

$$L_B = \frac{N}{V^2} \sum_k (\sigma_k \lambda_k) \begin{pmatrix} \varepsilon_f - E_0 & i\sigma_0 \\ i\sigma_0 & 0 \end{pmatrix} \begin{pmatrix} \sigma_{-k} \\ \lambda_{-k} \end{pmatrix}, \quad (2b)$$

$$L_I = \sum_{k,k',m} (c_{km}^\dagger f_{km}^\dagger) \begin{pmatrix} 0 & \sigma_{k-k'} \\ \sigma_{k-k'} & i\lambda_{k-k'} \end{pmatrix} \begin{pmatrix} c_{k'm} \\ f_{k'm}' \end{pmatrix} + \frac{N}{V^2} \sum_{k,k'} i\lambda_k \sigma_{-k'} \sigma_{k'-k}, \quad (2c)$$

$$L_C = N \left(\frac{\sigma_0^2}{V^2} - q_0 \right) (\varepsilon_f - E_0). \quad (2d)$$

Here σ and λ are real scalar bose fields related to the fluctuating parts of Vr and $(\partial\theta/\partial\tau + \lambda)$; $\sigma_0 = Va$ and ε_f are parameters determined by minimizing the free energy and related to the static parts of Vr and $(\partial\theta/\partial\tau + \lambda)$. q_0 is defined as Q_i/N . The model defined by Eqs. (2) is only equivalent to the original Anderson model if $q_0 = 1/N$, but to obtain a sensible $1/N$ expansion we must regard q_0 as a parameter independent of the explicit N in Eqs. (4), and only set $q_0 = 1/N$ at the end of a calculation.⁶

The mean-field theory corresponds to electrons moving in the band structure determined by L_F . This band structure has a gap centered at ε_f . To leading order in $1/N$ the parameters σ_0 and ε_f are (in the limit $\eta \gg 1$)

$$\varepsilon_f = W \exp(-|E_0|/\rho_0 V^2), \quad (3a)$$

$$\sigma_0^2 = q_0 V^2 (1 - n_f) (1 - n_f^2 \alpha/4) + O(q_0^3), \quad (3b)$$

$$n_f = (1 + \varepsilon_f/\rho_0 V^2)^{-1}. \quad (3c)$$

W is the energy of the bottom of the c -electron band, measured from the chemical potential, μ . ρ_0 is the c -electron density of states at $\varepsilon_k = \mu$. n_f is the mean-field, leading order in q_0 , number of f electrons. It is not the same as the physical number of f electrons.⁹ $\alpha = q_0/\rho_0 W$. We assume one has ~ 1 c electrons per spin channel. α is thus $O(q_0)$. ε_f , the smallest energy scale in the theory, is the effective Fermi energy for the low-lying quasiparticle excitations.⁶ σ_0 is the effective hybridization. We have assumed that the Fermi level lies in the lower band; essentially equivalent results are obtained if it lies in the upper one. It is convenient to work with the quasiparticle operators d_{k1} and d_{k2} which diagonalize H_0 and pertain to the lower and upper bands respectively. One finds $d_{ki} = \cos\theta_{ki} c_{ki} + \sin\theta_{ki} f_{ki}'$, where $(E_{ki}$ is the energy in band i) $\tan\theta_{ki} = (E_{ki} - \varepsilon_k)/\sigma_0 = \sigma_0/(E_{ki} - \varepsilon_f)$.

The density of states at the Fermi surface, ρ , is very large; the quasiparticles are therefore heavy fermions of mass enhancement $m^*/m = (\sigma_0/\varepsilon_f)^2 \gg 1$. The spectral weight of the c electrons at the Fermi surface is $m/m^* \ll 1$. The mass enhancement appears in the specific-heat coefficient γ , which may be written in the

usual Fermi-liquid form $\gamma = Nk_B m^* k_F/6$ or, if γ is normalized to the number of f -electron sites, $\gamma = \pi^2 k_B^2/3\varepsilon_f$.

Corrections to the mean-field theory are due to electron-boson interactions, which are small because the boson propagator contains a factor of $1/N$. However, we shall show that the electron-boson interaction leads to d -wave superconductivity in the model defined by Eqs. (2). We proceed by investigating divergences in the scattering amplitude, Γ , for two quasiparticles of opposite momenta near the Fermi surface. We note that the quasiparticle scattering amplitude is conventionally decomposed into spin singlet and triplet channels; this is meaningless in our spin- N model. An alternative procedure is to decompose the scattering amplitude into orbital angular momentum channels only. This is justified because the boson propagator does not carry spin.

We write the series for Γ in Fig. 1; Γ is a function only of angle between k and k' , φ . If $q = k' - k$, then $|q| = 2k_F \sin\varphi/2$. We define the interaction in the l th orbital-angular-momentum channel by

$$\Gamma_l = [(2l+1)/2] \int d\varphi \sin\varphi P_l(\cos\varphi) \Gamma(\varphi). \quad (4)$$

As shown in Fig. 1, Γ may be written as a ladder sum of particle-particle propagators linking the two-particle-irreducible (2PI) vertex Γ_0 . To leading order in $1/N$ it is given by the sum of graphs shown in Fig. 2. The resulting expressions are lengthy due to the matrix nature of the boson propagator, and will be presented in a longer paper. We note that Γ_0 is of order $1/N$; in fact, that part

FIG. 1. Expansion for the particle-particle scattering amplitude Γ in terms of the two-particle-irreducible (2PI) amplitude Γ_0 . The solid lines with arrows are electron Green's functions; they may be in either band. To leading order in $1/N$ they may be approximated by noninteracting Green's functions. $|k| = |k'| = k_F$; p_i and ε_i vary over all possible values.

FIG. 2. Expression for the 2PI amplitude Γ_0 . The wiggly line is the full boson propagator. The dashed line represents the matrix of bare-boson propagators, obtained by inverting L_B , Eq. 2(b). The bubble represents the various possible fermion loops that make up the leading-order boson self-energies; they are computed with use of noninteracting Green's functions, obtained from L_F [Eq. 2(a)] and are coupled to the boson propagators via the first term of L_I [Eq. 2(c)]. The momentum across the boson line is $q = k' - k$.

of Γ_0 which varies with the momentum transfer across the boson line is of order q_0/N . Thus we need only consider contributions to the full vertex Γ in which the smallness of the 2PI vertex is compensated by a logarithmic divergence arising from a particle-particle propagator. We find that logarithmic divergences only occur

$$\Gamma_0(q, \varepsilon=0) = \frac{1}{N\rho} \left[1 - (1 - n_f)^2 + n_f^2 \alpha \left(1 - \frac{4}{3} n_f + \frac{1}{2} n_f^2 \right) - \frac{n_f^2 \alpha}{12} f(q/2k_F) (3n_f^2 - 9n_f + 7) \right], \quad (6)$$

where $f(x) = (1/x) \ln |(1+x)/(1-x)|$.

For the Γ_{0l} we find $\rho\Gamma_{0s} > 0$, $\rho\Gamma_{0p} > 0$, and $\rho\Gamma_{0d} = -0.01\alpha/n_f^2 (3n_f^2 - 9n_f + 7)$; the Γ_{0l} are very small for $l > 2$. We therefore expect the boson-mediated interaction to give d -wave ($l=2$) superconductivity. We note that the s -channel interaction is repulsive because of the hard-core repulsion built into the model by the constraint 1(a). We also note that in an exchange model (such as the one we consider) $\rho\Gamma_{0l} = -A^l$, where A^l is the antisymmetric Landau parameter. Our results for the Landau parameters agree with those of Ref. 7, in the limit $n_f \rightarrow 1$.

Note that the quasiparticles involve the f' operators, which exist only in the radial gauge, and whose physical meaning is obscure. Note also that Γ is infrared divergent if calculated in the Cartesian gauge. Thus, one may wonder about the meaning of any computation of Γ . A gauge-invariant, physically meaningful, criterion for L -wave superconductivity is the divergence of the correlation function

$$\left\langle \sum_k Y_{Lm}(k) c_{km}^\dagger c_{km}^\dagger \sum_{k'} Y_{Lm}(\hat{k}') c_{k'm} c_{k'm} \right\rangle,$$

which involves only c operators and is therefore a physical quantity. We have computed this quantity to leading and next to leading orders in $1/N$ in both the radial and Cartesian gauge formulations. We obtain a series proportional to $\ln(\varepsilon_f/T) + \lambda \ln^2(\varepsilon_f/T)$. By assuming that the series can be summed to give $\ln(\varepsilon_f/T)[1 - \lambda$

when both intermediate lines are in the lower band and the energy ω and momentum p are such that $|\omega|$, $|E_{p1}| < \varepsilon_f$. In this region the 2PI vertex Γ_0 is essentially independent of $|\omega|$ and $|p|$ and also of ε , the energy across the boson line. $\Gamma_0(q, \omega)$ does have structure at small q and ω due to a zero-sound collective mode; because of the high velocity of this mode, it contributes only for a negligible region of q , and is thus unimportant for the pairing interaction. In the weak-coupling electron-phonon problem, we may, to logarithmic accuracy, replace the full interaction by its static limit, $\Gamma_0(q, \varepsilon=0)$, and cut off the energy integrals at $\varepsilon = \varepsilon_f$. We emphasize that unlike the usual electron-phonon problem, the upper cutoff comes from the curvature of the quasiparticle band as well as from the ω -space structure of the interaction $\Gamma_0(q, \omega)$.

After decomposing Γ_0 as in Eq. (4) and projecting onto the l orbital angular momentum channel, we may immediately write

$$\Gamma_l = \Gamma_{0l} [1 + (\rho\Gamma_{0l}/2l + 1) \ln(\varepsilon_f/T)]^{-1}. \quad (5)$$

The criterion for l -wave superconductivity is $\Gamma_{0l} < 0$.

We have computed $\Gamma_0(q, \varepsilon=0)$; assuming $\varepsilon_k \sim k^2$, and expanding for small q_0 , we find

$\times \ln(\varepsilon_f/T)]^{-1}$ we obtain a criterion for superconductivity which is identical to that obtained from the calculation of Γ above. This indicates that the superconducting instability found among quasiparticles defined in the radial gauge corresponds to a physical superconducting instability.

We now extend the Anderson model to include Coulomb interactions involving the c electrons. Two additional terms are possible: We write them $U_{cc} n_c(r) n_c(r')$ and $U_{cf} n_f(r) n_c(r')$. The ff interaction is already included in the Anderson model. We are interested in the short-range part of the interaction; U_{cc} and U_{cf} are therefore phenomenological Fermi-liquid parameters. We expect these residual interactions to be weak; thus $N\rho_0 U < 1$. (The factor N arises because ρ_0 is the density of states per spin.) We have shown that the term in U_{cf} is negligible compared to that in U_{cc} by using the constraint (1) to rewrite n_f in terms of Bose operators. A straightforward extension of the usual derivation of the Coulomb pseudopotential shows that the effects of U_{cc} may be incorporated by writing a new effective interaction, which in the $l=2$ channel is

$$\lambda_{\text{eff}} = -[\rho\Gamma_{0d} + (m/m^*)\mu^*/N]/(2l + 1). \quad (7)$$

Here $\mu^* = \mu/(1 + \mu \ln W/\sigma_0)$ is the $l=2$ projection of $N\rho_0 U_{cc}$, rescaled via the Anderson-Morel scaling. The factor (m/m^*) comes from the combination of the small

weight of the c electrons at the Fermi surface and the density-of-states enhancement.

Equation (7) immediately yields

$$T_c = \epsilon_f \exp(-1/\lambda_{\text{eff}}). \quad (8)$$

We make the following comments: (i) Although the attractive part of our interaction is of order $q_0/N \sim 1/N^2$, our calculation, which included only $1/N$ contributions to Γ_0 , is consistent. This is because the dressed-boson propagator may be written as the sum of two terms: one independent of the momentum q carried by the boson, and one, of order q_0 relative to the first, which does depend on q . Therefore any two boson contributions to Γ_0 would contribute to $\Gamma_{0l=0}$ in order $q_0/N = 1/N^2$ but would only contribute to $\Gamma_{0l \neq 0}$ in order $q_0/N^2 = 1/N^3$. (ii) The attractive pairing interaction is specific to the lattice: In the one-impurity version of our model, one finds that the s -channel interaction is repulsive and all other interaction amplitudes are zero. (iii) Equation (8) is closely related to the Patton-Zaringhala (PZ) formula¹⁰ and its generalization by Levin and Valls.¹¹ The PZ formula is a phenomenological expression for T_c in terms of Landau parameters under a particular assumption about extrapolating the quasiparticle interaction amplitude away from the forward scattering limit, and with the upper cutoff a parameter to be determined. Levin and Valls¹¹ have generalized the PZ formula to the "exchange model" case in which the interaction amplitude depends only on the momentum and frequency exchanged by the interacting particle, and have derived the generalized PZ formula for the T_c of ^3He from a microscopic model in which quasiparticles exchange paramagnons. Our model is an exchange model; however, we emphasize that in our approach, as well as that of Levin and Valls, the starting point is a microscopic model which yields an expression for the full frequency-dependent interaction $\Gamma(q, \omega)$ from which the interaction and (up to factors of order unity) the cutoff can be derived.¹² (iv) As in the usual weak-coupling theory of superconductivity, our calculation is valid to leading logarithmic order; thus we have determined the leading term (in $1/N$ in λ_{eff} and have determined the cutoff up to factors of order unity. (v) The values obtained for $\rho\Gamma_{0l}$ are nonuniversal, and depend on details of band structure, of hybridization, etc. (vi) Because the Coulomb term is multiplied by m/m^* , it is negligible in the limit $\eta \gg 1$, and superconductivity is possible despite the weakness of the pairing interaction. As $\eta \rightarrow \infty$, however, Eqs. 3(a), 3(b), and (8) show that the decrease in ϵ_f drives T_c to zero. As η decreases towards the intermediate valence regime $\eta \sim 1$, although ϵ_f increases, n_f and m^* decrease, and superconductivity is destroyed. Note that the leading corrections to λ_{eff} are proportional to $(1 - n_f)^2$ for $n_f \cong 1$. Thus, as η decreases from ∞ T_c must increase, pass through a maximum at some $\eta = \eta_{\text{max}}$, and then decrease. (vii) It seems possible to

vary η experimentally by applying pressure P . In Ce and U compounds, $d\eta/dP < 0$.² In CeCu_2Si_2 , $T_c(P)$ has the qualitative behavior expected if $\eta(P=0) > \eta_{\text{max}}$: It initially increases with pressure, then passes through a maximum and decreases.^{13,14} In UPt_3 ¹⁵ and UBe_{13} ,¹⁶ $T_c(P)$ monotonically decreases with increasing P , as expected if $\eta(P=0) < \eta_{\text{max}}$.

In conclusion, we have shown that in the Anderson lattice there is a nonphonon mechanism which leads to a weak, d -wave superconducting instability, at least within our $1/N$ model. The superconducting instability is specific to the heavy-fermion state, and is due to slave-boson exchange. We note that the slave boson does not carry spin: The superconductivity which we find is not associated with spin fluctuations. Rather, it is due to f -density fluctuations subject to the infinite- U constraint.

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