Interchange destabilization of collisionless tearing modes by temperature gradient
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Using a fluid theory, the stability of collisionless tearing modes in plasmas is analyzed in the presence of an inhomogeneous magnetic field, electron temperature and density gradients. It is shown that small scale modes, characterized by a negative stability parameter ($\Delta' < 0$), can be driven unstable due to a combination of the magnetic field and electron temperature gradients. The destabilization mechanism is identified as of the interchange type similar to that for toroidal Electron Temperature Gradient modes.

Tearing modes\textsuperscript{1,2} are instabilities that can occur in fusion plasmas in the presence of non-ideal effects (such as resistivity or inertia). They are responsible for change of the topology of magnetic fields\textsuperscript{4} and lead to the formation of magnetic islands through magnetic reconnection processes\textsuperscript{5}. Particles can then follow the perturbed field lines inside the magnetic islands connecting the inner and outer regions. This increases the radial transport from the core and causes a degradation in confinement\textsuperscript{4} and can possibly lead to disruption\textsuperscript{8}. The tearing mode instability has been observed in a wide variety of astrophysical and laboratory plasmas\textsuperscript{7} and is thought to be responsible for the reconnection processes in the Earth magnetotail\textsuperscript{18}. It has been extensively studied analytically within the framework of magnetohydrodynamics (MHD) since the seminal work by Furth et al.\textsuperscript{2}. The tearing mode is linearly excited by the radial gradient of the equilibrium parallel current. The radial domain is separated into an ideal region fully described by ideal MHD equations and a narrow resonant region inside which non-ideal effects take place and the perturbed parallel current in highly localized. The tearing mode stability is commonly parametrized by $\Delta'$, a parameter calculated from the solution of the tearing mode equations in the ideal outer region. It is defined as the jump in the logarithmic derivative of the parallel vector potential $A_\parallel$ across the non-ideal region, inside which $A_\parallel$ is assumed constant. This assumption is known as the constant-$\psi$ approximation\textsuperscript{2}, where $\psi$ is the parallel scalar potential of the magnetic field. Generally, large scale modes with low poloidal number $m$, may have $\Delta' > 0$. They are driven by the free energy in the outer region. The stability of such modes has been calculated in the collisionless limit in the framework of fluid theory\textsuperscript{9-11} as well as kinetic theory\textsuperscript{12-14}. On the other hand, small scale tearing modes with high $m$ are not much affected by the large scale current density gradient in the outer region. Such modes are characterized by a negative $\Delta' \approx -2k_y = -2m/r < 0$ where $k_y$ is the perpendicular wave vector and $r$ the minor radius. Therefore the high $m$ modes with $\Delta' > 0$ would be stable. It was shown previously that high $m$ tearing modes can be driven linearly unstable\textsuperscript{15-17} by the thermal force effects related to collisions\textsuperscript{18,19}. Such modes were called microtearing modes. It is important to note that the thermal force destabilization (due to the energy dependence of the Coulomb collision frequency) is related to the current contribution in the inner tearing mode layer but not to the outer ideal region (parameterized by the value of $\Delta'$).

Current and future tokamaks are characterized by weakly collisional scenarios which makes it important to understand the stability of tearing modes in such a limit. Recently a large body of gyrokinetic simulations have indicated the presence of an additional, collisionless, destabilization mechanism for small scale micro-tearing modes, likely related to magnetic gradients\textsuperscript{20-22}. In this paper we investigate the collisionless destabilization mechanism due to magnetic field inhomogeneity and plasma gradients with a fluid theory.

Near the tokamak rational surfaces one can introduce a slab-like geometry with a Cartesian coordinates system ($\hat{x}, \hat{y}, \hat{z}$). The magnetic field can then be represented in the form

$$B = B_0(x)\hat{z} + \nabla \psi \times \hat{z}$$

(1)

$$\psi = \frac{1}{2}B_0 x^2 + \bar{A}_\parallel(x, y, t)$$

(2)

where $B_0$ is the equilibrium magnetic field along the $z$-direction, $\bar{A}_\parallel$ is the $z$-component of the perturbed magnetic vector potential, $\psi$ is the auxiliary vector potential introduced to describe the magnetic shear effect for the helical perturbations, $k_y = m (\theta - \zeta/q)$, $L_s = qR/s$, $q$ is the safety factor, $s = (r/q)dq/dr$ is the magnetic shear, $R$ is the tokamak major radius and $x = r - r_s$ the distance to the resonant surface position $r_s$. With Eq. 2, the total parallel gradient operator along the magnetic field $\hat{\nabla}_\parallel = (\hat{B}_0 + \vec{B}) \cdot \hat{\nabla}/B$ can be split into the linear (shear magnetic field) and nonlinear parts as follows:

$$\nabla_\parallel = \frac{\hat{B}_0 \cdot \hat{\nabla}}{B_0} = ik_y \frac{x}{L_s}$$

(3)

$$\hat{\nabla}_\parallel = \frac{\vec{B} \cdot \hat{\nabla}}{B_0} = i\frac{k_y A_\parallel}{B_0} \frac{\partial}{\partial x}$$

(4)
The electron dynamics is described by standard fluid equations in the absence of collisions and neglecting the electrostatic potential. Similarly to Ref. 13, we neglect the contribution to the parallel current due to the electrostatic potential which is valid for small scale magnetic islands. The electron continuity equation takes the form

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) + \nabla (n_e V_{\parallel e}) = 0 \quad (5)$$

The parallel electron velocity $V_{\parallel e}$ is found from the electron parallel momentum balance equation, given by

$$m_e n_e \left[ \frac{\partial V_{\parallel e}}{\partial t} + \mathbf{b} \cdot (\mathbf{V} \cdot \nabla V) \right] = e n_e E_{\parallel} - \nabla p_e \quad (6)$$

where $m_e$ is the electron mass, $n_e$ the electron density, $\mathbf{b} = \mathbf{B}/B_0$ is the unit vector in the equilibrium magnetic field direction, $E_{\parallel} = -\partial A_{\parallel}/c\partial t$ is the parallel electric field and $p_e$ is the electron pressure. In the low frequency regime, i.e. $\omega \ll \omega_{ce}$ where $\omega_{ce} = m_e c/eB > 0$ is the electron cyclotron frequency and neglecting the electrostatic potential, the electron perpendicular velocity is the diamagnetic drift.

To the lowest order, it is given by

$$\mathbf{v}_{\perp e} = -\frac{c}{en_0 e} \frac{\mathbf{b} \times \nabla p_e}{B_0} \quad (7)$$

The second term in the continuity equation (5) can be written

$$\nabla \cdot (n_e \mathbf{v}_e) = \frac{1}{T_{0e}} \mathbf{v}_{De} \cdot \nabla p_e \quad (8)$$

where $\mathbf{v}_{De}$ is the electron magnetic drift velocity given by

$$\mathbf{v}_{De} = -\frac{2cT_{0e}}{eB_0} \mathbf{b} \times \nabla \ln B_0 \quad (9)$$

Linearizing Eqs. (5) and (6) gives

$$\frac{\partial \tilde{n}_e}{\partial t} + n_{0e} \mathbf{v}_{De} \cdot \nabla \tilde{p}_e + \nabla \| (n_{0e} \tilde{V}_{\parallel}) = 0 \quad (10)$$

$$m_e n_{0e} \left[ \frac{\partial \tilde{V}_{\parallel e}}{\partial t} - \frac{A_{\perp 0 e}}{n_{0e} e B_0} \mathbf{b} \cdot \nabla \ln B \cdot \nabla \tilde{V}_{\parallel e} \right] = -e n_{0e} \tilde{E}_{\parallel} - \nabla \| \tilde{p}_e - \nabla \| \tilde{p}_{0e} \quad (11)$$

where $\tilde{p}_e = n_{0e} T_{0e}$ and $\tilde{p}_{0e} = \tilde{n}_{0e} T_{0e} + n_{0e} \tilde{T}_e$, are respectively the equilibrium and perturbed pressure. Unless stated otherwise, all quantities in this text refer to electrons. The subscript $e$ to designate electrons will be dropped in the following.

In the simplest case of neglecting magnetic field gradient and temperature perturbations these equations read

$$- \omega \tilde{n} + n_{0e} k_{\parallel} \tilde{V}_{\parallel} = 0, \quad (12)$$

$$m_e n_{0e} \omega \tilde{V}_{\parallel} = \frac{en_{0e}}{c} \omega \tilde{A}_{\parallel} + k_{\parallel} T_{\tilde{n}}, \quad (13)$$

giving the following parallel electron response in terms of the perturbed magnetic potential

$$\tilde{V}_{\parallel} = -\frac{e}{cm_e k_{\parallel}^2} \omega^2 \tilde{A}_{\parallel}. \quad (14)$$

As it will be shown below, the limit in Eq.(14) reproduces the collisionless tearing mode instability studied by the kinetic theory in Refs. 12 and 13 and in the fluid theory in Refs. 9, 10 and 25. The reconnection here is driven by the parallel electron current due to inertia balanced by the inductive electric field and electron pressure perturbation. The parallel electron current is accompanied by electron density perturbation. When the gradient of the magnetic field is included, there is an additional contribution of the parallel current which comes from the compressibility of the perpendicular electron diamagnetic current in equation (10).

The pressure perturbation needs a closure for the temperature evolution. A reasonable closure in our case is to consider constant temperature along the perturbed magnetic surface. In the linear limit this takes the form

$$\nabla \| \tilde{T} + \nabla \| \tilde{T}_0 = 0 \quad (15)$$

The electron continuity and parallel momentum balance, Eqs. (10) and (11) together with the closure on the temperature (15) read

$$-(\omega - \omega_D) \tilde{n} + \omega_D \omega_{\ast T} \frac{en_{0}}{ck_{\parallel}} \tilde{A}_{\parallel} + n_{0e} k_{\parallel} \tilde{V}_{\parallel} = 0 \quad (16)$$

$$m_e n_{0e} (\omega - 2\omega_D) \tilde{V}_{\parallel} = \frac{en_{0}}{c} (\omega - \omega_{\ast n}) \tilde{A}_{\parallel} + k_{\parallel} T_{\tilde{n}} \tilde{V}_{\parallel} + k_{\parallel} T_{\tilde{n}} \tilde{T}_0 = 0 \quad (17)$$

$$\omega_{\ast T,n} = -\frac{ck_{\parallel} T_0}{eB_0} \frac{\partial \ln T_{\tilde{n}}}{\partial x} = -k_{\parallel} n_{0e} \frac{v_{th}}{L_{T,n}} \quad (19)$$

$$\omega_D = -\frac{ck_{\parallel} T_0}{eB_0} \frac{\partial \ln B_0}{\partial x} = -2k_{\parallel} n_{0e} \frac{v_{th}}{L_B} \quad (20)$$

Here $\rho_e = m_e c v_{th}/eB_0$ is the thermal electron Larmor radius, $v_{th} = \sqrt{T_0/m_e}$ is the electron thermal velocity, $L_{B,n,T}$ is the gradient length scale of the magnetic field, density and temperature respectively, defined by $L_G = -(1/G) \partial G/\partial x$.

The above system of fluid equations is closed with Ampère’s law. Projected onto the direction parallel to the equilibrium magnetic field, it takes the form

$$-\nabla \| \tilde{A}_{\parallel} = \frac{4\pi}{c} \tilde{J}_{\parallel}, \quad (21)$$

Here $\tilde{J}_{\parallel} = -en_{0e} \tilde{V}_{\parallel}$ is the perturbed electron parallel current which contains the destabilizing effect of the collisionless tearing mode. The perturbed electron parallel velocity is
calculated by coupling the system of Eqs. (16), (17) and (18)

\[ \bar{V}_\parallel = - \frac{e}{cm_e} k^2 \bar{\omega}_\parallel^2 \left( \omega_\perp^2 - (\omega - 2\omega_D)(\omega - \omega_D) \right) \bar{A}_\parallel \]

(22)

For small scale islands \( \partial / \partial x \gg k_y \), Eq. (21) is written

\[ \frac{\partial^2 \bar{A}_\parallel}{\partial x^2} = - \frac{\omega_{pe}^2}{c^2} k^2 \bar{\omega}_\parallel^2 \left( \omega_\perp^2 - (\omega - 2\omega_D)(\omega - \omega_D) \right) \bar{A}_\parallel \]

(23)

where \( \omega_{pe} = \sqrt{4\pi e^2 n_0 / m_e} \) is the electron plasma frequency.

The tearing mode dispersion relation is obtained by integrating Ampère’s law across the resonant layer. We employ the constant-\( \psi \) approximation which consists in assuming that in the non-ideal region the perturbed parallel vector potential does not vary significantly. The collisionless tearing mode dispersion relation then reads

\[ \Delta' + \frac{\omega_{pe}^2}{c^2} \int_{-\infty}^{\infty} dx \left( \frac{(\omega - \omega_\perp)(\omega - \omega_D) + \omega_D\omega_T}{\omega_\perp^2 - (\omega - 2\omega_D)(\omega - \omega_D)} \right) = 0 \]

(24)

Here \( \kappa_\parallel^2 = k_y s / qR \) and \( \Delta' \) is the tearing mode stability parameter that matches the solutions in the ideal outer region and non-ideal inner region. It is defined as the jump in the logarithmic derivative of \( \bar{A}_\parallel \), solution in the outer region, across the resonant layer, i.e.

\[ \Delta' = \lim_{\varepsilon \to 0} \frac{\partial \ln \bar{A}_\parallel}{\partial x} \bigg|_{r_s+\varepsilon}^{r_s-\varepsilon} \]

(25)

The x-integral in Eq. (24) is converging for complex \( \omega \). Using the properties of the complex logarithm one can obtain the expression

\[ \int_{-\infty}^{\infty} \frac{dx}{k^2 \Delta^2} = \begin{cases} \frac{i\pi}{|k_{\parallel \perp} \Omega|} & \text{for } \Im(\Omega) > 0 \\ -\frac{i\pi}{|k_{\parallel \perp} \Omega|} & \text{for } \Im(\Omega) < 0 \end{cases} \]

(26)

where we have introduced the notation \( \Omega^2 = (\omega - \omega_D)(\omega - 2\omega_D) \).

The expression (26) can be better understood by integrating in the complex plane and using the residue theorem. For this purpose, we extend the integration domain over a given real interval \([x_1, x_2]\) to the complex plane

\[ \int_{x_1}^{x_2} \frac{dx}{x^2 - \Omega^2} + \int_{\Gamma} \frac{dz}{z^2 - \Omega^2} = \int_{C} \frac{dz}{z^2 - \Omega^2} \]

(27)

where we have defined \( \Gamma \) as a half-circle that lies in the upper (resp. lower) complex half-plane for \( \Im(\Omega) > 0 \) (resp. \( < 0 \)) and \( C \) is the closed contour \( C = [x_1, x_2] \cup \Gamma \). The integral over the closed contour in both the upper and lower halves of the complex plane can be calculated by the residue theorem. Therefore, we decompose the integrand as follows

\[ \int_{C} \frac{dz}{z^2 - \Omega^2} = \frac{1}{2\Omega} \left( \int_{C} dz \left( \frac{1}{z - \Omega} - \frac{1}{z + \Omega} \right) \right) \]

(28)

Depending on the sign of the imaginary part of \( \Omega \) the first or the second term on the right hand side defines the value of the integral,

\[ \frac{1}{2\Omega} \int_{C} dz \left( \frac{1}{z - \Omega} - \frac{1}{z + \Omega} \right) = \frac{1}{2\Omega} 2\pi i, \text{ for } \Im(\Omega) > 0 \]

(29)

\[ \frac{1}{2\Omega} \int_{C} dz \left( \frac{1}{z - \Omega} - \frac{1}{z + \Omega} \right) = -\frac{1}{2\Omega} 2\pi i, \text{ for } \Im(\Omega) < 0 \]

(30)

Finally, taking the limit \( x_1 \to -\infty \) and \( x_2 \to \infty \) and noticing that within this limit the integral over \( \Gamma \) vanishes, we
obtain the expression (26). The dispersion relation in Eq. (24) is therefore expressed as

$$\Delta' = \omega_{pe}^2 \frac{c^2}{\pi} \frac{\pi}{|}\gamma| \omega_r = \omega_{sn} \quad \omega_r = \omega_{sn} (1 + \eta_e/2)$$

(32)

Such a solution has been found for the collisionless tearing mode using a fluid model in Refs. 10, 9, and 25. The dispersion relation in Eq. (32) obtained from fluid theory is not fully identical to the kinetic result,12,13 which is

$$\Delta' = \omega_{pe}^2 \frac{c^2}{\pi} \frac{2\sqrt{\pi}}{k_\parallel v_{th}} |\gamma| \omega_r = \omega_{sn} (1 + \eta_e/2)$$

(33)

where $\eta_e = L_n/L_T$. The kinetic $2\sqrt{\pi}$ and fluid $\pi$ coefficients are quite close. The difference in the real part of the frequency, $\omega_r = \omega_{sn} (1 + \eta_e/2)$ in kinetic theory and $\omega = \omega_{sn}$ in fluid theory, is due to our approximation in Eq. (15) on the closure which is not uniformly valid for all $k_\parallel (x)$.

It is interesting to note that the solution in Eq. (31) for $\omega_D = 0$ is in fact

$$\Delta' = \frac{\omega_{pe}^2}{c^2} \frac{\pi}{|k_\parallel v_{th}} |\gamma|. \quad \omega_r = \omega_{sn}$$

(34)

Thus it means that there are two solutions; with positive and negative gamma, for $\Delta' > 0$ and no solutions exist for $\Delta' < 0$.

In the general case with $\omega_D \neq 0$ and $\omega_{sT} \neq 0$, there are two solutions which may have the additional destabilization mechanism due to the magnetic drift gradient and the temperature gradient, provided $\omega_{sT}\omega_D > 0$. The modes with $\Delta' > 0$ can be driven by the free energy from the outer region. However, the high $m$ modes have negative $\Delta'$, but they can be driven by temperature and magnetic field gradient effects. This destabilizing mechanism can be easily illustrated for the marginal stability case $\Delta' = 0$. In this case the solution of Eq. (31) is found as

$$\omega = \frac{\omega_{sn} + \omega_D}{2} \pm \sqrt{\left(\frac{\omega_{sn} - \omega_D}{2}\right)^2 - \omega_{sT}\omega_D}, \quad \omega_r = \omega_{sn} (1 + \eta_e/2)$$

(35)

which has the instability for sufficiently large values of $\omega_{sT}\omega_D > 0$. Thus, the additional destabilization mechanism can be identified of the toroidal ETG (interchange) type27 rather than the tearing type due to $\Delta' > 0$. Furthermore, when $\Delta'$ is negative and small, the approximate solution of Eq. (31) has the same tendency as Eq. (35), as follows from numerical solutions.

In the rest of the paper all quantities are presented in normalized units. Frequencies are normalized to the electron transit frequency $\omega_t = v_{th}/R_{0}$, $\Delta'$ is normalized to $d_e^2/\rho_e$ where $d_e = c/\omega_{pe}$ is the electron skin depth.

When $\omega_D$ is finite Eq. (31) is solved numerically by looking for unstable solutions, i.e. solutions with $\Im(\omega) = \gamma > 0$. We find the zeros of the left hand-side of Eq. (31) by plotting the inverse of its modulus in the complex $(\omega_r, \gamma)$ plane and looking for its poles as outlines in Refs. 28–30. We find that for finite $\omega_D$ the temperature gradient generally destabilizes the mode while the density gradient modifies the mode growth rate and becomes stabilizing for stronger gradients around
1/L_n \approx 6$, as can be seen from the general tendency of the curves in Fig. 2a.

Figs. 1 and 2 show a scan of the normalized growth rate (a) and frequency (b) with the temperature and density gradient scale lengths respectively, at the low field side (i.e. \( L_B, L_n, T > 0 \)). The different lines correspond to different modes, at the same resonant surface \( q(x) = m/n = 2 \). The poloidal mode number \( m \) is determined by the values of \( \Delta \) and \( k_y \rho_e \). The magnetic field gradient is fixed to \( 1/L_B = 1 \). The hard blue line represents the mode \( m = 2 \) driven unstable due to \( \Delta > 0 \) with \( \Delta = 0.5 \) and \( k_y \rho_e = 2.2891 \times 10^{-4} \). The dashed orange line represents the \( \Delta < 0 \) mode \( m = 10 \), with \( \Delta = -1.1360 \) and \( k_y \rho_e = 0.0011 \) and the purple line represents the \( \Delta < 0 \) mode \( m = 20 \), with \( \Delta = -2.2718 \) and \( k_y \rho_e = 0.0023 \).

The numerical solution shows that, in agreement with Eq. (35), unstable solutions for high-\( m \) modes are present only if the combined effect of magnetic field inhomogeneity \( (\omega_D) \) and plasma temperature gradient \( (\omega_{aT}) \) is considered. This can be seen in Fig. 1a where at null temperature gradient the growth rate of both high-\( m \) modes (dashed orange and dotted purple lines) is equal to zero. Note that the growth rate and real frequency scale linearly with \( k_y \rho_e \). This indefinite increase would be terminated by the finite electron Larmor radius effects which are neglected here. Figure 2b shows that the mode’s frequency varies very little against temperature gradient variations and is linear in \( 1/L_n \), which is consistent with Eqs. (32) and (35). The instability however has an upper threshold in the density gradient at \( 1/L_n \approx 6 \) after which the mode is stable with \( \Im(\omega) = 0 \) and oscillates only with a real frequency. Furthermore, there is a threshold for the instability for low values of \( \omega_{aT}\omega_D \) such that

$$\frac{(\omega_{sT} - \omega_D)^2}{4} > \omega_{sT}\omega_D, \quad (36)$$

when the mode becomes stable. For \( \omega_{sT}\omega_D \to 0 \), only the mode with \( \Delta > 0 \) remains unstable as is shown on the insert in Fig. 1a.

In this paper, a fluid model has been used for the description of linear collisionless tearing modes taking into account magnetic field, plasma density and temperature gradients. The linear dispersion relation has been derived using the constant-\( \psi \) approximation. For a uniform magnetic field this dispersion equation reduces to the previous fluid and kinetic results that predict the instability of \( \Delta > 0 \) modes only. When the magnetic field gradient is included, our dispersion equation predicts an additional destabilization mechanism linearly driven by the combination of the finite temperature and magnetic field gradients persisting even for small scale tearing modes with \( \Delta < 0 \). Effects of large density gradients have been found to be stabilizing, which is in agreement with general tendencies of earlier results\(^ {9,14} \). The instability mechanism identified in our work requires the condition of the interchange type: \( \omega_D\omega_{aT} > 0 \), which is similar to the instability of ETG modes. The ETG instability was also suggested as a source of small scale magnetic islands due to the nonlinear energy transfer\(^ {34} \). A somewhat similar idea of the "mesoscopic" reconnection was also proposed in Ref. 35. Furthermore, it was suggested that the unstable ETG type modes can be responsible for the nonlinear excitation of the linearly stable microtearing modes\(^ {36} \). The defining feature of stable microtearing modes was their independence of the electrostatic potential \( \phi \) (which can be omitted for such modes) contrary to the ETG modes which involve essential perturbations of \( \phi \). In our work we show that in neglect of \( \phi \), the tearing type perturbations can be effectively destabilized by the interchange type mechanism related to the temperature and magnetic field gradients. We conjecture here that the interchange destabilization identified in our paper may also be operative in numerical simulations that demonstrate collisionless destabilization of micro-tearing modes. Note that when many modes with high-\( m \) are present and are unstable they naturally evolve towards a turbulent state via non-linear coupling\(^ {37} \). However, for low-\( m \) modes and if the mode is linearly stable the additional destabilization mechanism found in this work does not affect its stability. It has been shown in literature though that such modes can be excited non-linearly by microturbulence driven by pressure gradient as in Ref. 38 or by electromagnetic turbulence fluctuations as in Ref. 39.

It is important to note that our fluid model results in the non-analytic dispersion relation in Eq. (34). In fact, the same result is obtained in kinetic derivations of Refs. 12 and 13. In the latter case, the non-analytic nature of the dispersion relation is not apparent since the result was formulated in terms of the Plasma Dispersion Function which was defined for \( \Im(\omega) > 0 \) and analytically continued into the \( \Im(\omega) < 0 \) plane. In the kinetic theory, the non-analytic nature of the dispersion relation is related to the presence of the singular (continuous spectrum) eigen-functions which stipulates the use of the Landau pole rule (or Laplace transform for the initial value problem). In fluid theory, one can also introduce some dissipation which would also regularize the singular eigenfunctions. We should note however that our main result of the interchange destabilization for \( \omega_D\omega_{aT} > 0 \) is not affected as the \( \Delta = 0 \) limit given by Eq. (35) shows.

Some restrictions that limit direct application of our theory to the realistic tokamak geometry are essential to note. Our theory assumes the local (constant) value of \( \omega_D \). In the tokamak geometry, the main part of the \( \omega_D \) is oscillating in the poloidal direction (or equivalently along the magnetic field line) and the mean (constant part) only appears in the next order as a result of the magnetic well (Shafranov shift). This oscillation has been taken into account in Refs. 31–33 in the framework of resistive MHD where an average curvature is calculated to the \( \varepsilon^2 \) order after toroidal coupling of the central (low-\( m \), low-\( n \)) mode to its side bands. Its effect was found to be stabilizing for the large scale tearing mode. Our approximation is the same as the local \( \omega_D \) approximation made in the fluid theory of ITG/ETG modes\(^ {40,42} \) which...
shows good agreement with results from fully nonlocal kinetic simulations. We note also that linear drift-kinetic and gyro-kinetic simulations with local magnetic field gradient in Ref. 22 also show the growth rate of negative $\Delta'_{1}$ tearing mode increasing with the value of the local magnetic field gradient in very-low collisionality regimes. One can think that the local approximation may be acceptable for high $m$ modes, for which the mode spans a narrow poloidal region and any nonlocal corrections come as the next order effects. In general more accurate account of the structure of the magnetic field gradient would be required.

Another two approximations of our model, namely, the neglect of the electrostatic potential and the simplified temperature response model in Eq. 15 will also have to be abandoned in a more accurate model. The integral formulation in Ref. 45 which does take into account the effects of the electrostatic potential predicts stabilization with increasing values of density gradient (albeit no magnetic field gradient was included there). The isothermal approximation for electrons should be improved with more complete energy equation as in Ref. 46. The full electron energy equation could also include the Hammett-Perkins type closure. Such closure could be added to the gyrofluid models of the magnetic reconnection, e.g. those studied in Refs. 48 and 49. It would be interesting to investigate whether collisionless closures might also lead to the regularization of the singular eigen-functions. These questions and improvements of our model are left for future work.

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