Waves under location uncertainty

Valentin Resseguier, Bertrand Chapron, Fabrice Collard

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SCALIAN

OceanDataLab

Ifremer

Motivations

- Mimic interaction wave / (small-scale) current
    - Small-scale currents are hardly observed or simulated
    - Waves see them during very short time
- Quantification of associated modelling errors

  → Ensemble forecasts and data assimilation
- Improve the Scalian's wave simulator
- Study influence of swell / current interactions onto satellite altimetry

Part I
Scalian

SCALIAN

L@b
(~ 15 peoples)

Research, R&T, R&D

Expertise:

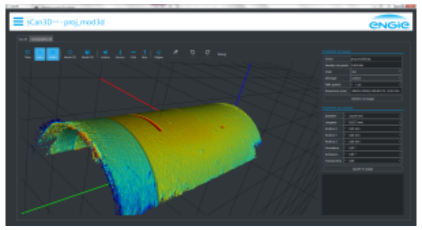
- Geophysical fluid dyn.
- Signal, data assimilation
- Machine Learning
- Multi-agents systems
- Drones

Other Business Units

~ 2400 people

CEN « Simulation » (~ 70 people)

R&D and engineering



Expertise:

- Radar, optronics, sonar
- Geophysical fluid dyn.
- Mechanical and thermal

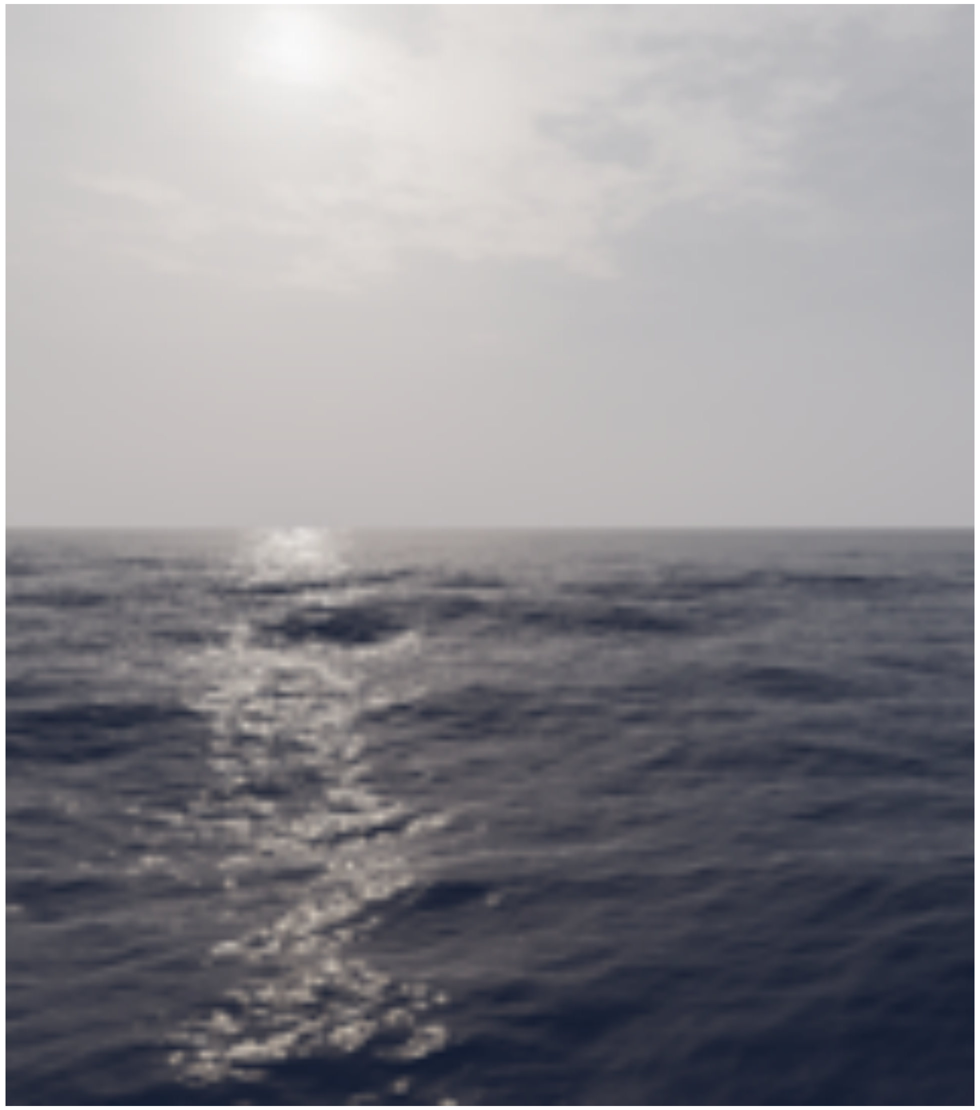
Business:

- Scientific softwares
- Simulations, HPC
- VR & AR

DGA, TOTAL, AREVA, EDF, AIRBUS DEFENCE & SPACE, VINCI, cea, ONERA, ENGIE, SHOM, MBDA, IRSN, THALES, Ifremer, TechnipFMC, Sogitec, ROBATEL Industries, NAVAL GROUP, ENEDIS, GRDF, RTE

SeaMotion-radar simulator

- Simple (mainly linear dynamics) and fast simulation of surface gravity waves
- Coupled with radar simulation
- Lot of physical phenomena taking into account
- Used to study satellite altimetry measurements



Part II
Waves LU

LU :Adding random velocity

$$\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$$

LU :Adding random velocity

Resolved large scales

$$\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$$

LU :Adding random velocity

Resolved large scales

White-in-time small scales

$$\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$$

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma} \dot{\boldsymbol{B}}$

Variance tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} \, (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$$

LU :Adding random velocity

Resolved large scales

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LU :Adding random velocity

Resolved large scales

White-in-time small scales

$$\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$$

References :

Mikulevicius and Rozovskii, 2004
Flandoli, 2011

Memin, 2014
Resseguier et al. 2017 a, b, c
Chapron et al. 2017
Cai et al. 2017

Holm, 2015
Holm and Tyranowski, 2016
Arnaudon et al., 2017

Cotter and al 2017
Crisan et al., 2017
Gay-Balmaz & Holm 2017
Cotter and al 2018 a, b

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma}\dot{\boldsymbol{B}}$

Variance tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x},\boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B}\,(\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$$

Wave:

$$a e^{\frac{i}{\epsilon}\phi}$$

Doppler frequency:

$$\omega_0 = \sqrt{g\|k\|}$$

Dispersion ratio

At the first order in steepness :

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma} \dot{\boldsymbol{B}}$

Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} \, (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

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$a e^{\frac{i}{\epsilon} \phi}$

Doppler frequency:

$\omega_0 = \sqrt{g||k||}$

Dispersion ratio

At the first order in steepness :

$$\frac{D\phi}{Dt} = \omega_0(k) = \sqrt{g||k||}$$

Dispersion ratio

At the first order
in steepness :

Large scales:

$\boldsymbol{w}$

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$a e^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g\|k\|}$

Stochastic transport

$$\frac{D\phi}{Dt} = \omega_0(k) = \sqrt{g\|k\|}$$

Large scales:

$\boldsymbol{w}$

Small scales:

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Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} \, (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

Wave:

$a e^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g\|k\|}$

Dispersion ratio

At the first order in steepness :

Stochastic transport

Frequency without currents

$$\frac{D\phi}{Dt} = \omega_0(k) = \sqrt{g\|k\|}$$

Case 0:
Stationary deterministic velocity

Random ray

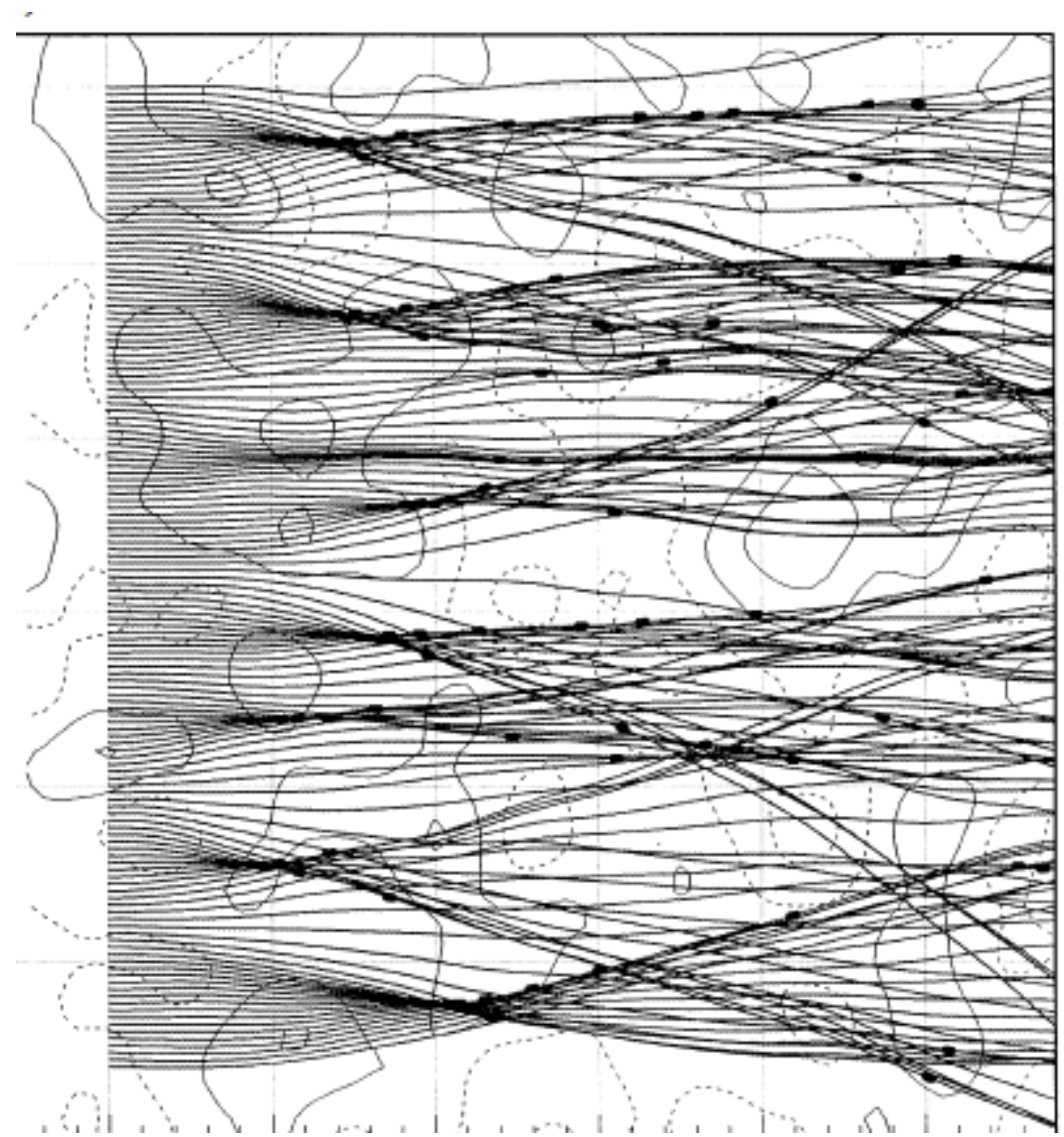
$$\dot{X}_R = u_g$$

Stretching of phase

$$\dot{k} = -\nabla w^T k$$

Conservation of action

$$\text{cst.} = A = \frac{a^2}{\omega_0}$$



White & Fornberg 1997

Large scale group velocity:

w $+\nabla_k \omega_0$

Wave:

$a e^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g\|k\|}$

Case 0:

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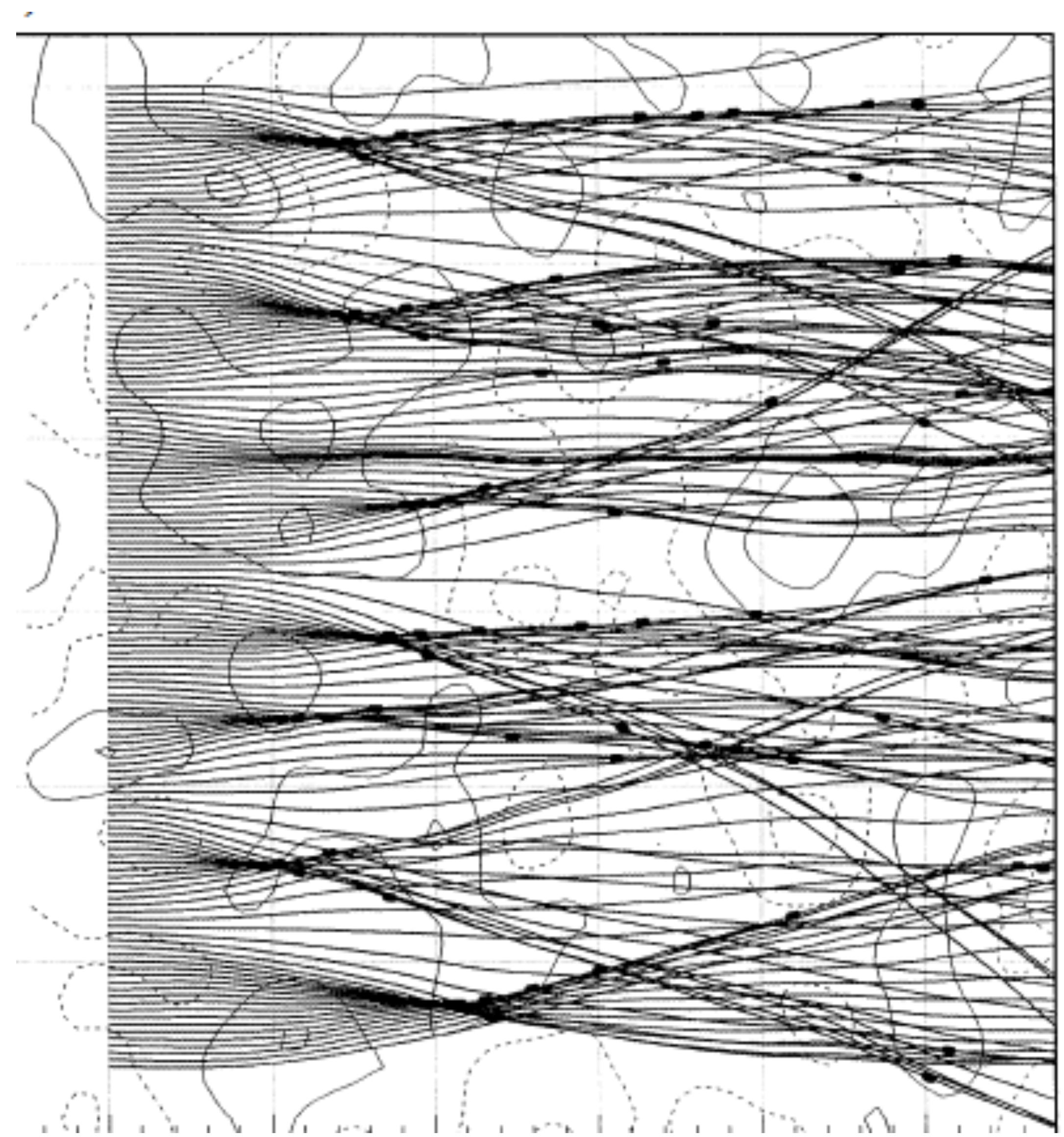


White & Fornberg 1997

Simple linear case 1:

stationary deterministic, divergence-free and linear in x large-scale velocity

Large scale group velocity:

$$w + \nabla_k \omega_0$$

Wave:

$$a e^{\frac{i}{\epsilon}\phi}$$

Doppler frequency:

$$\omega_0 = \sqrt{g\|k\|}$$

Stretching of phase

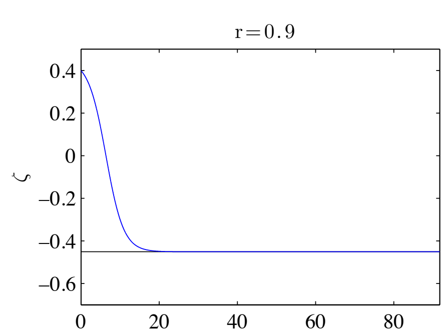
$$\dot{k} = -\nabla w^T k$$

Ratio effective rotation / strain rate

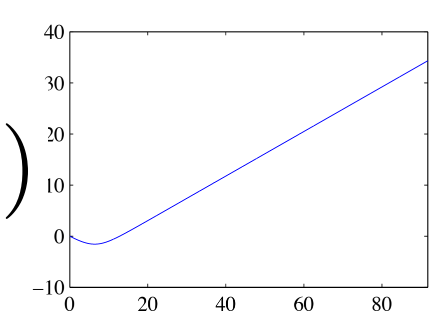
$$r = \frac{\omega}{\sigma_w}$$

$|r| < 1$

Hyperbolic

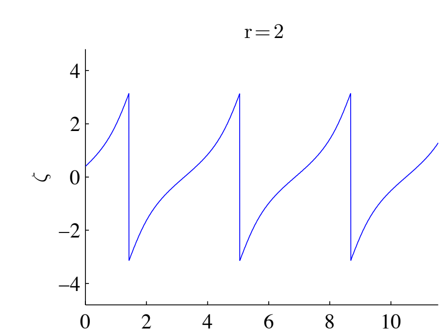


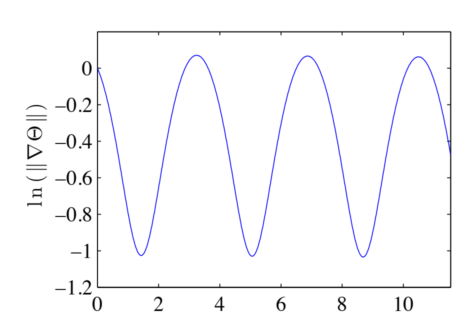
$ln\left(\|k\|\right)$



$|r| > 1$

Elliptic





$ln\left(\|k\|\right)$

$$\frac{D\zeta}{D\tau} = r - \cos(\zeta)$$

$$\frac{D}{D\tau}\; ln\left(\|k\|\right) = -\sin(\zeta)$$

Large scale group velocity:

$\boldsymbol{w}$ $+\nabla_k \omega_0$

Small scale group velocity:

$\boldsymbol{\sigma \dot{B}}$

Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma dB} \, (\boldsymbol{\sigma dB})^T\}}{dt}$

Wave:

$a e^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g \|k\|}$

Simple linear case 2:

no large-scale current
+ homogeneous and divergence-free small-scale velocity

$$\beta(x) e^{i(\omega_0 t + k \cdot \sigma B_t)}$$

Wave with Brownian phase

Stochastic stretching of phase

$$\dot{k} = -\nabla(\sigma \dot{B})^T k$$

Log-normal wave- number

Conservation of action

$$\text{cst.} = A = \frac{a^2}{\omega_0}$$

Log-normal amplitude

Large scale group velocity:

$\boldsymbol{w}$ $+\nabla_k \omega_0$

Small scale group velocity:

$\boldsymbol{\sigma} \dot{\boldsymbol{B}}$

Variance tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} \, (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$$

Wave:

$$a e^{\frac{i}{\epsilon}\phi}$$

Doppler frequency:

$$\omega_0 = \sqrt{g \|k\|}$$

Simple linear case 3:

divergence-free and linear-in-x large-scale current
+ homogeneous and divergence-free small-scale velocity

Stochastic stretching of phase

$$\dot{k} = -\nabla(w + \sigma \dot{B})^T k$$

Action

$$\text{cst.} = \frac{a^2}{\omega_0}$$

Large scale group velocity:

$w + \nabla_k \omega_0$

Small scale group velocity:

$\sigma \dot{B}$

Variance tensor:

$a = a(x,x) = \frac{\mathbb{E}\{\sigma dB\,(\sigma dB)^T\}}{dt}$

Wave:

$a e^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g\|k\|}$

Simple linear case 3:

divergence-free and linear-in-x large-scale current
+ homogeneous and divergence-free
small-scale velocity

Stochastic stretching of phase

$$\dot{k} = -\nabla(w + \sigma\dot{B})^T k$$

Action

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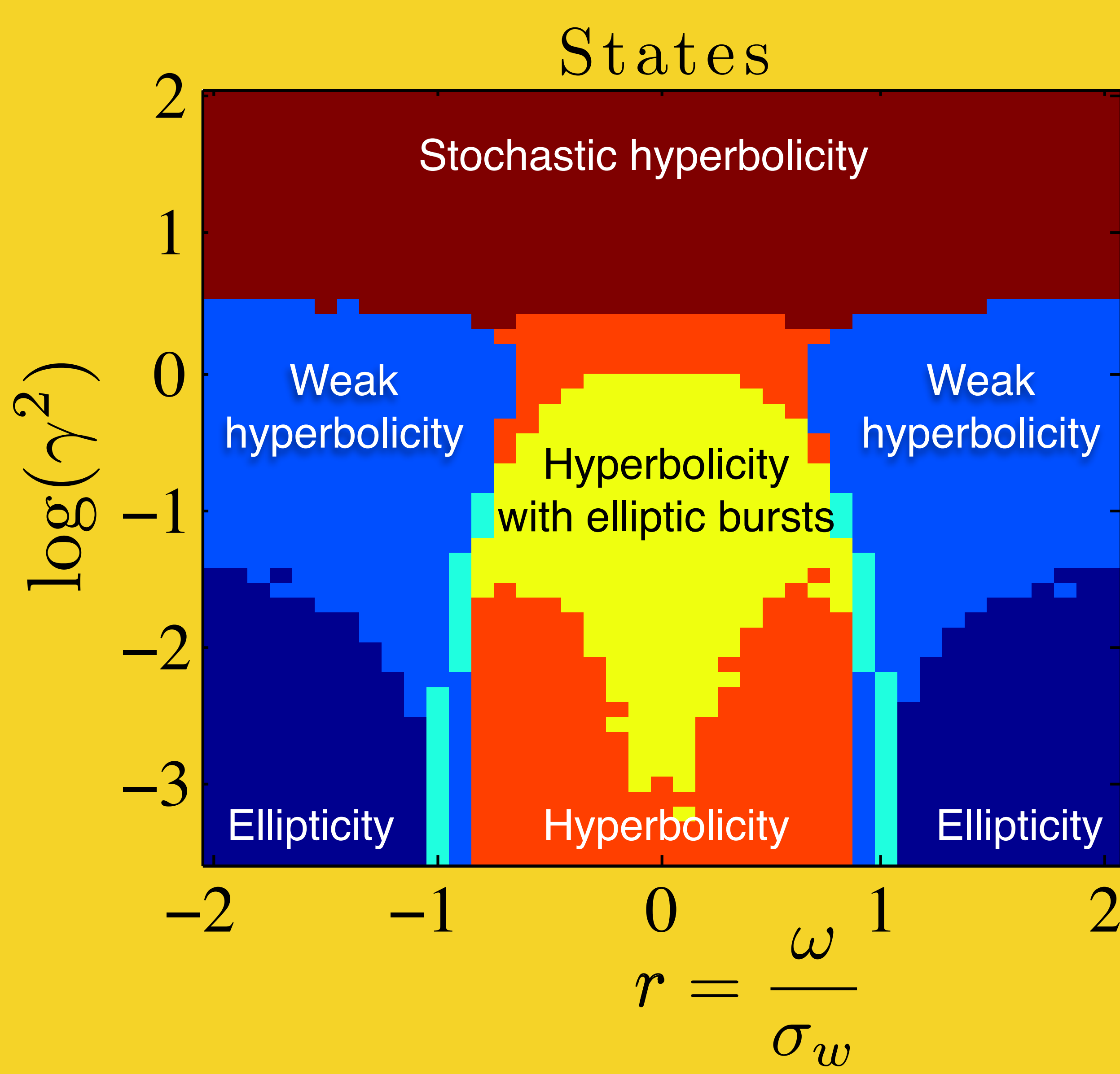

Large scale group velocity:

$w \; + \nabla_k \omega_0$

Small scale group velocity:

$\sigma \dot{B}$

Variance tensor:

$a = a(x,x) = \frac{\mathbb{E}\{\sigma dB \, (\sigma dB)^T\}}{dt}$

Wave:

$a e^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g\|k\|}$

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divergence-free and linear-in-x large-scale current
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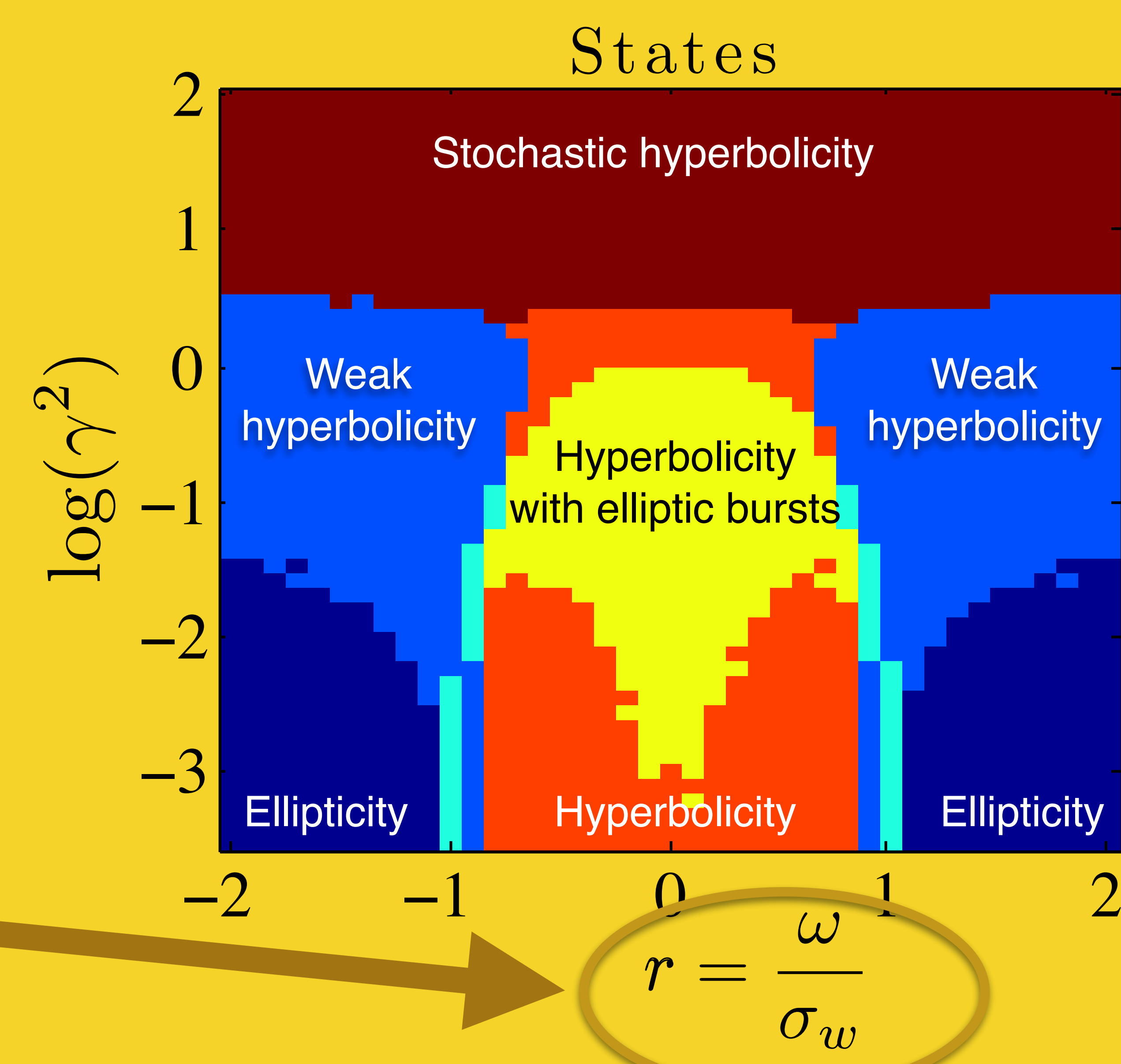
$$\text{cst.} = \frac{a^2}{\omega_0}$$



Ratio
effective rotation / strain rate

Large scale group velocity:

$\boldsymbol{w} + \nabla_k \omega_0$

Small scale group velocity:

$\boldsymbol{\sigma} \dot{\boldsymbol{B}}$

Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

Wave:

$a e^{\frac{i}{\epsilon} \phi}$

Doppler frequency:

$\omega_0 = \sqrt{g \|k\|}$

Simple linear case 3:

divergence-free and linear-in-x large-scale current
+ homogeneous and divergence-free
small-scale velocity

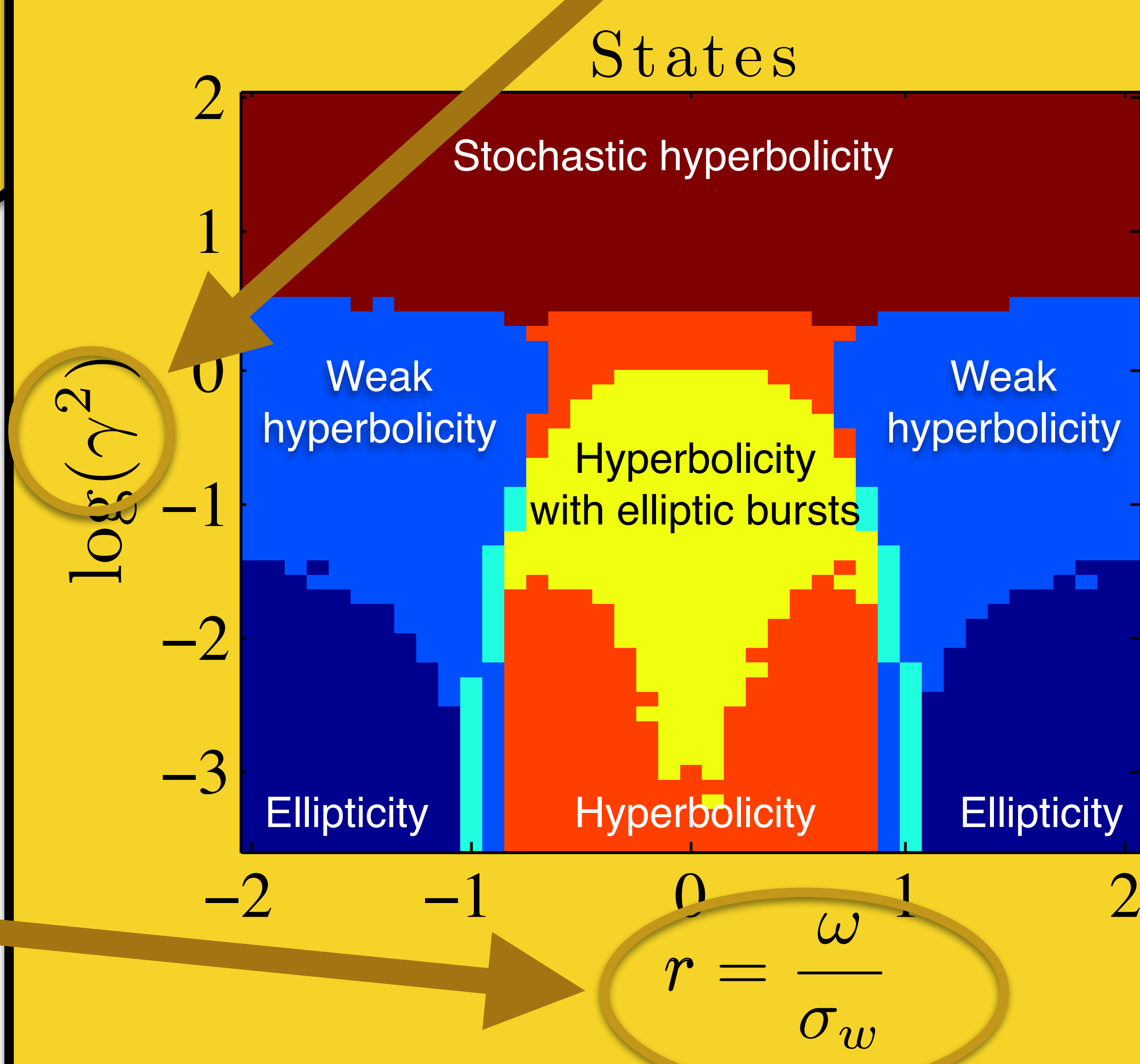
Stochastic stretching of phase

$$\dot{k} = -\nabla(w + \sigma \dot{B})^T k$$

Action

$$\text{cst.} = \frac{a^2}{\omega_0}$$

Depends on small scale's statistics



Ratio
effective rotation / strain rate

General case

Large scale group velocity:

$\boldsymbol{w} + \nabla_k \omega_0$

Small scale group velocity:

$\boldsymbol{\sigma \dot{B}}$

Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma dB} \, (\boldsymbol{\sigma dB})^T\}}{dt}$

Wave:

$a e^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g\|k\|}$

$$\dot{X}_R = u_g$$

Random ray

Stochastic stretching of phase

$$\dot{k} = \left(-\nabla(w + \sigma\dot{B})^T + (\nabla\sigma^T)^2\right) k$$

Wave- vector

Conservation of action

$$\dot{A} = -\nabla \cdot u_g \; A \quad \Big| \quad A = \frac{a^2}{\omega_0}$$

Amplitude

General case

Large scale group velocity:

$\boldsymbol{w} + \nabla_k \omega_0$

Small scale group velocity:

$\boldsymbol{\sigma} \dot{\boldsymbol{B}}$

Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} \, (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

Wave:

$a e^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g \|k\|}$

Fonction of k

$$\dot{X}_R = u_g$$

Random ray

Stochastic stretching of phase

$$\dot{k} = \left(-\nabla (w + \sigma \dot{B})^T + (\nabla \sigma^T)^2 \right) k$$

Wave- vector

Conservation of action

$$\dot{A} = -\nabla \cdot u_g \; A \quad \Big| \quad A = \frac{a^2}{\omega_0}$$

Amplitude

Large scale group velocity:

$\boldsymbol{w}$ $+\nabla_k \omega_0$

Small scale group velocity:

$\boldsymbol{\sigma \dot{B}}$

Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma dB}\,(\boldsymbol{\sigma dB})^T\}}{dt}$

Wave:

$a e^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g\|k\|}$

General case

$$\dot{X}_R = u_g$$

Fonction of k

Random ray

Stochastic stretching of phase

$$\dot{k} = \left(-\nabla(w + \sigma\dot{B})^T + (\nabla\sigma^T)^2\right) k$$

Fonction of x

Wave- vector

Conservation of action

$$\dot{A} = -\nabla \cdot u_g \ A \quad \Big| \quad A = \frac{a^2}{\omega_0}$$

Amplitude

General case

Large scale group velocity: $\boldsymbol{w} + \nabla_k \omega_0$

Small scale group velocity: $\boldsymbol{\sigma \dot{B}}$

Variance tensor: $\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma dB}\,(\boldsymbol{\sigma dB})^T\}}{dt}$

Wave: $a e^{\frac{i}{\epsilon}\phi}$

Doppler frequency: $\omega_0 = \sqrt{g\|k\|}$

Non-linear coupling

Fonction of k

$$\dot{X}_R = u_g$$

Random ray

Stochastic stretching of phase

$$\dot{k} = \left(-\nabla(w + \sigma\dot{B})^T + (\nabla\sigma^T)^2\right) k$$

Fonction of x

Wave- vector

Conservation of action

$$\dot{A} = -\nabla \cdot u_g \; A \quad \Big| \quad A = \frac{a^2}{\omega_0}$$

Amplitude

Conclusion

Conclusion and outlooks

- Method of characteristics : low computational costs
  We can simulate a lot of realisations, and initial conditions
- Understanding the dynamics with increasing complexity
- Tests against statistical physics results and satellite measurements.
  Test case : Wave spectra variations due to Agulhas courant
- Integration into the SeaMotion Radar
- Which are the swell / current effects on satellite measurements?
- Outlook : stochastic Wave Watch III

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