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Uncertainty quantification and ensemble forecast in coarse-grid or dimensionally-reduced computational fluid dynamics

Valentin Resseguier,
Etienne Mémin,
Bertrand Chapron
Motivations

• Rigorously identified subgrid dynamics effects

• Injecting likely small-scale dynamics

• Studying bifurcations and attractors

  Climate projections

• Quantification of modeling errors

  Ensemble forecasts and data assimilation
Contents

• Scalian
• Location uncertainty
• SQG under moderate uncertainty
Part I: SCALIAN

L@b (≈ 15 peoples)

Research, R&T, R&D

Expertise:
- Geophysical fluid dyn.
- Signal, data assimilation
- Machine Learning
- Multi-agents systems
- Drones

CEN « Simulation » (≈ 70 people)

R&D and engineering

Expertise:
- Radar, optronics, sonar
- Geophysical fluid dyn.
- Mechanical and thermal

Business:
- Scientific softwares
- Simulations, HPC
- VR & AR

Other Business Units

≈ 2400 people
Part II
Location uncertainty (LU)
Usual random CFD

- Random parameters, boundary conditions, forcing
- Random initial conditions
- Arbitrary Gaussian forcing
- Averaging, homogenization

Other (complementary) issues

- Underdispersive
  + need large ensemble

- Adding energy
  + wrong phase

- Assumptions and energy issues
Usual random CFD

- Random parameters, boundary conditions, forcing
- Random initial conditions
- Arbitrary Gaussian forcing
- Averaging, homogenization

Other (complementary) issues
- Underdispersive
- Need large ensemble
- Adding energy
- Wrong phase
- Assumptions and energy issues
LU : Adding random velocity

\[ v = w + \sigma \dot{B} \]
LU : Adding random velocity

Resolved large scales

\[ \nu = v + \sigma \dot{B} \]
LU : Adding random velocity

\[ \nu = w + \sigma \dot{B} \]

Resolved large scales

White-in-time small scales
Large scales: \( w \)
Small scales: \( \sigma \dot{B} \)

Variance tensor:
\[
a = a(x, x) = \mathbb{E}\left\{ \sigma dB \left( \sigma dB \right)^T \right\} \frac{dt}{dt}
\]

LU: Adding random velocity

\[
\nu = w + \sigma \dot{B}
\]

Resolved large scales

White-in-time small scales
LU : Adding random velocity

Large scales: \( w \)
Small scales: \( \sigma \dot{B} \)

Variance tensor:
\[
\alpha = \alpha(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\}/dt
\]

Resolved large scales

White-in-time small scales

\[
\nu = w + \sigma \dot{B}
\]

References:
Mikulevicius and Rozovskii, 2004
Flandoli, 2011
Memin, 2014
Resseguier et al. 2017 a, b, c
Chapron et al. 2017
Cai et al. 2017
Holm, 2015
Holm and Tyranowski, 2016
Arnaudon et al., 2017
Cotter et al. 2017
Gay-Balmaz & Holm 2017
Cotter and al 2017 a, b
Advection of tracer \( \Theta \)

\[
\frac{D\Theta}{Dt} = 0
\]
Advection of tracer $\Theta$

Large scales: $w$

Small scales: $\sigma \dot{B}$

Variance tensor:

\[ a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt} \]
Advection of tracer $\Theta$

\[
\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)
\]
Advection of tracer $\Theta$

Large scales: $w$
Small scales: $\sigma \dot{B}$

Variance tensor:
$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

$$\partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$
Advection
of tracer $\Theta$

Large scales:
- $w$
Small scales:
- $\sigma \dot{B}$
Variance tensor:
- $a = a(x,x) = \frac{1}{dt} \mathbb{E} \{ \sigma dB (\sigma dB)^T \}$

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$
Advection of tracer $\Theta$

\[ \partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \mathbf{\dot{B}} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} \alpha \nabla \Theta \right) \]

Drift correction

Large scales: $\mathbf{w}$
Small scales: $\sigma \mathbf{\dot{B}}$
Variance tensor:
\[
\alpha = a(x, x) = \frac{E\{\sigma dB (\sigma dB)^T\}}{dt}
\]
Advection of tracer $\Theta$

Multiplicative random forcing

Large scales:
$\boldsymbol{w}$

Small scales:
$\sigma \mathbf{B}$

Variance tensor:

$$a = a(x, x) = \mathbb{E}\left\{ \sigma dB \langle \sigma dB \rangle^T \right\}$$

Drift correction

$$\partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{\mathbf{B}} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$
Advection of tracer $\Theta$

Multiplicative random forcing

Large scales:
$w$

Small scales:
$\sigma \dot{B}$

Variance tensor:

$$a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} \frac{dt}{dt}$$

Drift correction

$$\partial_t \Theta + \omega^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$

Diffusion

8
Advection of tracer $\Theta$

Large scales: $\omega$
Small scales: $\sigma \dot{B}$

Variance tensor:
$$a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\}$$

Multiplicative random forcing

Drift correction

$$\partial_t \Theta + \omega^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$
Advection of tracer $\Theta$

Large scales: $w$
Small scales: $\sigma \dot{B}$

Variance tensor:
\[ a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt} \]

\[ \partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} \alpha \nabla \Theta \right) \]

Multiplicative random forcing

Balanced energy exchanges

Drift correction
A word about reduced order models

- Very fast simulation of very complex system (e.g. for industrial application)
- Physical model (PDE) simplified using observations

Reduced models under location uncertainty
- Rigorous and low-cost estimators
- Stabilization of the unstable modes
- Maintain variability of stable modes
- Uncertainty quantification

State of art
- Possible parametrization with eddy viscosity
- Impossible to parametrize with additional dissipation
  Need ad hoc closure like MQG
  (Sapsis and Majda, 2013a,b,c)
Part III
SQG under Moderate Uncertainty

SQG MU

Code available online
SQG

\[
\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \\
\text{Hyper-viscosity}
\]

\[
u = \left( \text{cst.} \nabla \perp \Delta^{-\frac{1}{2}} \right) b
\]

Reference flow:

deterministic

SQG

1024 x 1024
\[
\frac{D b}{D t} = -\alpha_{HV} \Delta^4 b \\
\mathbf{u} = \left( \text{cst.} \nabla^{\perp} \Delta^{-\frac{1}{2}} \right) b
\]

Reference flow:
- deterministic
- SQG

1024 x 1024
One realization: Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128
One realization: Stochastic destabilization
One realization:
Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128
One realization:
Stochastic destabilization

- Deterministic 128 x 128
- Deterministic 1024 x 1024
- Location Uncertainty 128 x 128

Spectrum
One realization: Stochastic destabilization

Location Uncertainty 128 x 128

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x128
One realization: Stochastic destabilization.
One realization:
Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128

\( t = 17 \) days
One realization: Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128
Ensemble: random coherent structures
Ensemble: random coherent structures
Ensemble: uncertainty quantification

![Ensemble Experiment](image)

- Bias LU
- Estim. error LU
- Estim. error RanIC

Spectrum of the errors and its estimation

$E(\kappa) (rad.m^{-1})$ vs $\kappa (rad.m^{-1})$
Ensemble: uncertainty quantification
Conclusion
Conclusion

LU models blindly describe unresolved triades

- Conserve energy
- Stabilization / destabilization in Reduced Order Model
- Instabilities triggered, possibly followed by extreme events
- Uncertainty quantification to address filter divergence
Related works, outlooks and application

• Bifurcations (SQG) and attractor (Lorenz 63) exploration

• Comparisons with data-driven parametrisation and SALT (Stochastic Advection by Lie Transport) (Holm and coauthors)

• Parametrization and tests based on higher-order statistics (curvature, energy flux through scales, bispectrum, …)

• (Surface gravity) wave / turbulence interaction

• **Data assimilation (DA)** :
  
  • Filtering / smoothing
    
    • **EnKF with LU model as a R&D tool** (for e.g. airplanes, drones)
    
    • **PF with reduced LU model for real-time monitoring and flow control** (for e.g. pollutant dispersion monitoring, drag and damage reduction in e.g. wind turbines)

  • Girsanov theorem for MLE and Bayesian estimations with e.g. satellite images