Uncertainty quantification and ensemble forecast in coarse-grid or dimensionally-reduced computational fluid dynamics
Valentin Resseguier, Etienne Mémin, Bertrand Chapron

To cite this version:
Valentin Resseguier, Etienne Mémin, Bertrand Chapron. Uncertainty quantification and ensemble forecast in coarse-grid or dimensionally-reduced computational fluid dynamics. ECCM - ECFD 2018 - 6th European Conference on Computational Mechanics - 7th European Conference on Computational Fluid Dynamics, Jun 2018, Glasgow, United Kingdom. pp.1-41. <hal-01891183>

HAL Id: hal-01891183
https://hal.archives-ouvertes.fr/hal-01891183
Submitted on 9 Oct 2018
Uncertainty quantification and ensemble forecast in coarse-grid or dimensionally-reduced computational fluid dynamics

Valentin Resseguier,
Etienne Mémin,
Bertrand Chapron
Motivations

• Rigorously identified sudgrid dynamics effects

• Injecting likely small-scale dynamics

• Studying bifurcations and attractors

  Climate projections

• Quantification of modeling errors

  Ensemble forecasts and data assimilation
Contents

• Scalian

• Location uncertainty

• SQG under moderate uncertainty
Part I: SCALIAN

L@b (~ 15 peoples)
- Research, R&T, R&D

Expertise:
- Geophysical fluid dyn.
- Signal, data assimilation
- Machine Learning
- Multi-agents systems
- Drones

CEN « Simulation » (~ 70 people)
- R&D and engineering

Expertise:
- Radar, optronics, sonar
- Geophysical fluid dyn.
- Mechanical and thermal

Business:
- Scientific softwares
- Simulations, HPC
- VR & AR

Other Business Units
- ~ 2400 people
Part II
Location uncertainty
(LU)
Usual random CFD

- Random parameters, boundary conditions, forcing
- Random initial conditions
- Arbitrary Gaussian forcing
- Averaging, homogenization

Other (complementary) issues

- Underdispersive
  + need large ensemble

- Adding energy
  + wrong phase

- Assumptions and energy issues
Usual random CFD

- Random parameters, boundary conditions, forcing
- Random initial conditions
- Arbitrary Gaussian forcing
- Averaging, homogenization

Other (complementary) issues

- Underdispersive
  + need large ensemble
- Adding energy
  + wrong phase
- Assumptions and energy issues
LU : Adding random velocity

\[ v = w + \sigma \dot{B} \]
LU : Adding random velocity

\[ \nu = w + \sigma \dot{B} \]

Resolved large scales
LU : Adding random velocity

\[ \nu = w + \sigma \dot{B} \]

Resolved large scales

White-in-time small scales
LU : Adding random velocity

Large scales: \( w \)
Small scales: \( \sigma \dot{B} \)

Variance tensor:
\[
a(x, x) = \mathbb{E}\left\{ \sigma dB (\sigma dB)^T \right\} \frac{dt}{dt}
\]

Resolved large scales

White-in-time small scales

\[
v = \mathbf{w} + \sigma \dot{\mathbf{B}}
\]
Large scales: \( w \)
Small scales: \( \sigma \dot{B} \)

Variance tensor:
\[
\alpha = \alpha(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T \} / dt
\]

Resolved large scales

\[ \nu = w + \sigma \dot{B} \]

White-in-time small scales

References:
- Mikulevicius and Rozovskii, 2004
- Flandoli, 2011
- Memin, 2014
- Resseguier et al. 2017 a, b, c
- Chapron et al. 2017
- Cai et al. 2017
- Holm, 2015
- Holm and Tyranowski, 2016
- Arnaudon et al., 2017
- Cotter and al 2017
- Crisan et al., 2017
- Gay-Balmaz & Holm 2017
- Cotter and al 2018 a, b
Advection of tracer $\Theta$

\[
\frac{D\Theta}{Dt} = 0
\]
Advection of tracer $\Theta$

Large scales: $w$
Small scales: $\sigma \dot{B}$
Variance tensor:

$$a = a(x, x) = E \left\{ \sigma dB \left( \sigma dB \right)^T \right\} dt$$
Advection of tracer $\Theta$

Large scales: $w$

Small scales: $\sigma \dot{B}$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$
Advection
of tracer $\Theta$

Large scales:
$w$
Small scales:
$\sigma \dot{B}$
Variance tensor:
$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

\[
\partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)
\]
Advection of tracer $\Theta$

Large scales: $w$
Small scales: $\sigma \dot{B}$
Variance tensor:
\[
a = a(x, x) = \frac{E\{\sigma dB (\sigma dB)^T\}}{dt}
\]
Advection of tracer $\Theta$

Large scales: $w$
Small scales: $\sigma \dot{B}$
Variance tensor:

$$a = a(x, x) = \frac{E\{\sigma dB (\sigma dB)^T\}}{dt}$$
Advection of tracer $\Theta$

Multiplicative random forcing

Large scales:
$\mathbf{w}$
Small scales:
$\sigma \dot{\mathbf{B}}$

Variance tensor:

$$a = a(x, x) = \frac{\mathbf{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Drift correction

$$\partial_t \Theta + \mathbf{w}^\star \cdot \nabla \Theta + \sigma \dot{\mathbf{B}} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$
Advection of tracer $\Theta$

Multiplicative random forcing

Large scales: $w$
Small scales: $\sigma \dot{B}$

Variance tensor:

$$a = a(x, x) = \mathbb{E}\{\sigma dB(x) \sigma dB(x)^T\}$$

$\partial_t \Theta + \mathcal{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \frac{1}{2} a \nabla \Theta$

Drift correction
Advection of tracer $\Theta$

Large scales:
$w$
Small scales:
$\sigma \dot{B}$
Variance tensor:
$a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} dt$

Multiplicative random forcing

$\partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta$

Drift correction

Diffusion
$\nabla \cdot \left( \frac{1}{2} \alpha \nabla \Theta \right)$
Advection of tracer $\Theta$

\[
\partial_t \Theta + \mathbf{\omega}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} \alpha \nabla \Theta \right)
\]

Large scales: $\mathbf{\omega}$
Small scales: $\sigma \dot{B}$
Variance tensor:
\[
a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} \frac{dt}{dt}
\]

Multiplicative random forcing
Balanced energy exchanges
Drift correction

Large scales:
Small scales:
Variance tensor:
A word about reduced order models

- Very fast simulation of very complex system (e.g. for industrial application)
- Physical model (PDE) simplified using observations

**Reduced models under location uncertainty**
- Rigorous and low-cost estimators
- Stabilization of the unstable modes
- Maintain variability of stable modes
- Uncertainty quantification

**State of art**
- Possible parametrization with eddy viscosity
- Impossible to parametrize with additional dissipation
  Need ad hoc closure like MQG (Sapsis and Majda, 2013a,b,c)
Part III
SQG under Moderate Uncertainty

SQG MU

Code available online
SQG

\[
\frac{D b}{D t} = -\alpha_{HV} \Delta^4 b \quad \text{Hyperviscosity}
\]

\[
u = \left( \text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b
\]

Reference flow:

deterministic

SQG

1024 x 1024
Hyper-viscosity Reference flow: deterministic

\[ \frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \text{ Hyper-viscosity} \]

\[ u = \left( \text{cst.} \ \nabla \right) \Delta^{-\frac{1}{2}} \]

Reference flow:
deterministic

SQG

1024 x 1024

\[ t = 17 \text{ days} \]
One realization: Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128
One realization:
Stochastic destabilization

\[ x(m) y(m) t = 17 \text{ days} \]

[Image of three graphs comparing deterministic and location uncertainty models]
One realization:
Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128

Spectrum
One realization:
Stochastic destabilization
One realization:
Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128
One realization:
Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128

$t = 17$ days
One realization:
Stochastic destabilization

- Deterministic 128 x 128
- Deterministic 1024 x 1024
- Location Uncertainty 128 x 128
One realization:
Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128

$\hat{b}(\kappa) = \kappa (\text{rad.m} - 1)$
Ensemble: random coherent structures
Ensemble: random coherent structures
Ensemble: uncertainty quantification

Bias LU

Estimated error LU

Estimated error RanIC

Spectrum of the errors and its estimation

$E(\kappa) (\text{rad.m})$

$\kappa (\text{rad.m}^{-1})$

Bias RanIC

Bias LU

Estimated error RanIC

Estimated error LU
Ensemble: uncertainty quantification
Conclusion
Conclusion

LU models blindly describe unresolved triades

- Conserve energy
- Stabilization / destabilization in Reduced Order Model
- Instabilities triggered, possibly followed by extreme events
- Uncertainty quantification to address filter divergence
Related works, outlooks and application

• Bifurcations (SQG) and attractor (Lorenz 63) exploration

• Comparisons with data-driven parametrisation and SALT (Stochastic Advection by Lie Transport) (Holm and coauthors)

• Parametrization and tests based on higher-order statistics (curvature, energy flux through scales, bispectrum, …)

• (Surface gravity) wave / turbulence interaction

• Data assimilation (DA):
  - Filtering / smoothing
    • EnKF with LU model as a R&D tool (for e.g. airplanes, drones)
    • PF with reduced LU model for real-time monitoring and flow control (for e.g. pollutant dispersion monitoring, drag and damage reduction in e.g. wind turbines)
  - Girsanov theorem for MLE and Bayesian estimations with e.g. satellite images