Uncertainty quantification and ensemble forecast in coarse-grid or dimensionally-reduced computational fluid dynamics
Valentin Resseguier, Etienne Mémin, Bertrand Chapron

To cite this version:
Valentin Resseguier, Etienne Mémin, Bertrand Chapron. Uncertainty quantification and ensemble forecast in coarse-grid or dimensionally-reduced computational fluid dynamics. ECCM - ECFD 2018 - 6th European Conference on Computational Mechanics - 7th European Conference on Computational Fluid Dynamics, Jun 2018, Glasgow, United Kingdom. pp.1-41. <hal-01891183>

HAL Id: hal-01891183
https://hal.archives-ouvertes.fr/hal-01891183
Submitted on 9 Oct 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Uncertainty quantification and ensemble forecast in coarse-grid or dimensionally-reduced computational fluid dynamics

Valentin Resseguier, Etienne Mémin, Bertrand Chapron
Motivations

• Rigorously identified subgrid dynamics effects
•Injecting likely small-scale dynamics
• Studying bifurcations and attractors
  Climate projections
• Quantification of modeling errors
  Ensemble forecasts and data assimilation
Contents

- Scalian
- Location uncertainty
- SQG under moderate uncertainty
Part I : SCALIAN

L@b (~ 15 peoples)

Research, R&T, R&D

Expertise:
- Geophysical fluid dyn.
- Signal, data assimilation
- Machine Learning
- Multi-agents systems
- Drones

Other Business Units
~ 2400 people

CEN « Simulation » (~ 70 people)

R&D and engineering

Expertise:
- Radar, optronics, sonar
- Geophysical fluid dyn.
- Mechanical and thermal

Business:
- Scientific softwares
- Simulations, HPC
- VR & AR
Part II
Location uncertainty (LU)
Usual random CFD

• Random parameters, boundary conditions, forcing

• Random initial conditions

• Arbitrary Gaussian forcing

• Averaging, homogenization

Other (complementary) issues

Underdispersive + need large ensemble

Adding energy + wrong phase

Assumptions and energy issues
Usual random CFD

- Random parameters, boundary conditions, forcing
- Random initial conditions
- Arbitrary Gaussian forcing
- Averaging, homogenization

Other (complementary) issues
- Underdispersive + need large ensemble
- Adding energy + wrong phase
- Assumptions and energy issues
LU : Adding random velocity

\[ v = w + \sigma \dot{B} \]
LU : Adding random velocity

\[ v = w + \sigma \dot{B} \]

Resolved large scales
LU : Adding random velocity

\[ \nu = w + \sigma \dot{B} \]

Resolved large scales

White-in-time small scales
LU : Adding random velocity

Large scales: $\nu$
Small scales: $\sigma \dot{B}$

Variance tensor:

$$a = a(x, x) = E\left\{\sigma dB \left(\sigma dB^T\right)\right\}_{dt}$$

Resolved
large scales

White-in-time
small scales

$$\nu = \omega + \sigma \dot{B}$$
LU : Adding random velocity

\[ \nu = w + \sigma \dot{B} \]

**Large scales:**
\[ w \]

**Small scales:**
\[ \sigma \dot{B} \]

**Variance tensor:**
\[ a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} \]

**Resolved large scales**

**White-in-time small scales**

References:
- Mikulevicius and Rozovskii, 2004
- Flandoli, 2011
- **Memin**, 2014
- Resseguier et al. 2017 a, b, c
- Chapron et al. 2017
- Cai et al. 2017
- **Holm**, 2015
- Holm and Tyranowski, 2016
- Arnaudon et al., 2017
- Cotter and al 2017
- Crisan et al., 2017
- Gay-Balmaz & Holm 2017
- Cotter and al 2018 a, b
Advection of tracer $\Theta$

$$\frac{D\Theta}{Dt} = 0$$

Large scales: $w$
Small scales: $\sigma \dot{B}$
Variance tensor:

$$a = a(x, x) = E\left\{ \sigma dB (\sigma dB)^T \right\}$$
Advection of tracer $\Theta$

Large scales: $\mathbf{w}$
Small scales: $\sigma \dot{B}$

Variance tensor:

\[ a = a(x, x) = \mathbb{E}\left\{ \sigma d\mathbf{B} (\sigma d\mathbf{B})^T \right\} dt \]
Advection of tracer $\Theta$

Large scales: $\boldsymbol{w}$
Small scales: $\sigma \dot{B}$
Variance tensor:

$$a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} \frac{dt}{dt}$$

$$\partial_t \Theta + \boldsymbol{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$
Advection of tracer $\Theta$

Large scales: $w$
Small scales: $\sigma \dot{B}$
Variance tensor:

\[ a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} \]

\[ \frac{dt}{dt} \]

\[ \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right) \]
Advection
of tracer $\Theta$

Large scales:
$w$
Small scales:
$\sigma \dot{B}$
Variance tensor:
$a = a(x, x) = E\{\sigma dB (\sigma dB)^T\}$

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$
Advection of tracer $\Theta$

Large scales: $w$
Small scales: $\sigma \dot{B}$

Variance tensor:

$$a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\}_{dt}$$

\[\partial_t \Theta + \omega^\star \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)\]

Diffusion

Drift correction
Advection of tracer $\Theta$

Multiplicative random forcing

Large scales: \( w \)
Small scales: \( \sigma \dot{B} \)

Variance tensor:
\[
a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}
\]

\[
\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)
\]

Drift correction

Diffusion
Advection of tracer \( \Theta \)

Large scales: \( \mathbf{w} \)

Small scales: \( \sigma \dot{\mathbf{B}} \)

Variance tensor:
\[
a = a(x, x) = \mathbb{E}\left\{ \sigma dB \left( \sigma dB \right)^T \right\}
\]

Drift correction

Multiplicative random forcing

\[
\partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{\mathbf{B}} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} \alpha \nabla \Theta \right)
\]
Advection of tracer $\Theta$

Large scales: $w$
Small scales: $\sigma \dot{B}$

Variance tensor:

$$a(x, x) = \operatorname{E}\{\sigma dB \sigma dB^T\}$$

\[
\frac{\partial}{\partial t} \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} \alpha \nabla \Theta \right)
\]
Advection of tracer $\Theta$

Large scales:
- $w$
Small scales:
- $\sigma \dot{B}$

Variance tensor:
- $a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\}/dt$

Multiplicative random forcing

Balanced energy exchanges

Drift correction

\[
\partial_t \Theta + \nabla \cdot (w^* \nabla \Theta) + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)
\]
A word about reduced order models

- Very fast simulation of very complex system (e.g. for industrial application)
- Physical model (PDE) simplified using observations

Reduced models under location uncertainty

- Rigorous and low-cost estimators
- Stabilization of the unstable modes
- Maintain variability of stable modes
- Uncertainty quantification

State of art

- Possible parametrization with eddy viscosity
- Impossible to parametrize with additional dissipation
  Need ad hoc closure like MQG
  (Sapsis and Majda, 2013a,b,c)
Part III
SQG under Moderate Uncertainty

SQG MU

Code available online
\[ \frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \text{ Hyper-viscosity} \]

\[ u = \left( \text{cst.} \cdot \nabla^\perp \Delta^{-\frac{1}{2}} \right) b \]

Reference flow:
- deterministic
- SQG
- 1024 x 1024

\[ t = 17 \text{ days} \]
SQG

\[ \frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyperviscosity} \]

\[ u = \left( \text{cst.} \nabla \perp \Delta^{-\frac{1}{2}} \right) b \]

Reference flow:

deterministic

SQG

1024 x 1024

\[ t = 17 \text{ days} \]
One realization: Stochastic destabilization

[Image of three graphs showing different scenarios at t = 17 days.]

- Deterministic 128 x 128
- Deterministic 1024 x 1024
- Location Uncertainty 128 x 128
One realization: Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128

Spectrum
One realization: Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128

Spectrum
One realization: Stochastic destabilization
One realization: Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128
One realization: Stochastic destabilization
One realization:
Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128
One realization:
Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128
Ensemble: random coherent structures
Ensemble: random coherent structures
Ensemble: uncertainty quantification

Spectrum of the errors and its estimation

Bias RanIC
Bias LU
Estim. error RanIC
Estim. error LU
Ensemble: uncertainty quantification
Conclusion
Conclusion

LU models blindly describe unresolved triades

• Conserve energy

• Stabilization / destabilization in Reduced Order Model

• Instabilities triggered, possibly followed by extreme events

• Uncertainty quantification to address filter divergence
Related works, outlooks and application

- Bifurcations (SQG) and attractor (Lorenz 63) exploration
- Comparisons with data-driven parametrisation and SALT (Stochastic Advection by Lie Transport) (Holm and coauthors)
- Parametrization and tests based on higher-order statistics (curvature, energy flux through scales, bispectrum, …)
- (Surface gravity) wave / turbulence interaction

**Data assimilation (DA):**

- **Filtering / smoothing**
  - EnKF with LU model as a R&D tool (for e.g. airplanes, drones)
  - PF with reduced LU model for real-time monitoring and flow control (for e.g. pollutant dispersion monitoring, drag and damage reduction in e.g. wind turbines)
- Girsanov theorem for MLE and Bayesian estimations with e.g. satellite images