Uncertainty quantification and ensemble forecast in coarse-grid or dimensionally-reduced computational fluid dynamics

Valentin Resseguier, Etienne Mémin, Bertrand Chapron

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Motivations

- Rigorously identified sudgrid dynamics effects
- Injecting likely small-scale dynamics
- Studying bifurcations and attractors

  → Climate projections
- Quantification of modeling errors

  → Ensemble forecasts and data assimilation

Contents

- Scalian
- Location uncertainty
- SQG under moderate uncertainty

Part I : SCALIAN

L@b
(~ 15 peoples)

Research, R&T, R&D

Expertise:

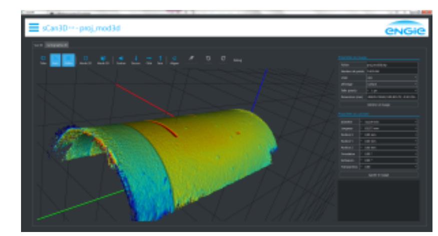
- Geophysical fluid dyn.
- Signal, data assimilation
- Machine Learning
- Multi-agents systems
- Drones

Other Business Units

~ 2400 people

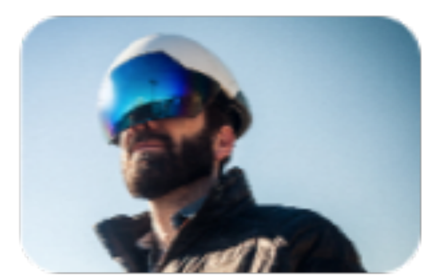
CEN « Simulation » (~ 70 people)

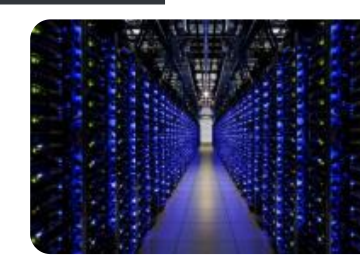
R&D and engineering



Expertise:

- Radar, optronics, sonar
- Geophysical fluid dyn.
- Mechanical and thermal





Business:

- Scientific softwares
- Simulations, HPC
- VR & AR

Part II
Location uncertainty
(LU)

Usual random CFD

- Random parameters, boundary conditions, forcing → Other (complementary) issues
- Random initial conditions → Underdispersive + need large ensemble
- Arbitrary Gaussian forcing → Adding energy + wrong phase
- Averaging, homogenization → Assumptions and energy issues

Usual random CFD

- Random parameters, boundary conditions, forcing → Other (complementary) issues
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LU :Adding random velocity

$$\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$$

LU :Adding random velocity

Resolved
large scales

$$\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$$

LU :Adding random velocity

Resolved large scales

White-in-time small scales

$$\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$$

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma}\dot{\boldsymbol{B}}$

Variance tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x},\boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B}\,(\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$$

LU :Adding random velocity

Resolved large scales

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LU :Adding random velocity

Resolved large scales

White-in-time small scales

$$\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$$

References :

Mikulevicius and Rozovskii, 2004
Flandoli, 2011

Memin, 2014
Resseguier et al. 2017 a, b, c
Chapron et al. 2017
Cai et al. 2017

Holm, 2015
Holm and Tyranowski, 2016
Arnaudon et al., 2017

Cotter and al 2017
Crisan et al., 2017
Gay-Balmaz & Holm 2017
Cotter and al 2018 a, b

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma}\dot{B}$

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$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x},\boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} \, (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

Advection of tracer $\boldsymbol{\Theta}$

$$\frac{D\Theta}{Dt} = 0$$

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma}\dot{\boldsymbol{B}}$

Variance tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x},\boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B}\,(\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$$

Advection of tracer Θ

Advection of tracer $\boldsymbol{\Theta}$

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma}\dot{\boldsymbol{B}}$

Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} \, (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

$$\partial_t \Theta + \boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta = \boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla}\Theta \right)$$

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma}\dot{B}$

Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x},\boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} dB\,(\boldsymbol{\sigma} dB)^T\}}{dt}$

Advection of tracer $\boldsymbol{\Theta}$

Advection

$$\partial_t \Theta + \boxed{\boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{B} \cdot \boldsymbol{\nabla}\Theta} = \boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla}\Theta \right)$$

Advection of tracer $\boldsymbol{\Theta}$

Large scales:

$\boldsymbol{w}$

Small scales:

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Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x},\boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B}\,(\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

Advection

Diffusion

$$\partial_t \Theta + \underbrace{\boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta}_{\text{Advection}} = \underbrace{\boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla}\Theta \right)}_{\text{Diffusion}}$$

Large scales:

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Small scales:

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Advection

Diffusion

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Drift correction

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma}\dot{B}$

Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x},\boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B}\,(\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

Advection of tracer $\boldsymbol{\Theta}$

Multiplicative random forcing

Advection

Diffusion

$$\partial_t \Theta + \boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta = \boldsymbol{\nabla} \cdot \left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla}\Theta\right)$$

Drift correction

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Advection of tracer $\boldsymbol{\Theta}$

Multiplicative random forcing

Advection

Diffusion

$$\partial_t \Theta + \boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{B} \cdot \boldsymbol{\nabla}\Theta = \boldsymbol{\nabla} \cdot \left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla}\Theta\right)$$

Drift correction

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma}\dot{\boldsymbol{B}}$

Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x},\boldsymbol{x}) = \dfrac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B}\, (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

Advection of tracer Θ

Multiplicative random forcing

Advection

Diffusion

$$\partial_t \Theta + \boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta = \boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla}\Theta \right)$$

Drift correction

Large scales:

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Small scales:

$\boldsymbol{\sigma}\dot{B}$

Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} dB\,(\boldsymbol{\sigma} dB)^T\}}{dt}$

Advection of tracer Θ

Multiplicative random forcing

Balanced energy exchanges

Advection

Diffusion

$$\partial_t \Theta + \boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{B} \cdot \boldsymbol{\nabla}\Theta = \boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla}\Theta \right)$$

Drift correction

A word about reduced order models

- Very fast simulation of very complex system (e.g. for industrial application)
- Physical model (PDE) simplified using observations

Reduced models under location uncertainty

- Rigorous and low-cost estimators
- Stabilization of the unstable modes
- Maintain variability of stable modes
- Uncertainty quantification

State of art

- Possible parametrization with eddy viscosity
- Impossible to parametrize with additional dissipation
  Need ad hoc closure like MQG
  (Sapsis and Majda, 2013a,b,c)

Part III
SQG under Moderate Uncertainty

SQG MU

Code available online

$$t = 17 \text{ days}$$

SQG

$$\frac{Db}{Dt} = \boxed{-\alpha_{HV}\Delta^4 b \quad \text{Hyper-viscosity}}$$

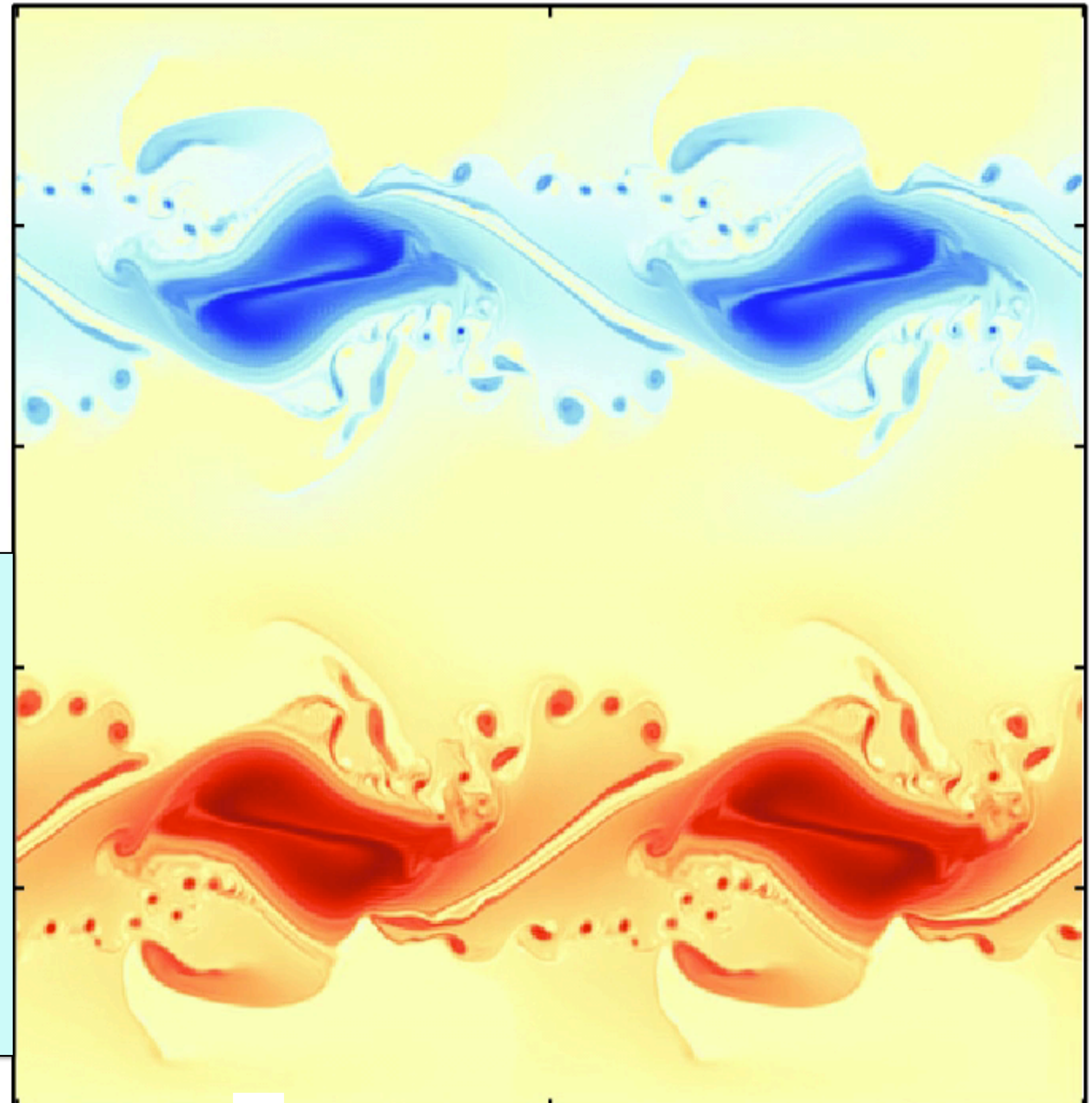
$$\boldsymbol{u} = \left(\text{cst}.\boldsymbol{\nabla}^{\perp}\Delta^{-\frac{1}{2}}\right) b$$

Reference flow:

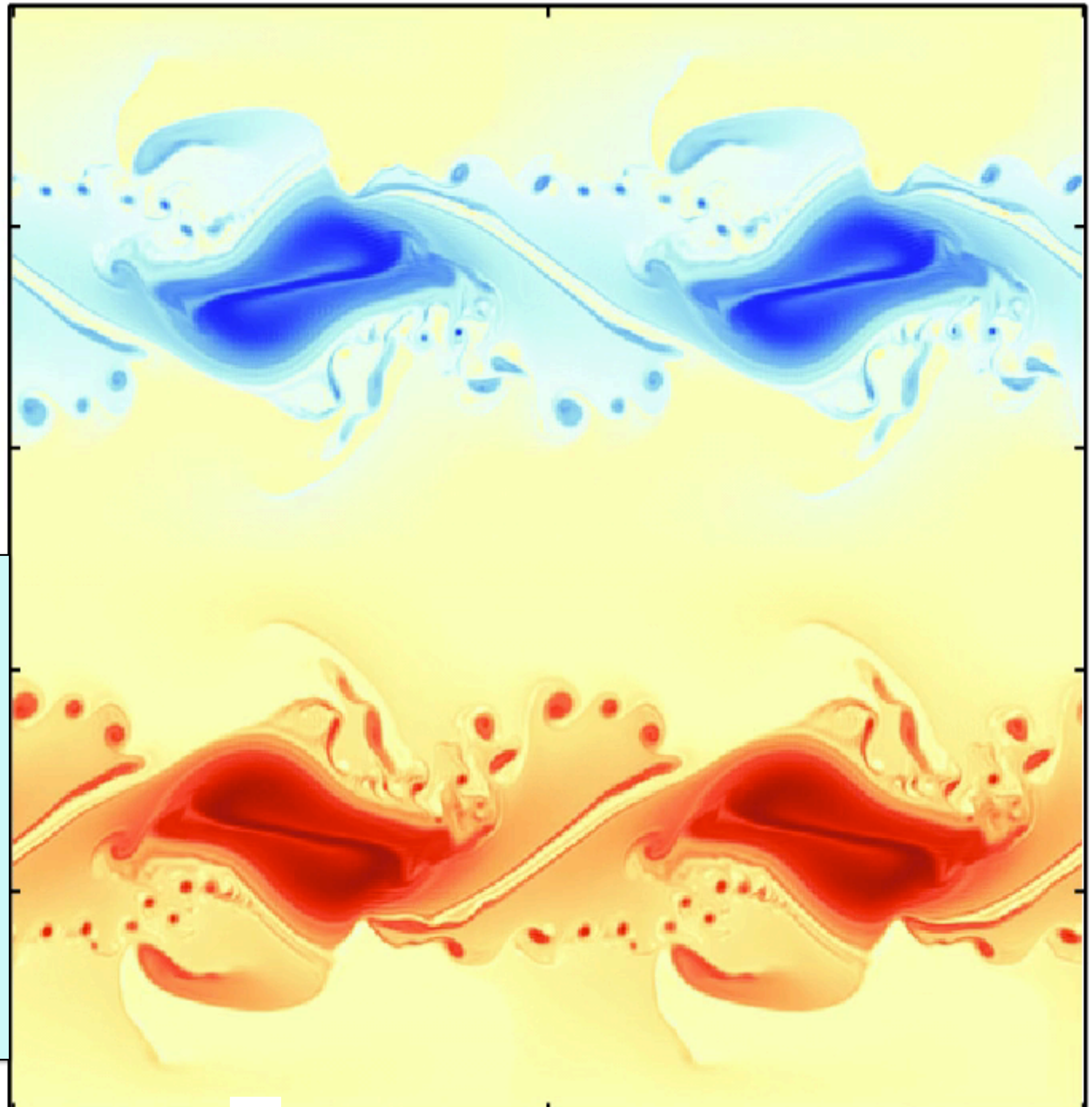
deterministic

SQG

1024 x 1024



$$t = 17 \text{ days}$$



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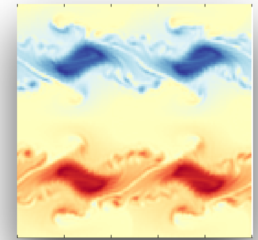
Reference flow:

deterministic

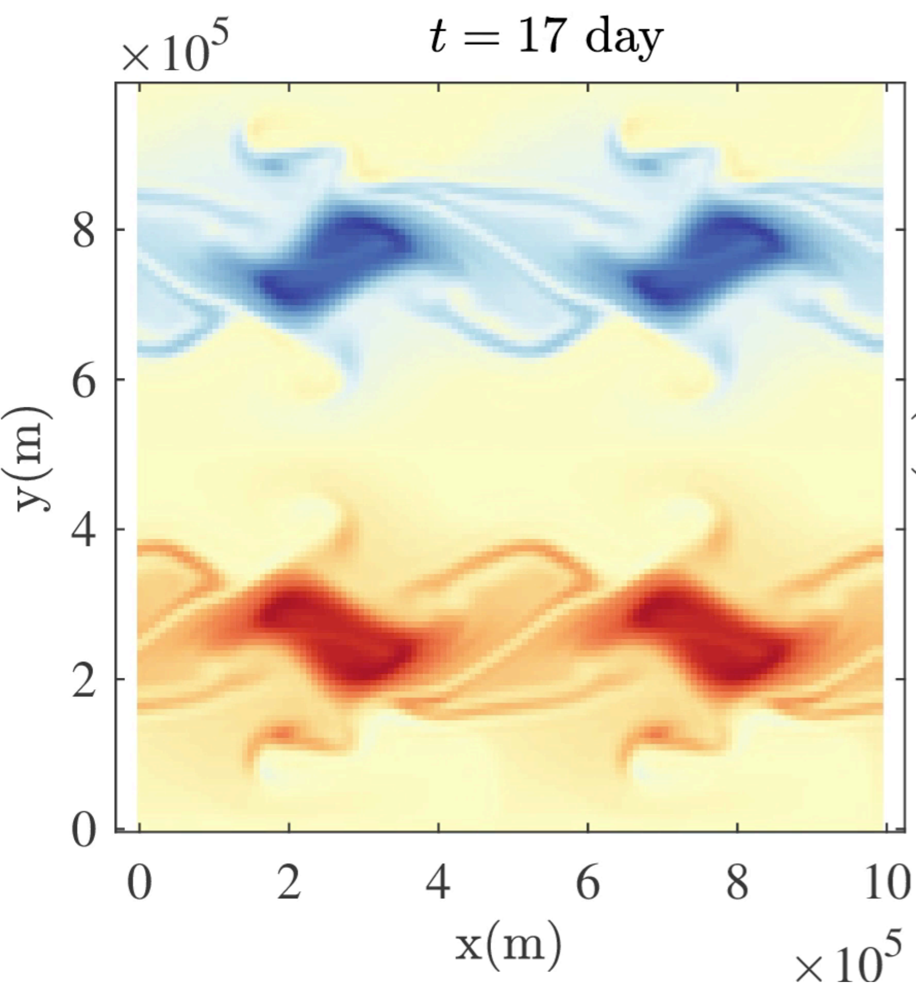
SQG

1024 x 1024

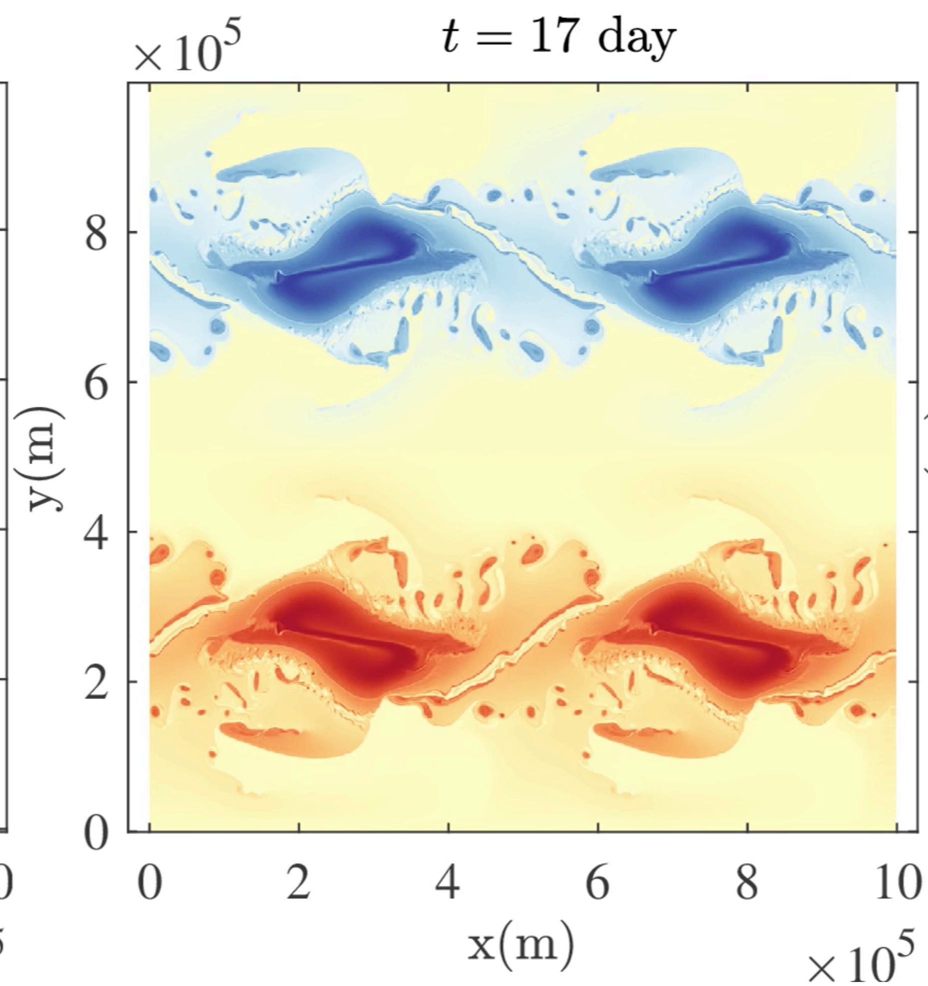


One realization : Stochastic destabilization

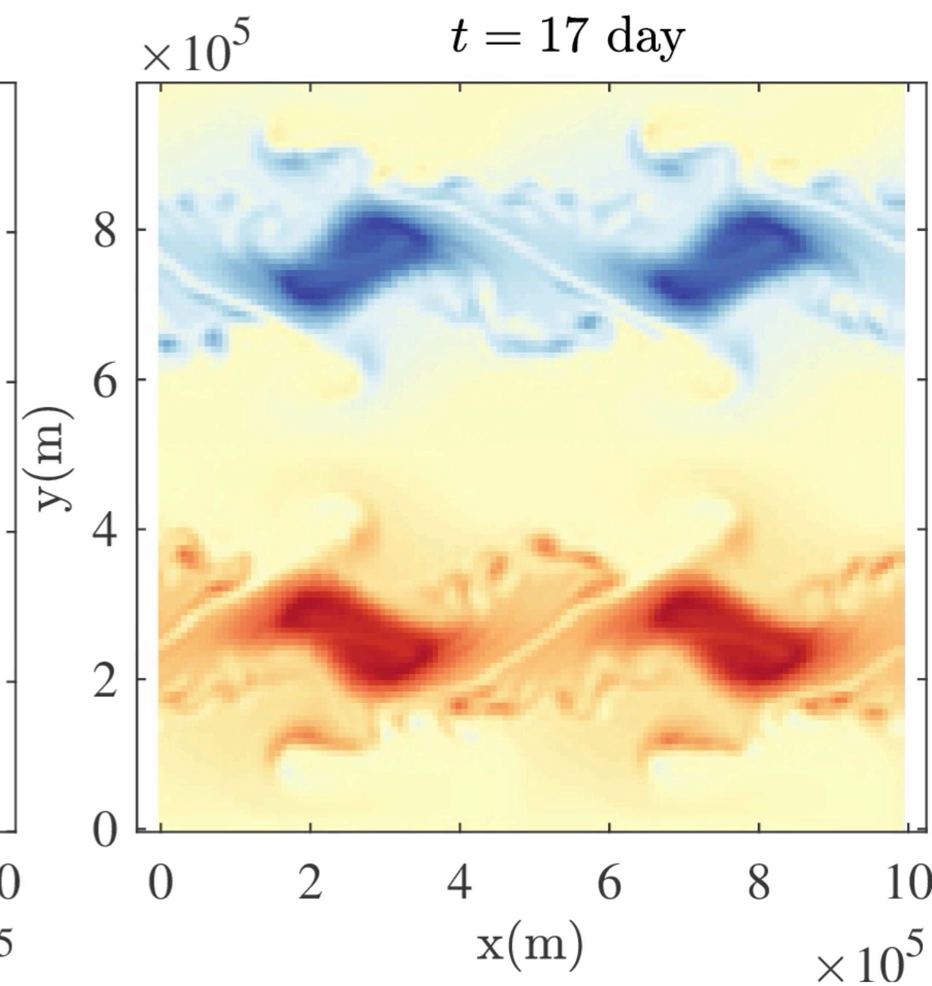
Deterministic 128 x 128

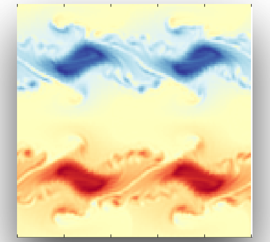


Deterministic 1024 x 1024



Location Uncertainty 128 x128





One realization : Stochastic destabilization

Deterministic 128 x 128

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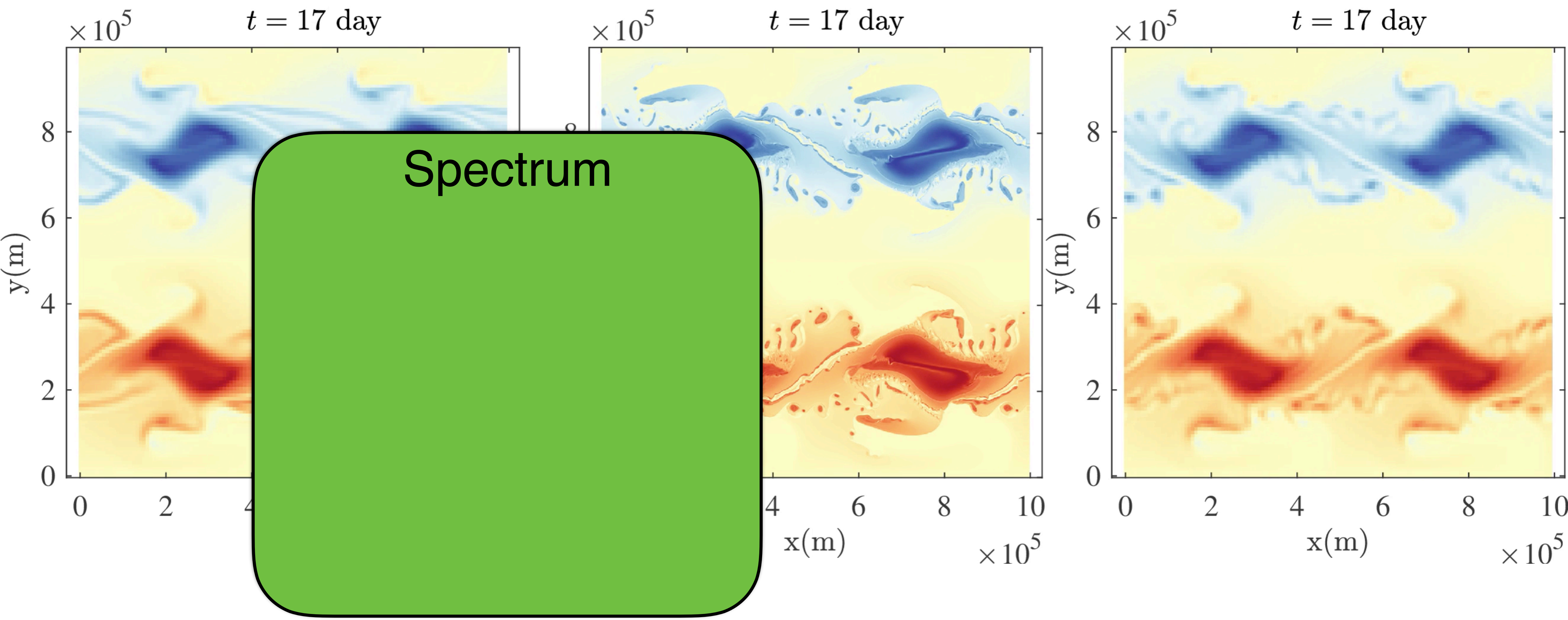
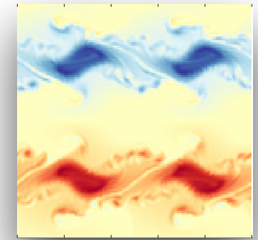
Location Uncertainty 128 x128





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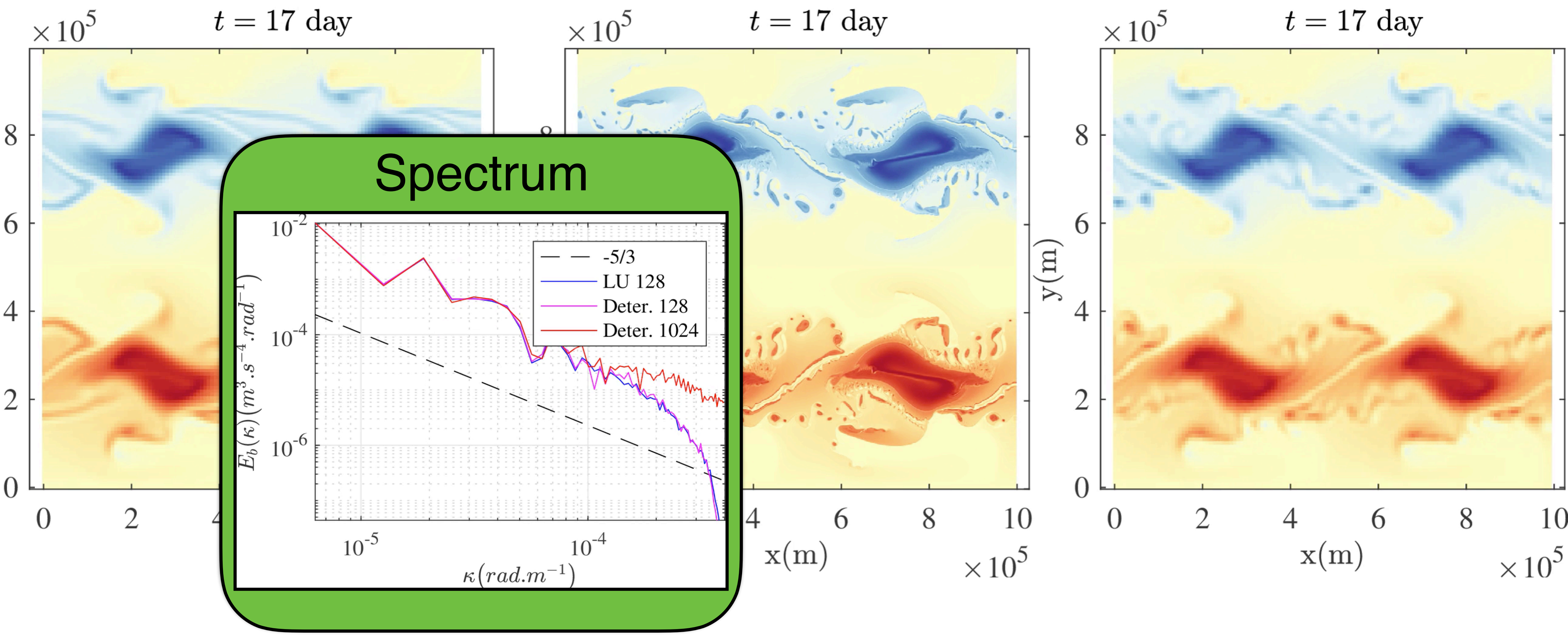
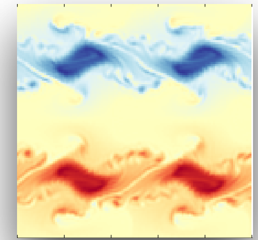
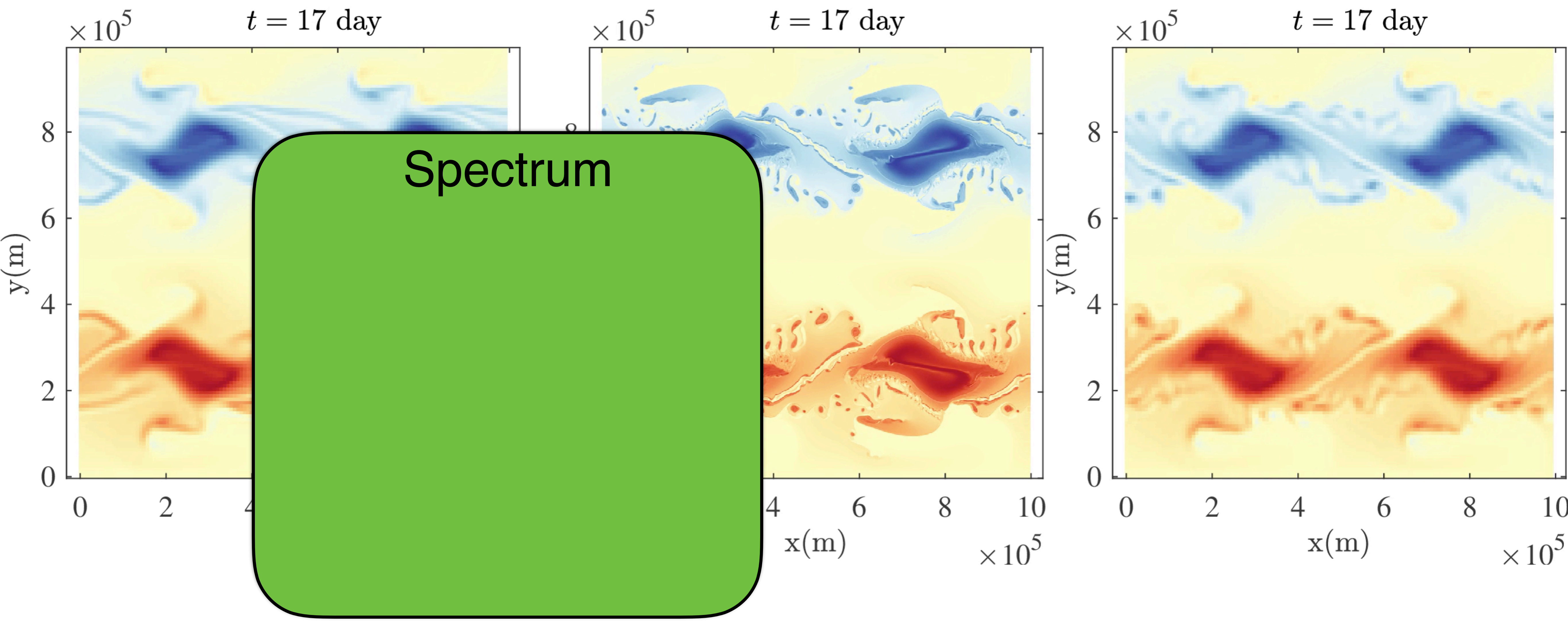
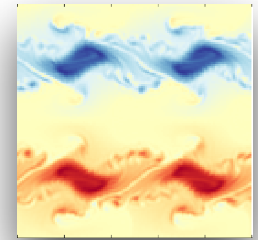




One realization : Stochastic destabilization

Deterministic 128 x 128 | Deterministic 1024 x 1024 | Location Uncertainty 128 x128





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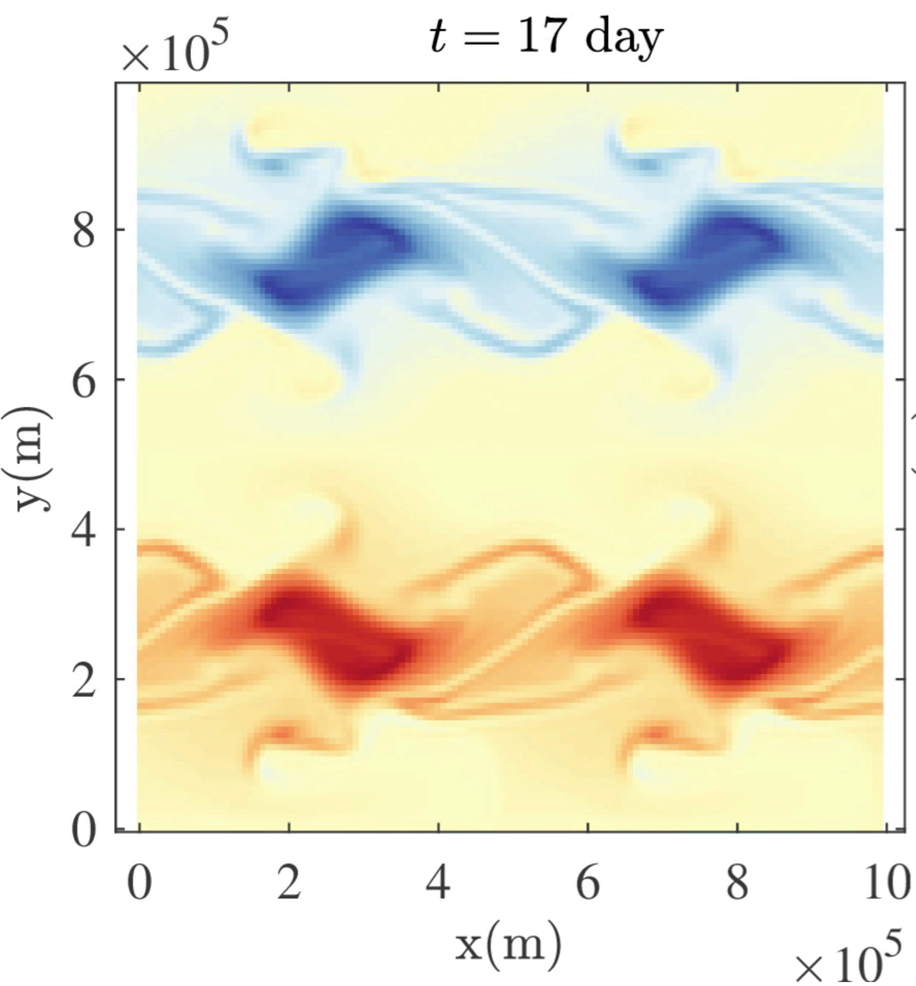
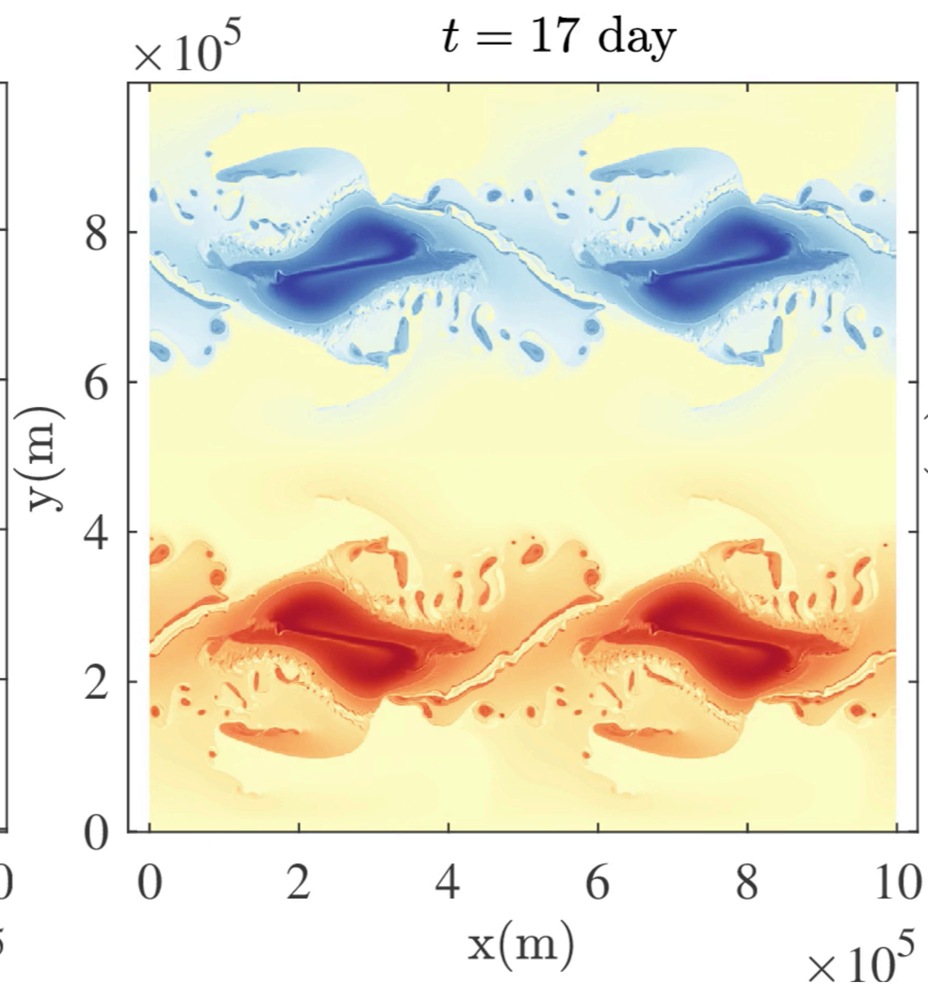
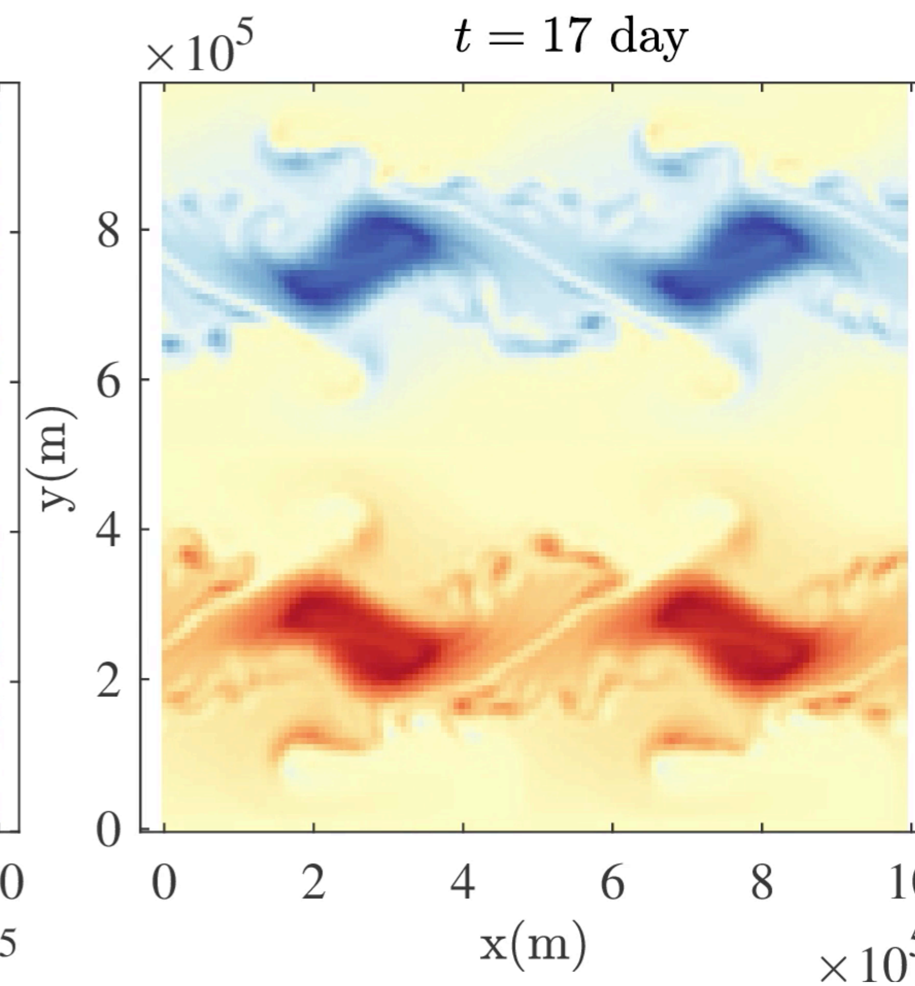
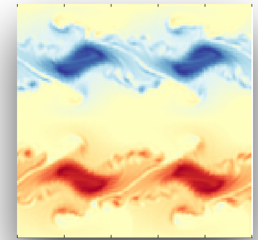


Deterministic 1024 x 1024



Location Uncertainty 128 x128





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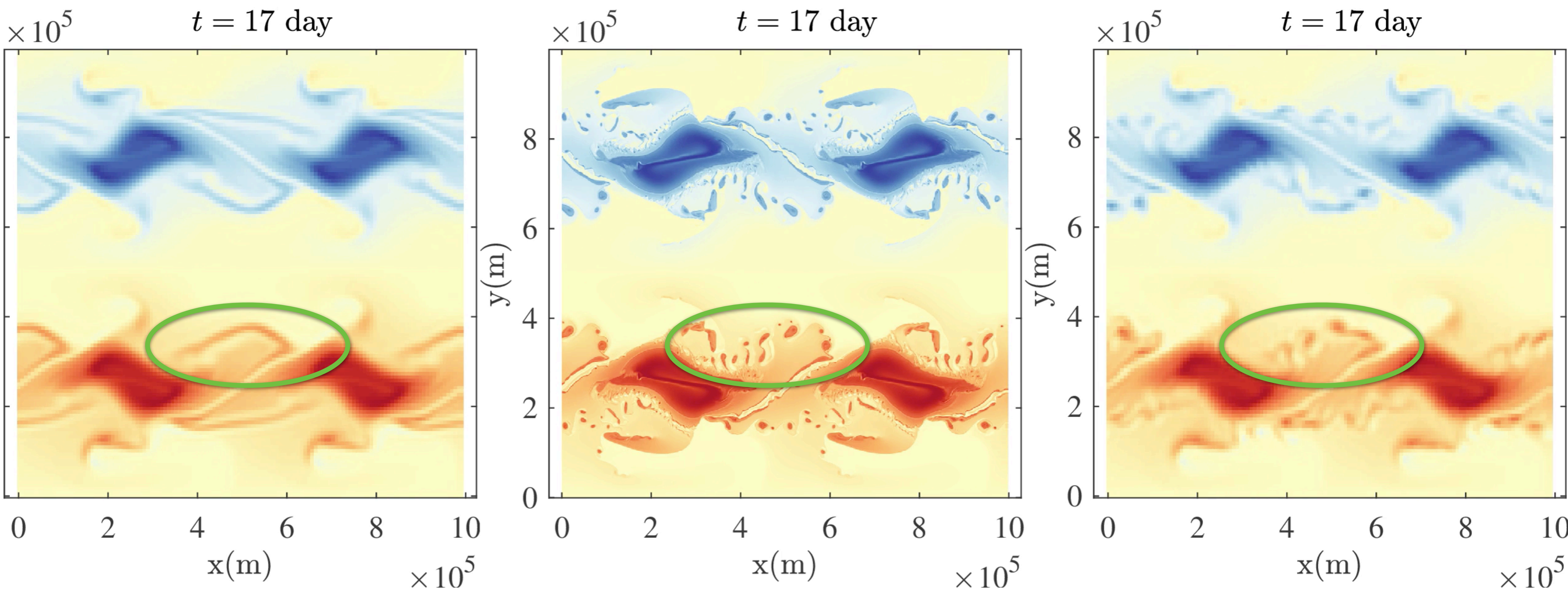

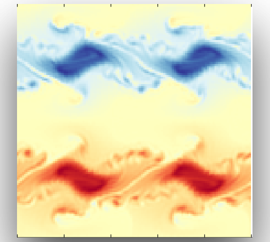

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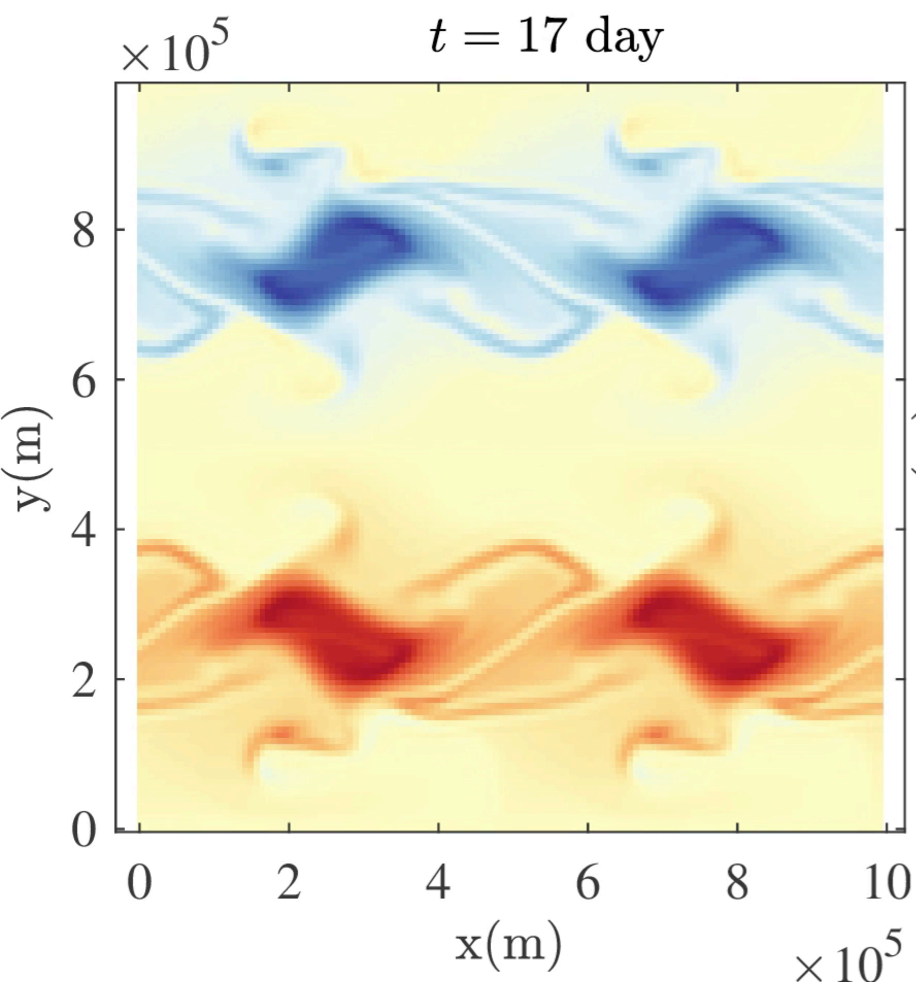
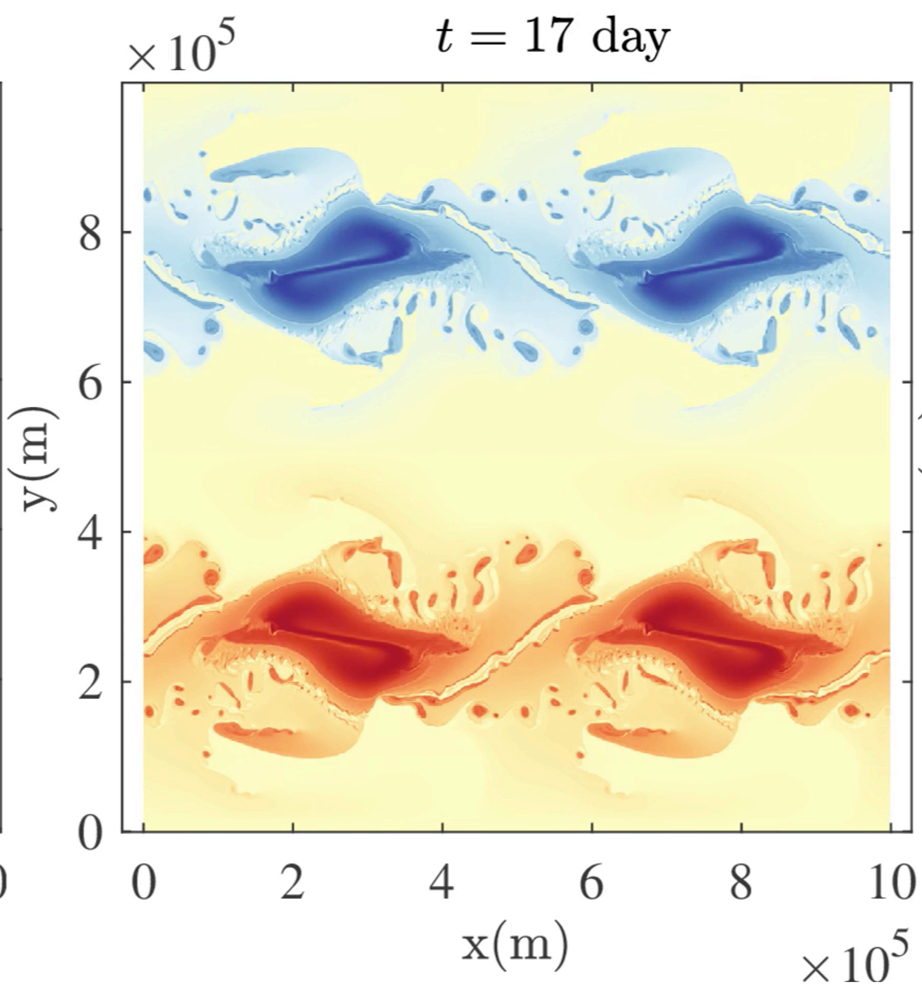


Deterministic 1024 x 1024



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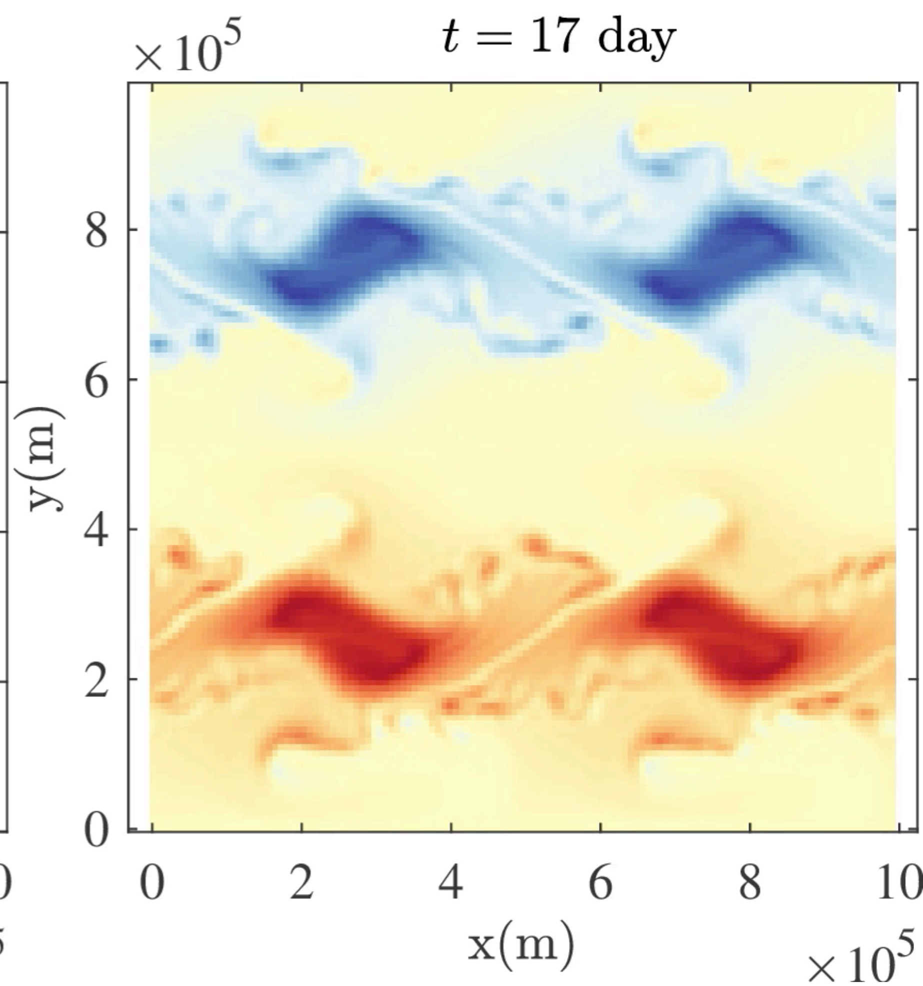

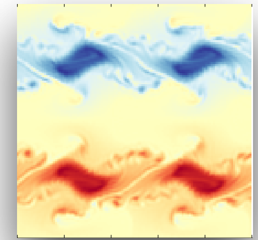

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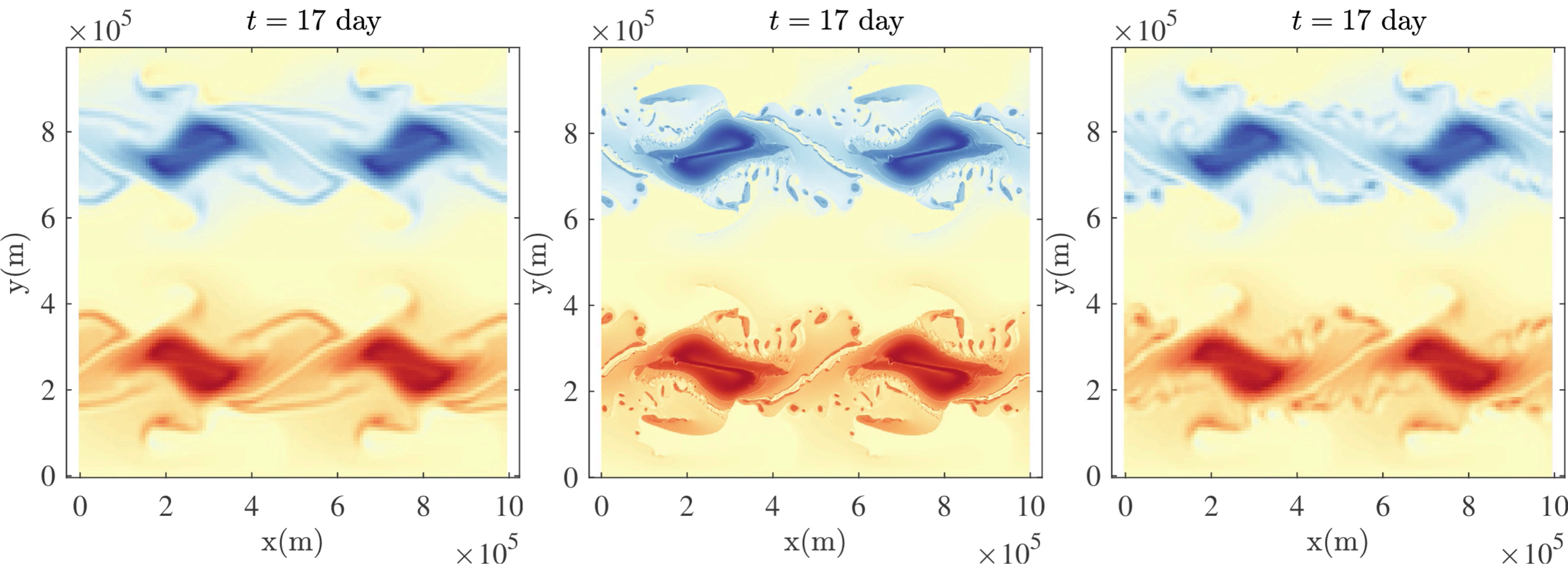
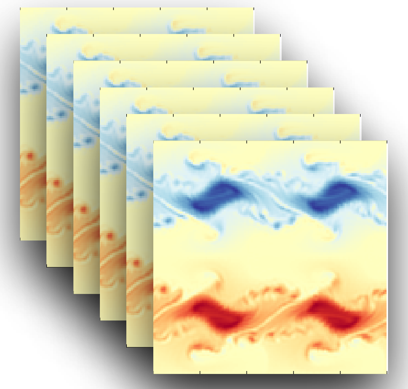
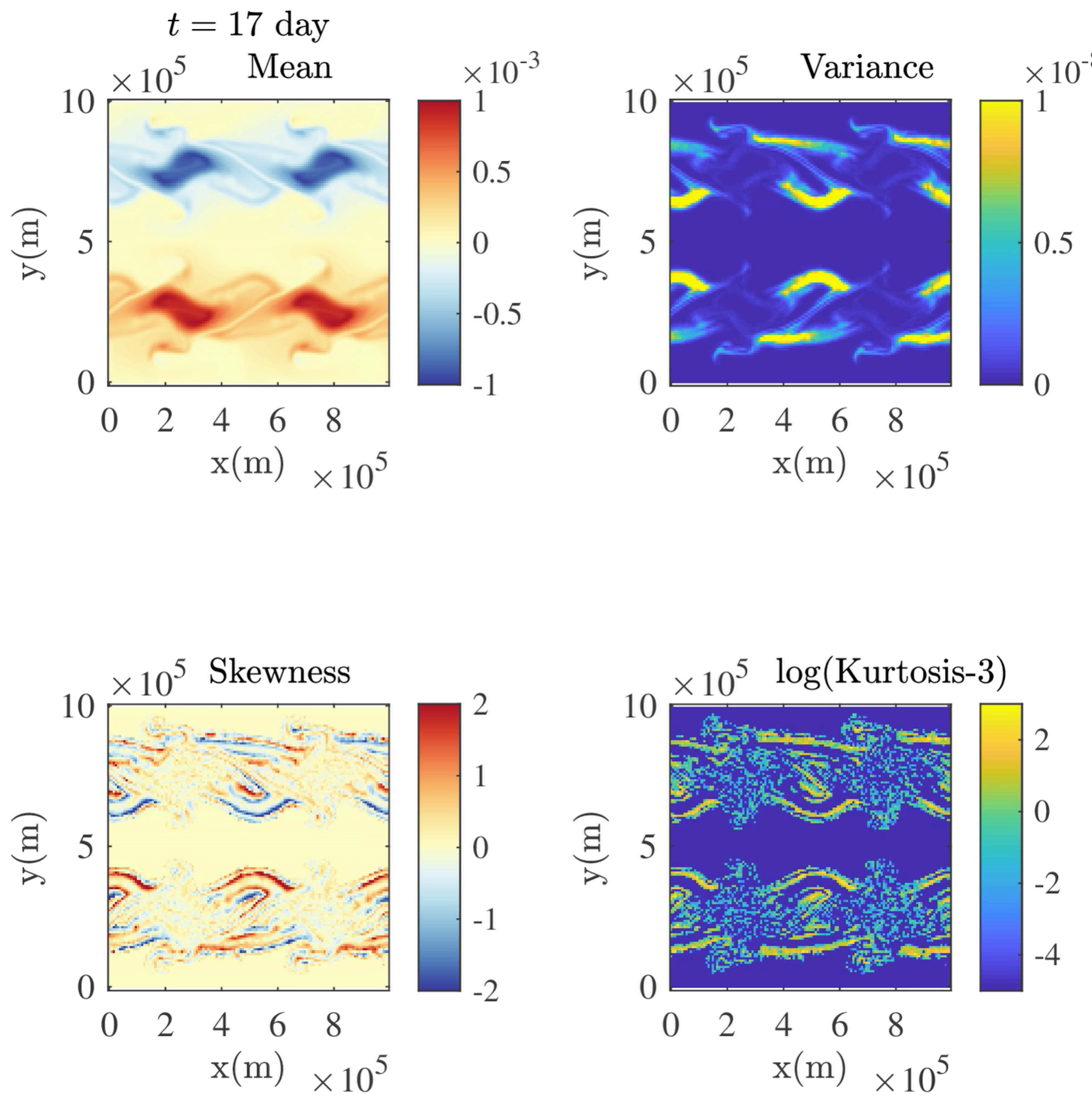
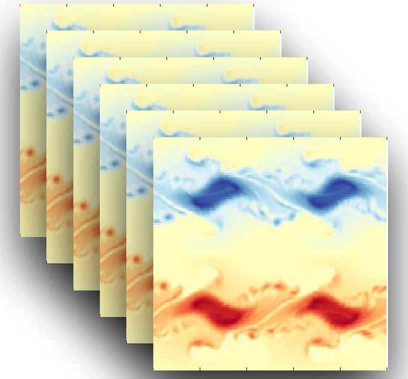
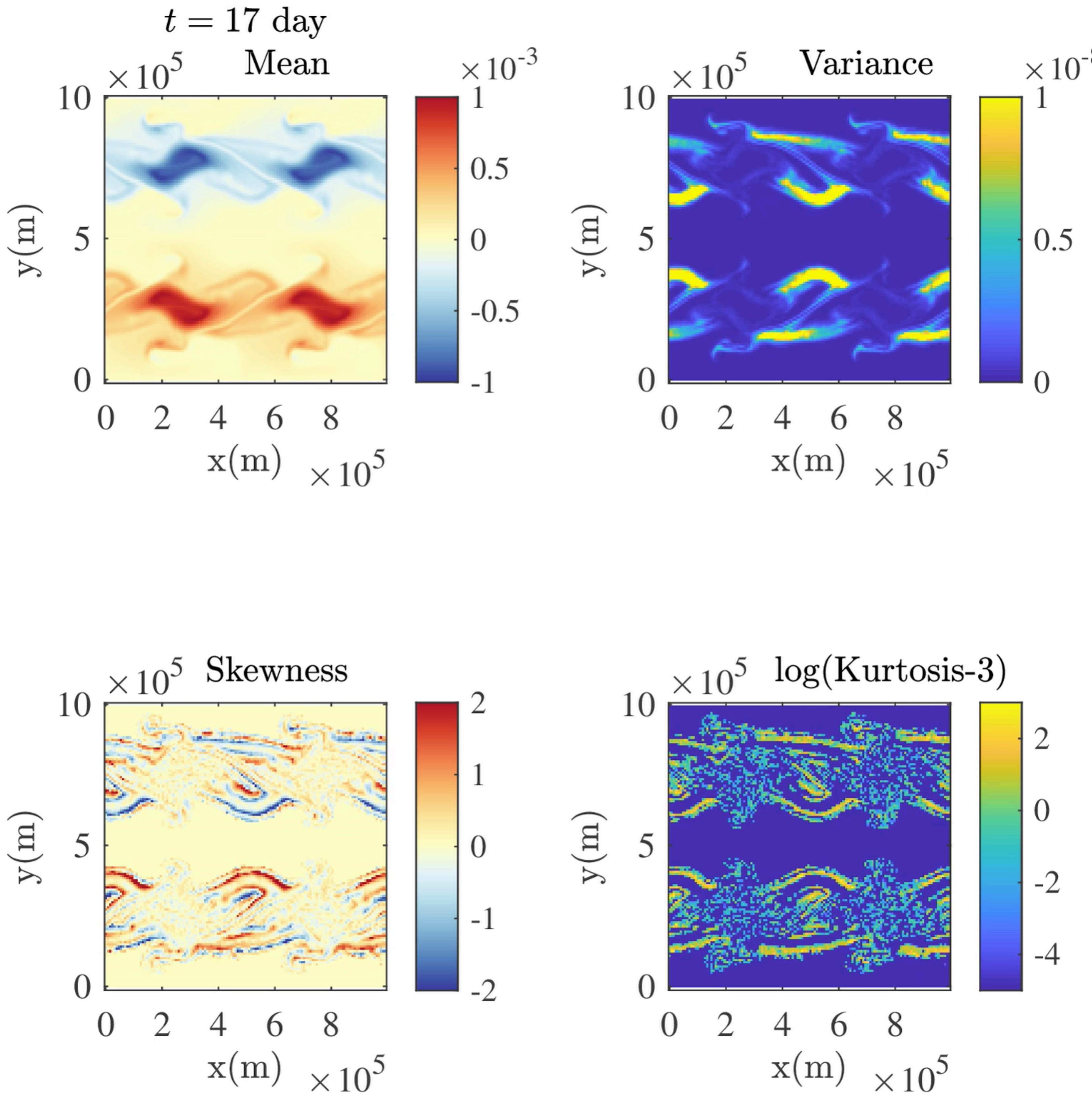




Ensemble : random coherent structures







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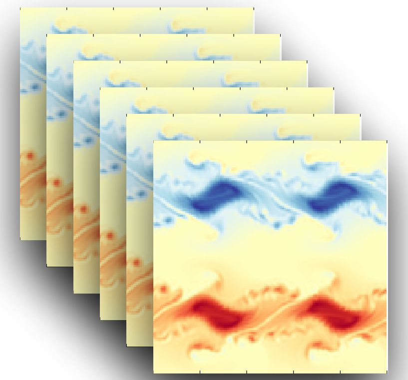
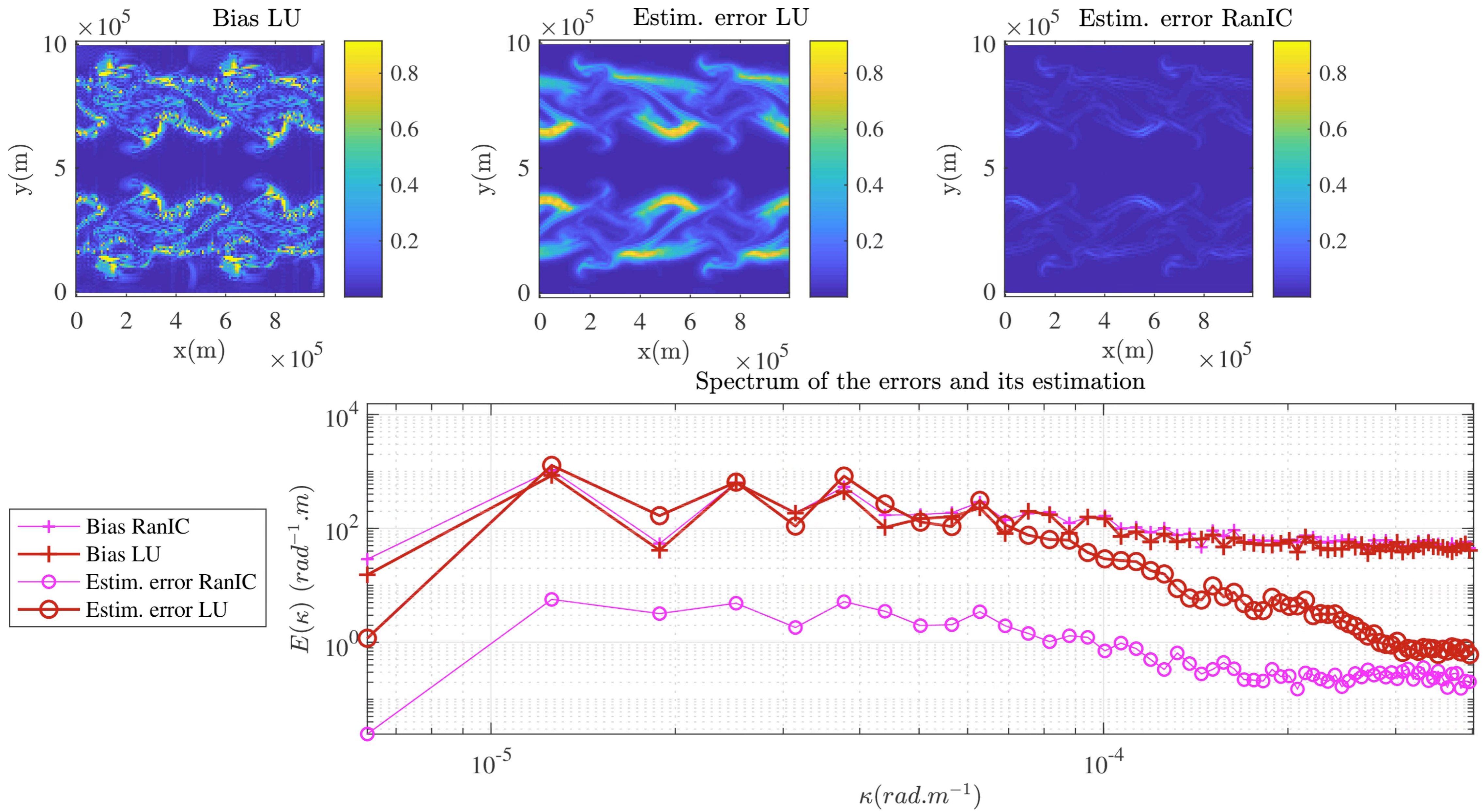
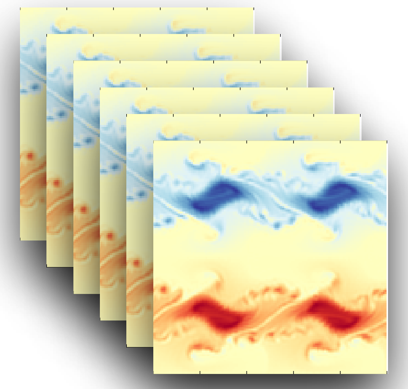


Ensemble : uncertainty quantification





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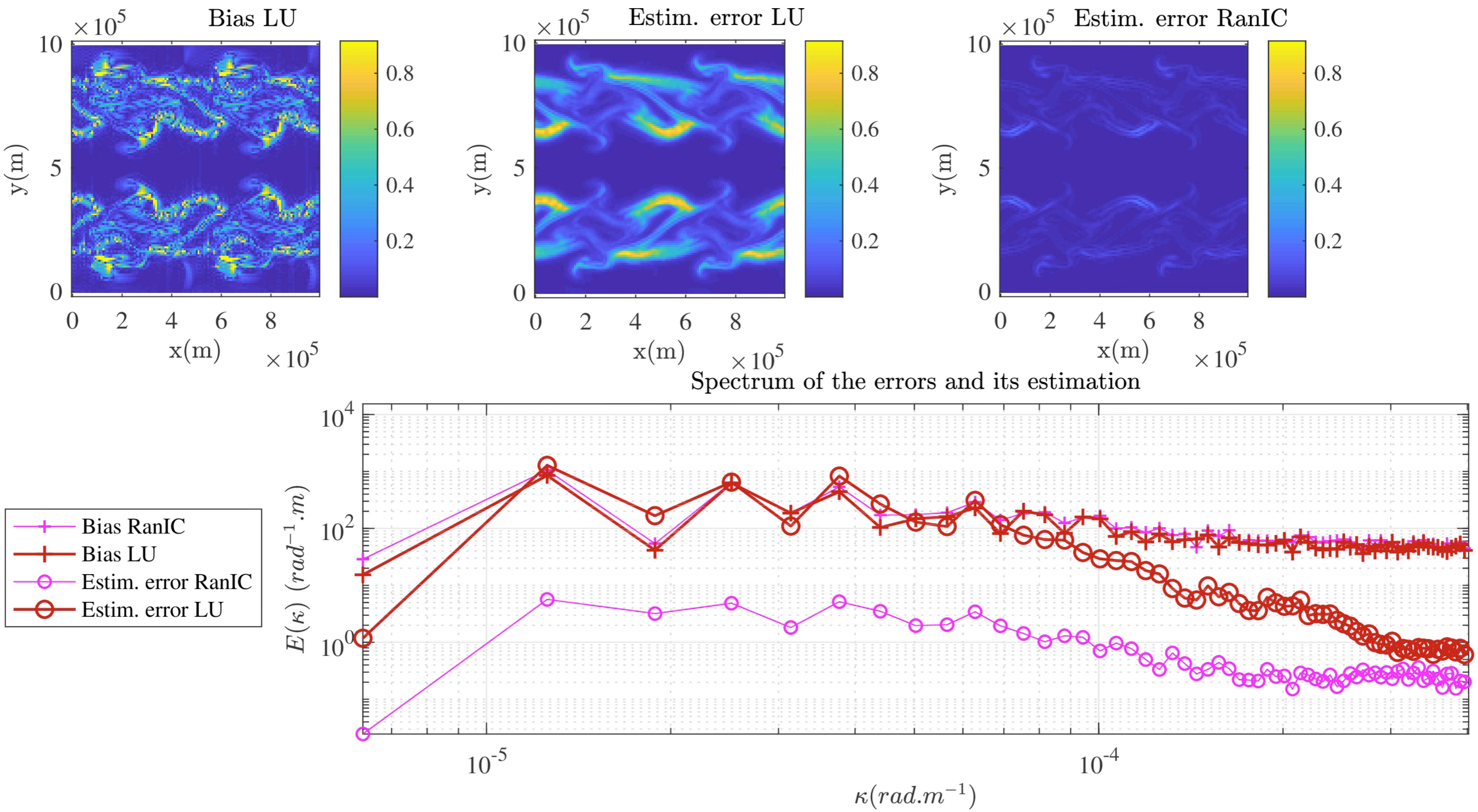

Conclusion

Conclusion

LU models blindly describe unresolved triades

- Conserve energy
- Stabilization / destabilization in Reduced Order Model
- Instabilities triggered,
  possibly followed by extreme events
- Uncertainty quantification to address filter divergence

Related works, outlooks and application

- Bifurcations (SQG) and attractor (Lorenz 63) exploration
- Comparisons with data-driven parametrisation
  and SALT (Stochastic Advection by Lie Transport) (Holm and coauthors)
- Parametrization and tests based on higher-order statistics
  (curvature, energy flux through scales, bispectrum, …)
- (Surface gravity) wave / turbulence interaction
- **Data assimilation (DA) :**
  - **Filtering / smoothing**
    - **EnKF with LU model as a R&D tool** (for e.g. airplanes, drones)
    - **PF with reduced LU model for real-time monitoring and flow control**
      (for e.g. polluant dispersion monitoring, drag and damage reduction in e.g. wind turbines)
  - Girsanov theorem for MLE and Bayesian estimations with e.g. satellite images