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Dynamics under location uncertainty: model derivation, modified transport and uncertainty quantification

Valentin Resseguier, Baylor Fox-Kemper
Etienne Memin, Bertrand Chapron
Motivations

• Rigorously identified subgrid dynamics effects

• Injecting likely small-scale dynamics

• Predict extreme events

• Quantification of modeling errors  Ensemble forecasts and data assimilation

• Studying different likely scenarios and attractors  Climate projections
Contents

I. Location uncertainty

II. SQG under moderate uncertainty
Part I
Location uncertainty
Adding random velocity

\[ \nu = w + \sigma \dot{B} \]
Adding random velocity

\[ \nu = \mathbf{w} + \sigma \mathbf{B} \]
Adding random velocity

\[ v = w + \sigma \dot{B} \]

Resolved large scales

White-in-time small scales
Adding random velocity

\[ \mathbf{v} = \mathbf{w} + \sigma \dot{\mathbf{B}} \]

Resolved large scales

White-in-time small scales
Adding random velocity

Large scales: \( w \)
Small scales: \( \sigma B \)
Variance tensor:

\[
a = a(x, x) = \mathbb{E}\{\sigma dB \sigma dB^T\}/dt
\]

\[
\nu = w + \sigma B
\]

Resolved large scales

White-in-time small scales

References:

- Mikulevicius and Rozovskii, 2004
- Flandoli, 2011
- Memin, 2014
- Resseguier et al. 2017 a, b, c
- Chapron et al. 2017
- Cai et al. 2017
- Holm, 2015
- Holm and Tyranowski, 2016
- Arnaudon et al. 2017
- Cotter and al 2017
- Crisan et al., 2017
- Gay-Balmaz and Holm 2017


Advection of tracer $\Theta$

$$\frac{D\Theta}{Dt} = 0$$
Advection of tracer $\Theta$

Large scales: $w$
Small scales: $\sigma \dot{B}$
Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB \, \sigma dB^T\}}{dt}$$
Advection of tracer $\Theta$

\[ \partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right) \]
Advection of tracer $\Theta$

\[ \partial_t \Theta + \mathbf{w}^\ast \cdot \nabla \Theta + \sigma \dot{\mathbf{B}} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} \mathbf{a} \nabla \Theta \right) \]
Advection of tracer $\Theta$

\[ \partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{\mathbf{B}} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right) \]

Large scales: $\mathbf{w}$
Small scales: $\sigma \dot{\mathbf{B}}$

Variance tensor:
\[ a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} \]
Advection of tracer $\Theta$

Large scales:
$w$
Small scales:
$\sigma \dot{B}$
Variance tensor:
$a = a(x, x) = \frac{E \{ \sigma dB (\sigma dB)^T \}}{dt}$

$$\frac{\partial \Theta}{\partial t} + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$

Drift correction
Advection of tracer $\Theta$

Large scales: $w$
Small scales: $\sigma \dot{B}$

Variance tensor:

$\begin{align*}
a &= a(x, x) = \\
\mathbb{E}\{\sigma dB (\sigma dB)^T\} \\
dt
\end{align*}$

$\frac{\partial_t \Theta}{\partial t} + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$

Drift correction

Multiplicative random forcing

Diffusion
Advection of tracer $\Theta$

\[ \partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \mathbf{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} \alpha \nabla \Theta \right) \]

- Large scales: $\mathbf{w}$
- Small scales: $\sigma \mathbf{B}$
- Variance tensor: $\alpha = \alpha(x, x) = \mathbb{E}\{\sigma dB \sigma dB^T\}/dt$

Drift correction

Multiplicative random forcing

Diffusion
Advection of tracer $\Theta$

Large scales:
$w$
Small scales:
$\sigma \dot{B}$

Variance tensor:
$a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\}/dt$

\[
\partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} \alpha \nabla \Theta \right)
\]

Drift correction
Multiplicative random forcing
Advection of tracer $\Theta$

\[ \partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{\mathbf{B}} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right) \]

- Large scales: $\mathbf{w}$
- Small scales: $\sigma \dot{\mathbf{B}}$
- Variance tensor: $a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} \frac{dt}{dt}$

Drift correction
Multiplicative random forcing
Balanced energy exchanges
Derived random models

Large scales:
\( w \)
Small scales:
\( \sigma \dot{B} \)
Variance tensor:
\[
a = a(x, x) = \mathbb{E}\left\{ \sigma dB (\sigma dB)^T \right\} \frac{dt}{dt}
\]

Conservations (mass, linear momentum, …)

\( \frac{D}{Dt} \)

Navier-Stokes
Derived random models

Large scales: \( w \)
Small scales: \( \sigma \dot{B} \)
Variance tensor:
\[
a = a(x, x) = \mathbb{E}\left\{ \sigma dB (\sigma dB)^T \right\} dt
\]

Conservations (mass, linear momentum, ...)
Derived random models

Large scales: \( w \)
Small scales: \( \sigma \dot{B} \)

Variance tensor:
\[
a = a(x, x) = \frac{1}{d} \mathbb{E} \left\{ \sigma dB (\sigma dB)^T \right\}
\]

Conservations (mass, linear momentum, ...)

\[
\frac{D}{Dt}
\]

Navier-Stokes

Simplifications

Data

Reduced Order Model
Derived random models

Large scales: \( w \)
Small scales: \( \sigma \dot{B} \)
Variance tensor: 
\[
a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\} \frac{dt}{dt}
\]

Conservations (mass, linear momentum, …)

\[
\frac{D}{Dt}
\]

Boussinesq

Navier-Stokes
Derived random models

Conservations
(mass, linear momentum, …)

Large scales:
\( w \)
Small scales:
\( \sigma B \)
Variance tensor:
\( a = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\}/dt \)

Data
Reduced Order Model
Lorenz 63

Simplifications

\[ \frac{D}{Dt} \]

Boussinesq

Navier-Stokes

Variance tensor:
\[ \dot{\mathbf{B}} = a(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\}/dt \]
Derived random models

Large scales:
\[ w \]
Small scales:
\[ \sigma \dot{B} \]

Variance tensor:
\[
\alpha = \alpha(x, x) = \mathbb{E}\left\{ \sigma dB (\sigma dB)^T \right\} dt
\]

Conservations
(mass, linear momentum, …)

\[ \frac{D}{Dt} \]

Data
Reduced Order Model

Lorenz 63

Boussinesq

Navier-Stokes

Simplifications
Derived random models

Large scales: $w$
Small scales: $\sigma \dot{B}$

Variance tensor:
$$ a = a(x, x) = \frac{1}{dt} \mathbb{E}\{\sigma dB (\sigma dB)^T\} $$

Conservations (mass, linear momentum, …)

Data

Reduced Order Model

Lorenz 63

Boussinesq

Navier-Stokes

Uncertainty

$$ \frac{a}{2UL} $$

Simplifications
Derived random models

Large scales: $w$
Small scales: $\sigma \dot{B}$

Variance tensor:

\[ a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt} \]

Conservations (mass, linear momentum, ...)

\[ \frac{D}{Dt} \]

Reduced Order Model

Lorenz 63

Boussinesq

Navier-Stokes

QG

QG MU

Uncertainty

\[ \frac{a/2}{UL} \]

Simplifications
Derived random models

Large scales: \( \mathbf{w} \)
Small scales: \( \mathbf{\sigma B} \)

Variance tensor:
\[
\mathbf{a} = a(x, x) = \mathbb{E}\{\mathbf{\sigma dB} (\mathbf{\sigma dB})^T\} / dt
\]

Conservations (mass, linear momentum, …)

\[
\frac{D}{Dt}
\]

Data

Reduced Order Model

Lorenz 63

\( \frac{a/2}{UL} \)

SQG MU

QG MU

QG

Uncertainty

Boussinesq

Navier-Stokes

Simplifications
Derived random models

Large scales: \( \mathbf{w} \)
Small scales: \( \sigma \mathbf{B} \)
Variance tensor:
\[
a = a(x, x) = \frac{E\{\sigma dB (\sigma dB)^T\}}{dt}
\]

Conservations (mass, linear momentum, …)

\[
\frac{D}{Dt}
\]

Simplifications

Uncertainty:
\[
\frac{a/2}{UL}
\]

Derived random models:

Lorenz 63

\[
\dot{B} = a(x, x) = E\{\sigma dB (\sigma dB)^T\}
\]

SQG MU

SQG SU

Boussinesq

Navier-Stokes
Derived random models

Large scales: $w$
Small scales: $\sigma \dot{B}$

Variance tensor:

$\sigma = \sigma(x, x) = \mathbb{E}\{\sigma dB (\sigma dB)^T\}/dt$

Conservations (mass, linear momentum, …)

$\frac{D}{Dt}$

Reduced Order Model

Data

Lorenz 63

Uncertainty

Simplifications

SQG MU

SQG SU

Boussinesq

Navier-Stokes
Part II
SQG under Moderate Uncertainty

SQG MU

Code available online
$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b$  

$\mathbf{u} = \left( \text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$

Reference flow:

deterministic

SQG

1024 x 1024

$t = 17$ day
\[ \frac{D b}{D t} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity} \]

\[ u = \left( \text{cst.} \nabla \frac{1}{\Delta} \right) b \]

Reference flow:

deterministic

SQG

1024 x 1024
One realization: Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128
One realization: Stochastic destabilization
One realization: Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128
One realization: Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128

Spectrum
One realization: Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128

$t = 17$ days
One realization: Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128
One realization: Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128
One realization: Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

Location Uncertainty 128 x 128
Ensemble: random coherent structures
Ensemble: random coherent structures
Ensemble : uncertainty quantification

Spectrum of the errors and its estimation

- Bias RanIC
- Bias LU
- Estim. error RanIC
- Estim. error LU
Ensemble : uncertainty quantification

---

**Bias LU**

- $x(m)$
- $y(m)$

**Estim. error LU**

- $x(m)$
- $y(m)$

**Estim. error RanIC**

- $x(m)$
- $y(m)$

---

**Spectrum of the errors and its estimation**

- $E(\kappa)$ (rad m$^{-1}$)
- $\kappa$ (rad m$^{-1}$)

---

Legend:

- Bias RanIC
- Bias LU
- Estim. error RanIC
- Estim. error LU
Conclusion

Models under location uncertainty blindly describe unresolved triades

• Conserve energy

• Model derivation

• Instabilities triggered, possibly followed by extreme events

• Uncertainty quantification to address filter divergence
Related works and outlooks

- Bifurcations (SQG) and attractor (Lorenz 63) exploration
- Stabilization / destabilization of Reduced Order Model
- Comparisons with data-driven $\mathbf{\sigma}$ and Stochastic Lie Derivative approaches (Holm and coauthors)
- Parametrization and tests based on higher-order statistics (curvature, energy flux through scales, bispectrum, …)
- Mimic barotropization
- Girsanov theorem for MLE and Bayesian estimations with satellite images
- Learning $\mathbf{\sigma}$ on SWOT data
- Filtering / smoothing with (reduced) models under location uncertainty