What MDL can bring to Pattern Mining

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Introduction

Patterns are subsets of attributes that describe an object.

Pattern Mining. Objective: find a small set of patterns that are well interpretable by experts.

Input data: binary table G x M, where G is a set of objects, M is a set of attributes, and I is a relation between them.

Interpretation of gIm: object $g \in G$ has attribute $m \in M$.

Background Knowledge: Assumptions on Interestingness

Idea: use measures that reflect knowledge of experts about 'interestingness" of patterns

Examples of interestingness measures for concept *(A,B)*

$$area(A,B) = |A| \cdot |B|$$

[area] "interesting patterns are those that take the biggest area in dataset"

$$length(A,B) = |B| \, I(|A| \geq q)$$

[length] "Interesting patterns are the most detailed ones that are quite frequent in dataset where $I(\cdot)$ is the indicator*, *q* is a threshold.

$$sep(A,B) = \frac{|A||B|}{\sum_{g \in A}|g'| + \sum_{m \in B}|m'| - |A| \cdot |B|}$$

[separation] "Interesting patterns are separated the best from the context"

$$^{*} \quad I(cond) = \begin{cases} 1 & \textbf{if } cond \textbf{ is } True \\ 0 & \textbf{otherwise} \end{cases}$$

combined measures, etc.

Pattern Mining. What kind of patterns we should compute?

Total number of patterns is 2^M

Types of patterns in terms of Formal Concept Analysis

FCA. Basic Notions

A **formal context** [Ganter and Wille, 1999; Wille, 1982] is a triple *(G,M,I)*, where G is a set objects, *M* is a set attributes, $I \subseteq G \times M$ is a relation called **incidence relation.**
The **derivation operator** (·)' is defined for $Y \subseteq G$ and $Z \subseteq M$ as follows:

$$Y' = \{ m \in M \mid gIm \text{ for all } g \in Y \}; \quad Z' = \{ g \in G \mid gIm \text{ for all } m \in Z \}$$

A **(formal) concept** is a pair *(Y, Z)*, where $Y \subseteq G$, $Z \subseteq M$ and $Y' = Z$, $Z' = Y$. *Y* is called the (formal) **extent** and *Z* is called the (formal) **intent** of the concept *(Y, Z)*.

A **concept lattice** (or Galois lattice) is a partially ordered set of concepts, the order ≪ is defined as follows: *(Y, Z)* ≪ *(C, D)* iff $Y \subseteq C$ ($D \subseteq Z$), a pair *(Y, Z)* is a subconcept of *(C, D)* and *(C, D)* is a superconcept of *(Y, Z)*.
Formal concepts ordered by **generality relation** $(A_1, B_1) \ll (A_2, B_2)$ iff $A_1 \subseteq A_2$ make a lattice, called **concept lattice.**

Types of patterns (defined for concept (A,B)):

Closed itemises (intents): *B*.
Minimal generators are minimal subsets $B_i \subseteq B : B_i' = A$.
Generators are any patterns between minimal generators and closed itemises

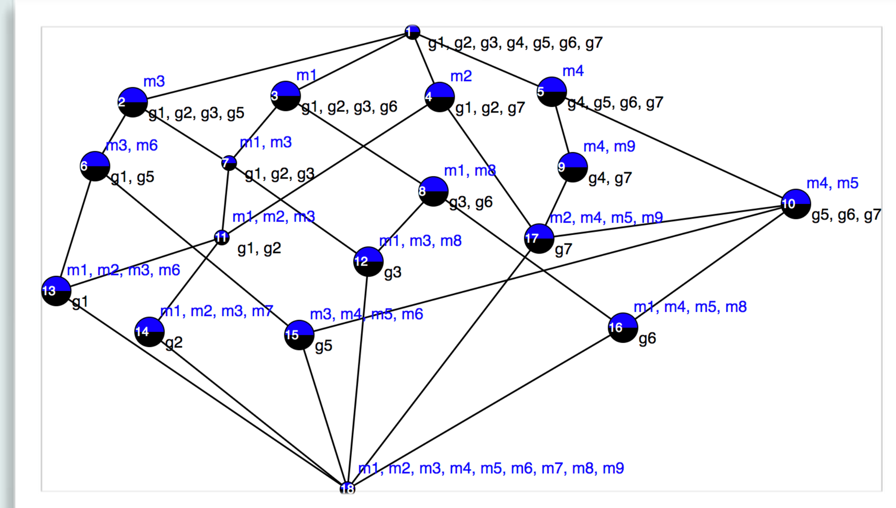
Example

Formal context

	Objects	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
g_1	dog	X	X	X			X			
g_2	cat	X	X	X				X		
g_3	frog	X		X					X	
g_4	car				X					X
g_5	ball			X	X	X	X			
g_6	chair	X			X	X			X	
g_7	fur coat		X		X	X				X

m_1: 4 legs
m_2: wool
m_3: change size
m_4: cold-resistant
m_5: do release CO_2
m_6: black-white
m_7: yellow-braw
m_8: green
m_9: gray

Concept lattice (partially ordered full set of formal concepts)



For a formal concept *({g₁, g₂}, {m₁, m₂, m₃})*

- closed patterns $\{m_1, m_2, m_3\}$;
- minimal generators $\{m_1, m_2\}$, $\{m_2, m_3\}$;
- generators $\{m_1, m_2\}$, $\{m_2, m_3\}$, $\{m_1, m_2, m_3\}$.

The most interesting concepts w.r.t. given assumptions:

(area) $(\{g_1, g_2\}, \{m_1, m_2, m_3\})$, $(\{g_1, g_2, g_3\}, \{m_1, m_3\})$, $(\{g_5, g_6, g_7\}, \{m_4, m_5\})$; area = 6
(length, frequency ⩾ 2): $(\{g_1, g_2\}, \{m_1, m_2, m_3\})$; length = 3
(separation): $(\{g_1, g_2\}, \{m_1, m_2, m_3\})$, $(\{g_1, g_2, g_3\}, \{m_1, m_3\})$; separation = 6/13.

Background Knowledge

Input data → **Compute patterns** → **Reorder patterns** → **Filter patterns**

Minimal Description Length (MDL) Principle. Basic Definitions

The main principle: the best set of patterns is the set that best compresses the database [Vreeken et al., 2011].

Objective: *L(D, CT) = L(D | CT) + L(CT | D)*, where L(D | CT) is the length of the dataset encoded with the code table *CT* and *L(CT | D)* is the length of the code table *CT* computed w.r.t. *D*.

Key notions:
- **Encoding length:** new length that "compresses", i.e. the most frequently used ones have the shortest encoding length.
- **Code table:** a set of selected patterns with their encoding lengths.
- **Disjoint covering:** principle of compression by patterns.

Total length: $L(D, CT) = L(CT \mid D) + L(D \mid CT)$
Code table length w.r.t. data: $L(CT \mid D) = \sum_{i \in CT} code(i) + len(i)$
Data length w.r.t. code table: $L(D \mid CT) = \sum_{d \in D} \sum_{i \in cover(d)} len(i)$

CT computed w.r.t. D

Item-sets	Encoding length
m_3	
m_1	
m_2	
m_4	
m_6	
m_7	
m_8	
m_9	
m_5	

D computed w.r.t. CT

Data with covering	Encoding length
$(m_1)(m_2)(m_3)(m_6)$	
$(m_1)(m_2)(m_3)(m_7)$	
$(m_1)(m_3)(m_8)$	
$(m_3)(m_4)(m_9)$	

MDL in practice: greedy algorithm (Krimp)

Initial state

CT Itemsets	Usage
m_3	4
m_1	3
m_2	2
m_4	1
m_6-m_7	1
m_5	0

Data with covering
$(m_1)(m_2)(m_3)(m_6)$
$(m_1)(m_2)(m_3)(m_7)$
$(m_1)(m_3)(m_8)$
$(m_3)(m_4)(m_9)$

Candidate set, area
$m_1m_2m_3$, 6
m_1m_3, 6
$m_1m_2m_3m_6$, 4
$m_1m_2m_3m_7$, 4
$m_1m_3m_8$, 3
$m_3m_4m_9$, 3

An intermediate state

CT Itemsets	Usage
$m_1m_2m_3$	2
m_3	2
m_1, m_4	1
m_6-m_9	1
m_2, m_5	0

Data with covering
$(m_1m_2m_3)(m_6)$
$(m_1m_2m_3)(m_7)$
$(m_1)(m_3)(m_8)$
$(m_3)(m_4)(m_9)$

Candidate set, area
$m_1m_3m_8$, 3
$m_3m_4m_9$, 3
m_1m_3, 2

Add ordered candidates one by one if they allow for reducing the total length

Final state

CT Itemsets	Usage
$m_1m_2m_3$	2
$m_1m_3m_8$	2
$m_3m_4m_9$	1
m_6,m_7	1
m_1-m_5	0
m_8-m_9	0

Data with covering
$(m_1m_2m_3)(m_6)$
$(m_1m_2m_3)(m_7)$
$(m_1m_3m_8)$
$(m_3m_4m_9)$

Reduction in the number of patterns*

dataset	nmb. of obj.	nmb. of attr.	nmb. of concepts total	nmb. of concepts MDL	dataset	nmb. of obj.	nmb. of attr.	nmb. of concepts total	nmb. of concepts MDL
auto	205	135	67 557	19.26	horse colic	368	83	173 808	101
breast	699	16	642	9.04	iris	150	19	107	13
car	1 728	25	12 617	94	led7	3 200	24	1 937	152
chess	28 056	58	152 753	1 675	mushroom	8 124	90	181 945	211
dermatology	366	49	16 324	47	nursery	12 960	30	176 536	392
ecoli	336	29	694	25	page blocks	5 473	44	715	45
flare	1 389	38	16 303	106	pima indians	768	38	1 609	50
glass	214	46	4 704	50	ticTacToe	958	29	42 685	160
heart	303	50	36 708	54	wine	178	68	13 170	52
hepatitis	155	52	199 954	44	zoo	101	42	4 563	17

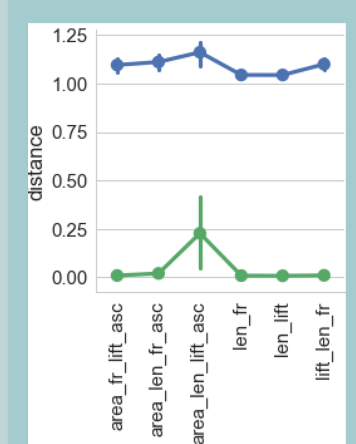
Significant reduction in the number of patterns (up to 5% of the formal concepts).

* datasets from LUCS-KDD repository [4]

MDL: is there a place for background knowledge?

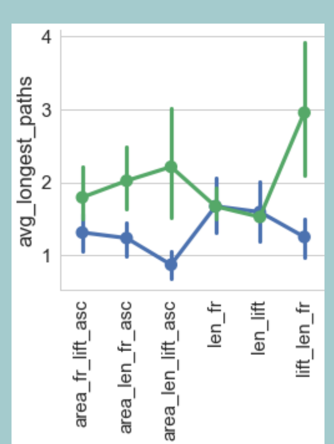
Idea: MDL as an additional filtering stage in pattern selection.

MDL-optimal (blue) vs top-n (green) closed itemsets



Non-redundancy
Distance to the 1st NN

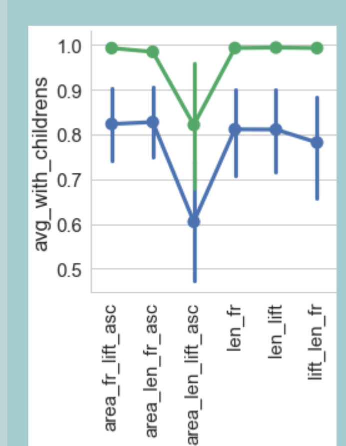
Top-n concepts have a lot of "twins", while MDL-optimal ones are pairwise distinctive (w.r.t. Euclidean distance).



Non-redundancy
Average length of the longest paths built from posets (lattices)

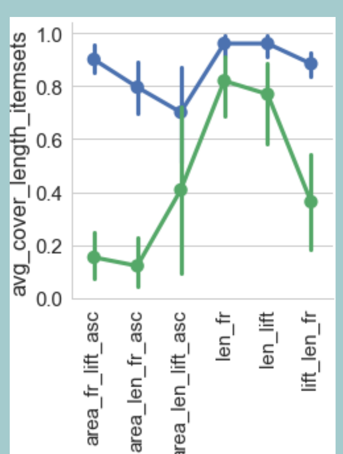
A long path is an indicator of redundancy, since in that case patterns characterize the same objects at different levels of abstraction. Short paths correspond to "flat" structures with more varied patterns.

Pattern mining with area len_sep and area_sep lift, lift_len_fr can be significantly improved by the application of MDL.



Non-redundancy
Average number of itemsets with children

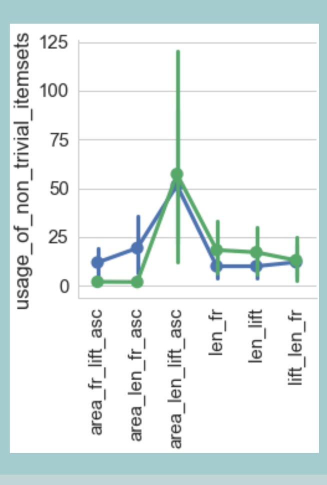
Characterizes the uniqueness of patterns in a set. It indicates just an amount of itemsets having at least one more general itemset.



Data coverage
The rate of covered "crosses" in object-attribute relation

A subset of selected patterns can be considered as a concise representation of a dataset. Thus, it is important to know how much information is lost by compression. It can be measured by the rate of covered attributes. Values close to 1 correspond to the lossless compression

MDL ensures better covering and allows for the biggest gain for area-based orderings.



Typicality (representativeness)

It is measured by the usage of patterns, i.e. the frequency of the occurrence of patterns in the greedy covering, so the usage does not exceed the frequency.

It is not obvious which values are better. The high values of usage correspond to a subset of common patterns, while low values indicates that a subset contains less typical, but still interesting (w.r.t. interestingness measures) patterns.

The usage of MDL-optimal patterns is almost the same for different orders while the usage of top-n is dependent on ordering.

Used measures for ordering candidate sets.
The ordered list of candidates is used for greedy covering of data in Krimp

area_fr_lift = frequency(B)·lift(B)
area_len_lift = len(B)·lift(B) = |B|·lift(B)
area_len_fr = len(B)·frequency(B)

len_fr(B) sort by |B|,then by frequency(B)
len_lift(B) sort by |B|,then by lift(B)
lift_len_fr(B) sort by lift(B), |B| and frequency(B)

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