

Single Tree Detection from Airborne Laser Scanning Data using a Marked Point Process Based Method

Junjie Zhang, Gunho Sohn, Mathieu Brédif

► **To cite this version:**

Junjie Zhang, Gunho Sohn, Mathieu Brédif. Single Tree Detection from Airborne Laser Scanning Data using a Marked Point Process Based Method. ISPRS Workshop on Virtual City Modeling (VCM), May 2013, Regina, Canada. pp.41–46, 10.5194/isprsannals-II-3-W1-41-2013 . hal-01882675

HAL Id: hal-01882675

<https://hal.archives-ouvertes.fr/hal-01882675>

Submitted on 27 Sep 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1 **Title:**

2 A Hybrid Framework for Single Tree Detection from Airborne Laser Scanning Data: A Case Study in
3 Temperate Mature Coniferous Forests in Ontario, Canada

4

5 **Authors:**

6 Junjie Zhang (1), Gunho Sohn *(1), Mathieu Brédif (2)

7

8 **Affiliations:**

9 (1) GeoICT lab, Department of Earth and Space Science & Engineering, York University, 4700

10 Keele Street, Toronto, ON M3J 1P3, Canada

11 (2) Université Paris Est, IGN, MATIS 73, avenue de Paris, 94165 Saint-Mandé, France

12 * Corresponding author.

13

14 **Contact information:**

15 **JZ:** +1 416 736 2100 ext. 22611; junjiez@yorku.ca

16 **GS:** +1 416 650 8011; gsohn@yorku.ca

17 **MB:** +33 1 43 98 83 19; mathieu.bredif@ign.fr

18

19 **Abstract**

20 This study presents a hybrid framework for single tree detection from airborne laser scanning (ALS) data
21 by integrating low-level image processing techniques into a high-level probabilistic framework. The
22 proposed approach modelled tree crowns in a forest plot as a configuration of circular objects. We took
23 advantage of low-level image processing techniques to generate candidate configurations from the canopy
24 height model (CHM): the treetop positions were sampled within the over-extracted local maxima via local
25 maxima filtering, and the crown sizes were derived from marker-controlled watershed segmentation using
26 corresponding treetops as markers. The configuration containing the best possible set of detected tree
27 objects was estimated by a global optimization solver. To achieve this, we introduced a Gibbs energy,
28 which contains a data term that judges the fitness of the objects with respect to the data, and a prior term
29 that prevents severe overlapping between tree crowns on the configuration space. The energy was then
30 embedded into a Markov Chain Monte Carlo (MCMC) dynamics coupled with a simulated annealing to
31 find its global minimum. In this research, we also proposed a Monte Carlo-based sampling method for
32 parameter estimation. We tested the method on a temperate mature coniferous forest in Ontario, Canada
33 and also on simulated coniferous forest plots with different degrees of crown overlap. The experimental
34 results showed the effectiveness of our proposed method, which was capable of reducing the commission
35 errors produced by local maxima filtering, thus increasing the overall detection accuracy by
36 approximately 10% on all of the datasets.

37 **Keywords:**

38 LiDAR, Forestry, Single Tree Detection, Local Maxima Filtering, Marker-controlled Watershed
39 Segmentation, Stochastic Model, Energy Minimization, MCMC

40 **1 Introduction**

41 Remote sensing techniques have become an integral part of forest inventory to provide accurate, precise
42 and timely forest and tree characteristics at different scales to support practices of forest management and
43 planning (Dubayah and Drake, 2000; Naesset et al., 2004; Tomppo et al., 2002; Wulder, 1998; Xie et al.,
44 2008). Among these techniques, small-footprint airborne laser scanning (ALS), also known as airborne
45 LiDAR, has rapidly gained popularity in forest inventory in recent decades. The unique capability of ALS
46 to directly measure the 3D structural information of trees and the elevation of the terrestrial surface under
47 the canopy in forests makes ALS an alternative to traditional passive optical remote sensing technology,
48 or even the preferred method, to derive certain forest parameters, such as canopy height, crown
49 dimensions, stand volume, basal area, and above-ground biomass (Bortolot and Wynne, 2005; Hyypä
50 and Inkinen, 1999; Means et al., 2000; Næsset, 1997; Naesset, 2002).

51 Characterization of forest resources using ALS can be broadly categorized into area-based approaches
52 (ABAs) and individual-tree-based approaches (ITDs) (Hyypä et al., 2008). ABAs rely on the statistical
53 principle and predicts forest attributes based on parametric regression or nonparametric imputation
54 models built between field measured variables and features derived from ALS data (Maltamo et al., 2006;
55 Naesset, 2002). ABAs can perform under a low ALS point density, and is the method currently applied in
56 operational forest inventory to provide a wall-to-wall estimation of forest attributes (Naesset, 2004; White
57 et al., 2013). ITDs measure or predict tree-level variables on the basic unit of the individual trees from
58 ALS data and then aggregate them to obtain stand-level forest inventory results (Hyypä et al., 2012).

59 Despite the added costs and amount of information to store and process high-density ALS data, ITDs are
60 of significant interest in forest inventory and is a motivating research topic. The primary advantage of
61 ITDs over ABAs is the supply of tree lists and the ability to directly derive the true stem distribution
62 series, which would result in better prediction for timber assortments (Vastaranta et al., 2011a).

63 Generally, this information is invaluable in forest planning-related simulation and optimization, logging

64 operation planning and wood supply logistics (Vastaranta et al., 2011b), e.g., detection of harvest trees
65 and forest growth determination (Yu et al., 2004). Another advantage of ITDs is that they can reduce the
66 amount of or potentially replace the expensive fieldwork required for ABAs (Hyypä et al., 2008;
67 Vastaranta et al., 2012). Additionally, tree species classification based on ITD has been reported in recent
68 studies (Brandtberg, 2007; Heinzl and Koch, 2011; Orka et al., 2009; Suratno et al., 2009), which could
69 potentially improve the prediction of species-specific forest attributes (Heurich, 2008; Yao et al., 2012;
70 Yu et al., 2010). Furthermore, the combination of ITD and ABA, called the semi-ITD method, to improve
71 the estimation accuracy has also been viewed as a future method for forest inventory (Breidenbach et al.,
72 2010; Hyypä et al., 2012; Vastaranta et al., 2012). Therefore, individual tree detection techniques are
73 still of significant importance from the practical forestry viewpoint.

74 Accordingly, numerous methods have been proposed to detect single trees from ALS data. Most of the
75 methods focus on the generation of the canopy height model (CHM), which provides an accurate
76 representation of the outer surface of the tree canopy. The peaks and valleys on the CHM generated from
77 high-density ALS data are better estimations of treetop positions and crown edges than can be obtained
78 from aerial photographs or satellite imageries. Therefore, many studies have extended methods developed
79 for passive optical imageries to detect single trees from ALS data. Those methods include, but are not
80 limited to, local maxima filtering (Popescu et al., 2002; Wulder et al., 2000), region growing (Erikson,
81 2003; Solberg et al., 2006), valley following (Gougeon, 1995; Leckie et al., 2003), template matching
82 (Korpela et al., 2007; Pollock, 1996), watershed segmentation and its variance marker-controlled
83 watershed segmentation (Chen et al., 2006; Pyysalo and Hyypä, 2002; Wang et al., 2004), and multi-
84 scale segmentation (Brandtberg and Walter, 1998; Brandtberg et al., 2003).

85 Among the proposed methods, local maxima filtering (LM) and marker-controlled watershed
86 segmentation (MCWS) are the most commonly used and are ready for operational application because of
87 their rapid implementation while maintaining the capability to produce relatively accurate results
88 (Kaartinen et al., 2012). Popescu et al. (2002) have been the first to test a variable window local maxima

89 filtering on the CHMs, attempting to overcome errors of omission and commission associated with fixed
90 window local maxima filtering (Hyypä et al., 2001).

91 Once the treetops are detected, MCWS is well suited to delineate the tree crown segments from the CHM.
92 MCWS, which possesses the advantages of other segmentation methods of region growing and edge
93 detection, was introduced by Meyer and Beucher (1990) to overcome the over-segmentation problem of
94 ordinary watershed segmentation. In MCWS, user-specified markers are used as the marker function to
95 perform the segmentation; for additional details, see Gonzalez and Woods (2008). In the resultant
96 segmentation, there will be one segment corresponding to each marker; in the case of single tree
97 detection, one tree crown will be captured by one treetop. This result indicates the detection accuracy of
98 MCWS, subject to the accuracy of the pre-determined local maxima as true treetops in the previous stage.

99 The issue with LM is the selection of the filter window size and the determination of the relationship
100 between the crown size and the tree height. In the comparison of tree detection algorithms (Kaartinen et
101 al., 2012), the local maxima-based approach tends to produce high commission errors. Especially in
102 coniferous forests, spurious treetops are detected within the tree crowns from large branches. In other
103 cases, local maxima filtering produces a low commission error, and the omission error often increases
104 because small tree crowns are more likely to be undetected (Gebreslasie et al., 2011).

105 Probabilistic methods represent another branch of powerful tools in image analysis. These methods have
106 proven to hold great promise in solving inverse problems, including image segmentation, image
107 restoration, and feature extraction (Descombes and Zerubia, 2002). In particular, stochastic models have
108 evolved from random fields to object processes, and the work has shifted from an early focus on ‘low-
109 level’ tasks that aim to de-noise, sharpen, and segment images to solving ‘high-level’ tasks of feature
110 recognition, i.e., describing an image by its content (Van Lieshout, 2009). Additional details on low-level
111 and high-level image analysis tasks can be found in Sonka et al. (2008).

112 Marked point processes, detailed in Van Lieshout (2000), are among the most efficient stochastic models
113 used to exploit the random variables whose realizations are configurations of geometric objects or shapes.
114 Generally, in these processes, after a probability distribution measuring the quality of each object
115 configuration is defined in the configuration space, the maxima density estimator is searched for by the
116 Markov Chain Monte Carlo (MCMC) sampler (Hastings, 1970) coupled with conventional simulated
117 annealing (Metropolis et al., 1953). This process has led to convincing experimental results in various
118 image analysis and feature extraction applications, such as road networks extraction (Lacoste et al., 2005),
119 road mark detection (Tournaire and Paparoditis, 2009), and 3D building reconstruction (Lafarge et al.,
120 2008; Ortner et al., 2008; Tournaire et al., 2010).

121 Likewise, several stochastic models have been proposed to detect tree crowns from remote sensing data.
122 Descombes and Pechersky (2006) have presented a three-state Markov Random Field (MRF) model to
123 detect the tree crowns from aerial imageries. This approach addressed the problem as an image
124 segmentation problem and works on the pixel level. Each pixel is assigned to one of the following three
125 states: (i) *vegetation*, (ii) *background*, and (iii) *center of trees*. Although the MRF was defined on the
126 pixel level, the label update was performed on the object level using elliptical templates of crowns.
127 Furthermore, Perrin (2005, 2006) has employed marked point processes to detect tree crowns in
128 plantations from color infrared (CIR) aerial imageries. Tree crowns in the remote sensing image are
129 modeled as a configuration of discs or ellipses. In both of the studies, tree crowns were detected by
130 maximizing a Bayesian criterion, such as *Maximum A Posteriori* (MAP), which became an energy
131 minimization problem and was solved in a simulated annealing framework.

132 These stochastic models provide a powerful framework to allow the inclusion of spatial interactions
133 between objects in the prior while enabling a measure of consistency between objects and the image in
134 the data term. However, the inherited property of stochastic models requires exploration of a large
135 configuration space searching for the optimal configuration, especially for non-data-driven models, which

136 do not employ any low-level information that can be extracted from the images. The optimization process
137 is typically lengthy and computationally expensive.

138 This study presents a hybrid framework used to detect single trees from ALS data by integrating the low-
139 level image processing techniques, i.e., LM and MCWS, into a high-level probabilistic model. The
140 proposed model aims to improve the detection accuracy compared with traditional LM. Moreover, this
141 model samples in a reduced configuration space by utilizing image features extracted by LM and MCWS,
142 which potentially accelerate the optimization process compared with classical stochastic models, e.g.,
143 marked point processes. The estimation of parameters is another issue. In most cases, the parameters are
144 tuned by trial and error. We address the problem of parameter estimation by proposing a Monte Carlo
145 based method.

146 This paper is organized as follows. Section 2 describes the study area and the data used in the study.
147 Section 3 is dedicated to the formulation of our proposed model. We provide an overview of the general
148 framework of energy modeling for the stochastic model, followed by detailing the model design from the
149 configuration space definition to the energy formulation, parameter estimation and model optimization.
150 Finally, an accuracy assessment method is included. The experimental results of the parameter estimation
151 and tree detection on real and simulated ALS data are given in Section 4, and Section 5 presents a
152 discussion on the proposed model and the achieved results. Conclusions and certain perspectives for
153 future studies are outlined in Section 6.

154 **2 Materials**

155 **2.1 Study Area**

156 The study area is a temperate mature coniferous forest located in the Great Lakes-St. Lawrence region
157 approximately 60 km east of Sault Ste. Marie, Ontario, Canada (**Figure 1(a)**). The natural vegetation
158 dominant in the coniferous forest is eastern white pine (*Pinus strobus*) and jack pine (*Pinus banksiana*),
159 mixed with some red pine (*Pinus resinosa*) and black spruce (*Picea mariana*). The forest has an

160 intermediate dense canopy with some open space. The canopy height is homogenous with an average
161 height of approximately 20 m. There are some small white pines and shrubs growing in the understory
162 with a height of approximately 2-3 m (**Figure 1(b)-(c)**).

163 *****Approximate position of Figure 1 *****

164 **2.2 Field Survey**

165 To test the proposed single tree detection model, three plots with sizes of 82×95 m, 50×50 m and $80 \times$
166 80 m were selected, and a field survey was conducted in August 2009. The forest mensuration campaign
167 determined the tree height (h_i , m) with a Vertex hypsometer and the diameter at breast height (DBH) with
168 a DBH tape. The positions of trees with a height greater than 5m ($h_i \geq 5$) were determined using GPS
169 and the total station. The crown width and species were also measured and recorded. The stem densities
170 of trees with a value of $h_i \geq 5$ are 154/ha, 160/ha and 190/ha, with increasing values for the three study
171 plots.

172 **2.3 Airborne Laser Scanning Data**

173 The ALS data were acquired over the study area by a Riegl LMS-Q560 laser scanner during the same
174 period as the field work. The flight was performed at a height of approximately 300 m above the ground
175 with a maximum scanning angle of 22.5° , rendering a swath width of approximately 300 m. The flight
176 line was designed to pass over the planned forest plots; therefore, they were located in the middle part of
177 the swath, and the obscure effect of the crowns can be minimized for the plots of interest. The device
178 recorded full-waveforms that were processed into discrete point clouds with up to 5 returns per pulse. The
179 data collection configuration yielded a high point density of approximately 30 points per m^2 over the
180 forested area. The returns were classified as ground and vegetation points using TerraScan software
181 (TerraSolid Ltd, Helsinki, Finland). The CHM with a resolution of 0.5 m was derived as the difference
182 between the digital surface model (DSM) and the digital elevation model (DEM), interpolated from
183 vegetation points and ground points, respectively (Hyypä et al., 2001).

184 **2.4 Simulated ALS Data**

185 Vauhkonen et al. (2012) noted that the performance of the ITD algorithms typically depends on the tree
186 density and the spatial distribution of trees, i.e., clustering patterns. To test the robustness of the proposed
187 model more thoroughly, simulated ALS data of coniferous forest plots with a higher stem density than
188 real forest plots and different degrees of crown overlap were also prepared in our study. First, three forest
189 plots, each with a size of 100×100 m, were generated with a hard-core process in which the crown
190 overlap was controlled by the interaction distance specified in the hard-core process. The smaller the
191 interaction distance in the hard-core process, the more likely the tree objects will be overlapped in the
192 resultant plots. **Figure 2(a)-(c)** show the three resulting point processes. With an increasing degree of
193 crown overlap, the tree density in the plots also increases. The stem densities of trees with a value of $h_i \geq$
194 5 in the three forest plots are 186/ha, 234/ha and 261/ha, respectively.

195 *****Approximate position of Figure 2*****

196 ALS point clouds of individual trees were then selected according to the crown size from a coniferous
197 tree template library and placed in each position to synthesize the ALS data of the forest plot. The tree
198 template library was prepared from ALS data acquired from the study area we surveyed. A more detailed
199 procedure can be found in Zhang and Sohn (2010). The generated ALS point clouds viewed from the
200 nadir direction are shown in **Figure 2(d)-(f)**. The plots from left to right show forest plots with separated,
201 touching and overlapping tree crowns, respectively.

202 In the simulated forest plots, the tree position, height and crown size are exactly known, therefore
203 providing ideal reference data to examine the performance of our proposed model under different forest
204 conditions. The simulated ALS data can also be used to validate the parameter estimation method
205 proposed in Section 3.5 and to investigate the influence that the degree of crown overlap has on the
206 parameter setting in the proposed model.

207 **3 Methodology**

208 **3.1 General Framework of Energy Modeling for the Stochastic Models**

209 In a probabilistic framework, feature extraction or object detection from remotely sensed data can be
210 viewed as an inverse problem. In object oriented stochastic models, features or objects are represented as
211 a configuration of geometric shapes or objects. To find the best configuration \mathbf{x} based on the observed
212 data \mathbf{y} (the image), we must find the configuration $\hat{\mathbf{x}}$ maximizing the posterior probability, according to
213 the following equation:

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \Omega} \mathbb{P}(X = \mathbf{x} | Y = \mathbf{y}) \quad (1)$$

214 where Ω is the configuration space in which \mathbf{x} resides. X and Y are two random variables.

215 The probability of the model can also be specified in the form of a Gibbs energy $U(\mathbf{x})$, which implicitly
216 depends on the constant value \mathbf{y} given by the observation:

$$\mathbb{P}(X = \mathbf{x} | Y = \mathbf{y}) = \frac{1}{Z} e^{-U(\mathbf{x})} \quad (2)$$

217 where Z is a normalizing constant such that $Z = \int_{\mathbf{x} \in \Omega} e^{-U(\mathbf{x})}$. The issue is then reduced to the energy
218 minimization problem of finding the *Maximum A Posteriori* estimator $\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \Omega} \mathbb{P}(X = \mathbf{x} | Y = \mathbf{y})$,
219 which is equivalent to finding the configuration minimizing the Gibbs energy $U(\cdot)$, i.e., $\hat{\mathbf{x}} =$
220 $\arg \min_{\mathbf{x} \in \Omega} U(\mathbf{x})$. Generally, an MCMC embedded simulated annealing is used to find the optimal
221 configuration $\hat{\mathbf{x}}$. The optimization process is particularly interesting because the complex computation of
222 the normalizing constant Z is avoided.

223 **3.2 Overall Workflow of the Proposed Model**

224 The flow chart of the proposed method is shown in Figure 3. As our primary contribution, the blue blocks
225 show the process how we construct a constrained configuration space for tree detection, by taking

226 advantages of low-level image processing techniques, which is detailed in Section 3.3. The red block
227 involves techniques of energy formulation and parameter estimation, which are covered in Section 3.4
228 and Section 3.5, respectively. The optimization process illustrated by the yellow blocks is described in
229 Section 3.6.

230 *****Approximate position of Figure 3*****

231 3.3 Configuration Space Definition of the Proposed Model

232 Let us first recall the configuration space definition in the marked point process. In remote sensing
233 images, the distribution of tree crowns in forests can be represented by a marked point process of disks.
234 The associated space \mathcal{S} can be written according to the following equation:

$$\mathcal{S} = \mathcal{P} \times \mathcal{M} = [0, X_M] \times [0, Y_M] \times [r_m, r_M] \quad (3)$$

235 where X_M and Y_M are the width and height of the image \mathcal{I} , respectively, and (r_m, r_M) are the minimum
236 and maximum radii of the disks in the configuration, respectively. Note that $x = (p, r) \in \mathcal{S}$ is a tree
237 object, where $p \in \mathcal{P}$ is its position and $r \in \mathcal{M}$ its radius. The configuration space Ω of the marked point
238 process of the tree crowns can be written according to the following equation:

$$\Omega = \bigcup_{n=0}^{\infty} \Omega_n, \Omega_n = \{\{x_1, \dots, x_n\} \subset \mathcal{S}\} \quad (4)$$

239 that contains all of the configurations of a finite number of tree objects x_i of \mathcal{S} .

240 In this study, we seek to construct a constrained configuration space $\Omega_T \subset \Omega$ in which the optimal or near
241 optimal configuration resides. We will then limit the search for the optimal configuration in the
242 constrained space Ω_T , which could significantly reduce the computation demand of random sampling in Ω
243 in the optimization process.

244 We begin by constructing a CHM image, representing the height of the tree crowns above ground from
245 the classified ALS data. Then, we extract the local maxima as potential treetops from the CHM using
246 local maxima filtering with a variable window size method adapted from Popescu et al. (2002). Our rule
247 is to detect as many true treetops and reduce omission errors in the first stage. Therefore, the filters of the
248 LM are set with relative small size empirically based on the *priori* knowledge about the plots to over-
249 populate initial ‘treetops’. Let T represents the set of extracted local maxima: $T = \{t_1, \dots, t_N\}, \forall i \in$
250 $\{1, \dots, N\}, t_i \in \mathcal{P}$, where N is the total number of local maxima extracted. The true treetops within the set
251 of local maxima T are noted as $T^o \subset T$.

252 Given any subset of local maxima $C \subset T$, they can be used as markers in marker-controlled watershed
253 segmentation to obtain a partition $S(C) = \{s_{C_1}, \dots, s_{C_{n(C)}}\}$ of the CHM, where s_{C_i} is the corresponding
254 segment of the local maxima $t_{C_i} \in C$. $S(C)$ is a low-level presentation of the CHM image, and the set of
255 segments are assumed to be a reasonable approximation of the tree crowns with respect to the set of local
256 maxima C , where $n(c)$ is the number of local maxima in C .

257 A tree object $x_{C_i} = (t_{C_i}, r_{C_i})$ is then defined by its location and radius on the segment s_{C_i} , where the tree
258 location is the corresponding local maximum t_{C_i} , and the radius r_{C_i} is calculated as the average radius of
259 the segment s_{C_i} . A configuration $\mathbf{x}(C) = \{x_{C_1}, \dots, x_{C_{n(C)}}\}$ is then constructed from the set of local maxima
260 C . The entire procedure of configuration construction is illustrated in **Figure 4**.

261 *****Approximate position of Figure 4*****

262 We note all of the configurations generated from the subsets of local maxima T as $\Omega_T = \{\mathbf{x}(C), C \subset T\}$.
263 Apparently, Ω_T is a discrete subspace of the configuration space Ω , which cardinality is
264 $\text{card}(\{\mathbf{x}(C), C \subset T\}) = \text{card}(\{C, C \subset T\}) = 2^{\text{card}(T)}$. In this manner, we build a constrained
265 configuration space Ω_T from which to sample the optimal configuration.

266 **3.4 Energy Formulation**

267 As previously mentioned, the Gibbs energy $U(\mathbf{x})$ is defined on the configuration space to measure the
268 goodness or cost of each object configuration. The Gibbs energy can be further expressed as a weighted
269 sum of a prior term $U_p(\mathbf{x})$ that favors a specific spatial pattern in configuration \mathbf{x} and a data term $U_d(\mathbf{x})$
270 that quantifies the quality of the configuration with respect to the data, according to the following
271 equation:

$$U(\mathbf{x}) = \alpha U_d(\mathbf{x}) + (1 - \alpha) U_p(\mathbf{x}) \quad (5)$$

272 where $\alpha \in [0,1]$ specifies the relative weights of the two energy terms.

273 We intend to make simple and effective choices for the design of each energy term. The basic
274 assumptions are the geometric properties of trees in mature coniferous forests in which treetops are
275 typically located in the central part of tree crowns, and tree crowns are of a circular shape when viewed
276 from the nadir direction (Chen et al., 2006; Gleason and Im, 2012). We also tend to penalize certain
277 patterns in the configurations in the prior term that tree crowns should not severely overlap.

278 **3.4.1 Data Term**

279 The data term is in accordance with the aforementioned assumption, indicating the likelihood of the tree
280 objects relative to the low-level segments obtained from the CHM image. Certain geometric features are
281 extracted from the underlying segment of each object, and energy functions are proposed to measure how
282 well those features support the object as a plausible tree.

283 We incorporate the following two energy functions to reflect the assumption: symmetric function $U_d^s(x)$
284 and area ratio function $U_d^a(x)$. The data term is a weighted sum of the two energy functions, subject to a
285 hard constraint on the object radii, according to the following equation:

$$U_d(\mathbf{x}) = \begin{cases} \sum_{x \in \mathbf{x}} (w_1 U_d^s(x) + (1 - w_1) U_d^a(x)) & \text{if } r(x) \in [r_m, r_M] \\ +\infty & \text{otherwise} \end{cases} \quad (6)$$

286 where w_1 is the weight regulating the relative importance of the symmetric and area ratio functions in the
287 data term.

288 **(i) Symmetric Function $U_d^s(x)$**

289 A symmetric function is defined as a measure of how well a treetop is located in the central part of the
290 crown and the degree to which the tree crown is of a symmetric circular shape. For a given tree object x
291 with corresponding segment s_x , the radii from the treetop point T to the edge of the segment in 8
292 directions with constant angular intervals \overline{TP}_i ($i = 1, \dots, 8$) are first extracted (see **Figure 5**). The average
293 and standard deviation of the 8 radii are noted as $r(x)$ and $\Delta r(x)$. The asymmetric ratio $R_{sym}(x) \in [0, 1]$
294 of object x is calculated as the coefficient of variance of the radii according to the following equation:

$$R_{sym}(x) = \frac{\Delta r(x)}{r(x)} \quad (7)$$

295 A sigmoid function is then used to define the symmetric function to penalize asymmetric tree crowns
296 given by Eq. (8):

$$U_d^s(x) = \frac{1}{1 + \exp\left(-\frac{R_{sym}(x) - \mu_s}{\lambda_s}\right)} - 1 \quad (8)$$

297 where μ_s and λ_s are parameters set to control the position and slope of the sigmoid function, respectively.
298 The larger the asymmetric ratio $R_{sym}(x) \in [0, 1]$, the higher the symmetric function score $U_d^s(x) \in$
299 $[-1, 0]$, which indicates that the treetop is more likely to be a false treetop.

300 *****Approximate position of Figure 5*****

301 **(ii) Area Ratio Function $U_d^a(x)$**

302 Another area ratio term $U_d^a(x)$ is included to re-enforce the assessment of the geometric features of the
303 objects in the configuration.

304 Likewise, an area ratio $R_{area} \in [0, 1]$ is first calculated. The ratio is computed as the proportion of the
305 intersection of object x and the underlying segments s_x to the entire area of the segments $A(s_x)$ by Eq.
306 (9). As the area ratio increases, the degree of the geometric feature of the object increases, in accordance
307 with the hypothesis (see **Figure 6**).

$$R_{area}(x) = \frac{A(x \cap s_x)}{A(s_x)} \quad (9)$$

308 Based on the area ratio of the object, the area ratio function is defined according to the following
309 equation:

$$U_d^a(x) = \frac{1}{1 + \exp - \left(\frac{R_{area}(x) - \mu_a}{-\lambda_a} \right)} - 1 \quad (10)$$

310 where μ_a and λ_a are used to control the position and slope of the sigmoid function, respectively.

311 *****Approximate position of Figure 6*****

312 3.4.2 Prior Term

313 The prior term introduces a *priori* knowledge concerning the layout of the objects. In most mature
314 coniferous forest stands, tree crowns will not overlap too severely. However, overlap between objects
315 should not be totally prohibited. A repulsive term is then defined as a soft penalizing function to penalize
316 severe overlaps in the configuration.

317 (i) Overlap Function $U_p^o(\mathbf{x})$

318 To define the overlap function, we first introduce a symmetric neighborhood relationship between
319 objects. We say two objects $x_i = (t_i, r_i)$ and $x_j = (t_j, r_j)$ are overlapping if the distance between them is

320 smaller than the sum of their radii, noted as $d(t_i, t_j) < r_i + r_j$, and we write $x_i \sim x_j$. Then, an overlap ratio
321 $R_{overlap} \in [0, 1]$ is calculated as the ratio of the overlap area between the two objects normalized by the
322 area of the smaller object, according to the following equation (see **Figure 7**):

$$R_{overlap}(x_i, x_j) = \frac{A(x_i \cap x_j)}{\min(A(x_i), A(x_j))} \quad (11)$$

323 The overlap score $O(x_i, x_j)$ on $x_i \sim x_j$ is then given according to the following equation:

$$O(x_i, x_j) = \frac{1}{1 + \exp - \left(\frac{R_{overlap}(x_i, x_j) - \mu_o}{\lambda_o} \right)} \quad (12)$$

324 where μ_o and λ_o are set to control the position and slope of the sigmoid function, respectively.

325 The overlap function of configuration X can be expressed according to the following equation:

$$U_p^o(\mathbf{x}) = \sum_{x_i \sim x_j} O(x_i, x_j), \forall x_i, x_j \in \mathbf{x}, i \neq j \quad (13)$$

326 *****Approximate position of Figure 7*****

327 Compared with a classical Marked Point Process, limiting the search space to configurations generated
328 from a subset of a finite set of seed points T (the pre-extracted local maxima) prevents multiple detection
329 problems. The global energy does not have to be designed to prevent the selection of multiple instances of
330 the same tree because duplicated trees are not part of the search space. Thus, the prior term contains only
331 the overlap function and is written according to the following equation:

$$U_p(\mathbf{x}) = U_p^o(\mathbf{x}) \quad (14)$$

332 **3.5 Parameter Estimation**

333 Parameters in the model can be distinguished into the following three categories: physical parameters,
334 weights and thresholds. The physical parameters r_m and r_M are size constraints specifying the range of the

335 tree crown radius in the forest plots. These parameters are set as 1.0 m and 6.0 m, respectively, according
336 to the range of tree sizes in the test sites.

337 The weights α and w_1 are assigned to tune the relative importance that we want to grant to different
338 energy terms or functions in the combination (see Eq. (5) and (6)). Both α and w_1 are set to 0.5 because
339 we place equal importance on those functions in all of our experiments.

340 To reduce the hand-tuned parameters and to avoid a “trial-and-error” test for parameter setting in most
341 practices, we propose a parameter estimation method to estimate the threshold pair (μ, λ) in the sigmoid
342 functions (Eq. (8), (10), and (12)) in the energy terms. In each function, the threshold pair (μ, λ) controls
343 the tolerance and slope of the sigmoid function, respectively, which plays a significant role in the model.
344 For example, if we set a smaller μ_s value in the symmetric function (Eq. (8)), trees with asymmetric
345 crowns will be penalized more effectively. For a sigmoid function, a smaller value of λ results in a steeper
346 slope, and the associated energy function has an increased discriminative behavior of a step function (see
347 **Figure 8**).

348 *****Approximate position of Figure 8*****

349 We address two issues in the parameter estimation of the energy minimization model. First, the energy
350 terms are designed to penalize false tree objects or implausible configurations with respect to the data
351 term and the prior term. False tree objects or implausible configurations between the objects should
352 receive high energy scores. The parameter estimation is performed by fitting the sigmoid functions to the
353 posterior probability of the features derived from false tree objects or implausible configurations based on
354 the logistic regression model. Second, collection of a large sample size is required to model the posterior
355 probability of those aforementioned features through the Bayesian theorem. In this study, we propose a
356 Monte Carlo-based method, which enables generation of a sufficient number of samples and leads to the
357 estimation of the parameters in the logistic functions.

358 For example, we will examine the symmetric function $U_d^S(\mathbf{x})$. Let us denote the feature, the asymmetric
359 ratio in this case, extracted from a tree object x_i as d . A random variable $Y = \{0,1\}$ takes the value of 1 if
360 x_i is a true tree object or 0 otherwise. Given an observation d , the probability that the random variable is
361 derived from a false tree object can be given by the posterior probability, according to the following
362 equation:

$$p(Y = 0|d) = \frac{p(d|Y = 0)p(Y = 0)}{p(d)} \quad (15)$$

363 The higher the posterior probability of the object being a false tree, the higher the energy we assign to the
364 object through the energy functions.

365 According to the Bayesian theorem, the posterior probability can be rewritten as the following:

$$p(Y = 0|d) = \frac{1}{1 + L_i^o P_i^o} \quad (16)$$

366 where L_i^o is the likelihood ratio, and P_i^o is the prior ratio, according to the following equations:

$$L_i^o = \frac{p(d|Y = 1)}{p(d|Y = 0)} \quad (17)$$

$$P_i^o = \frac{p(Y = 1)}{p(Y = 0)} \quad (18)$$

367 The likelihood ratio L_i^o can be calculated by modeling the likelihood distributions of features derived from
368 true and false tree objects. A Monte Carlo sampling is utilized to estimate the likelihood distributions. We
369 generate a configurations $\mathbf{x}(T_i)$ from a random subset of local maxima $T_i \subset T$, and $\mathbf{x}(T_i)$ is then
370 compared with the reference configuration $\mathbf{x}(T^o)$. Each tree object $x_j^{T_i}, j = 1, \dots, n(T_i)$ in configuration
371 $\mathbf{x}(T_i)$ is then labeled as *true* or *false*. We repeat this process for n ($n = 50$ in our experiments) time, to
372 collect enough samples for features of the *true* and *false* tree objects.

373 The Monte Carlo-based method produces a pool of samples sufficient to model the likelihood
374 distributions of different features. The maximum likelihood method is applied to model the likelihood
375 distributions of the asymmetric ratio, area ratio, and overlap ratio for true and false trees. In practice, we
376 set the prior ratio P_i^o to 2, which is empirically based on the general detection accuracy achieved by LM-
377 based approaches. The modeled distributions and fitted functions are shown in **Figure 9**.

378 *****Approximate position of Figure 9*****

379 **3.6 Model Optimization**

380 In model optimization, we aim to find the configuration of objects that minimizes the global energy $U(\mathbf{x})$
381 in the configuration space Ω_T that we have proposed. This discrete configuration space can be effectively
382 explored using a Markov Chain Monte Carlo sampler coupled with simulated annealing.

383 An MCMC sampler consists in simulating a discrete Markov chain (X_t) , $t \in \mathbb{N}$ on the configuration space
384 Ω_T , which converges towards an invariant measure specified by the energy $U(\mathbf{x})$. The sampler performs
385 transitions for one state of the chain to another by proposing a local change of the current configuration.

386 In our application, a configuration of trees $\mathbf{x}(T^k)$ can be solely determined by a subset of local maxima
387 $T^k \subset T$ given the CHM image. Once treetops are set as the local maxima T^k , the tree sizes are decided
388 and directly derived from the corresponding marker-controlled watershed segments. Therefore, finding
389 the optimal configuration of trees $\mathbf{x}(T^*)$ is equivalent to determining the optimal set of local maxima
390 $T^* \subset T$. The transition of the chain can be managed by a birth-and-death process in which a local maxima
391 is added to or removed from the current set of local maxima T^k to generate a new configuration $\mathbf{x}(T^{k+1})$,
392 from a previous configuration $\mathbf{x}(T^k)$. More specifically,

- 393 • In a birth process, a local maxima u is randomly selected from $T \setminus T^k$ and added to the current local
394 maxima set T^k to generate a new configuration $\mathbf{x}(T^{k+1})$, with $T^{k+1} = T^k \cup \{u\}$.

- 395 • In a death process, a local maxima v is randomly selected and removed from the current local
396 maxima set T^k to generate a new configuration $\mathbf{x}(T^{k+1})$, with $T^{k+1} = T^k \setminus \{v\}$.

397 The move between the configurations is symmetric and accepted with the following probability:

$$\min\left(1, \exp - \left(U\left(\mathbf{x}(T^{k+1})\right) - U(\mathbf{x})\right)\right) \quad (19)$$

398 Otherwise, the previous set of local maxima is kept: $T^{k+1} = T^k$.

399 A simulated annealing is then embedded in the MCMC to find the optimal configuration with the
400 minimum global energy $U(\mathbf{x})$. To perform the simulated annealing, the Gibbs energy $U(\mathbf{x})$ is replaced
401 with $U_{T_t} = U(\mathbf{x})/T_t$. T_t is the temperature parameter, which tends toward zero as t approaches ∞ . A
402 logarithmic decrease ensures the convergence to the global optimum for all of the initial configurations
403 \mathbf{x}_0 . In practice, a geometric cooling scheme is preferred to accelerate the process and to give an
404 approximate solution close to the optimal one, for example, use $T_t = T_0 \alpha^t$ with α close to 1, typically
405 $\alpha = 0.98$.

406 **3.7 Accuracy Assessment**

407 To evaluate the performances of the proposed model, the detected trees are compared with the reference
408 data. The comparison results of all of the aggregated trees from the detected trees and the reference data
409 can be classified into the following three categories: the correctly detected trees (correct), trees in the
410 detection results that have no corresponding reference tree (commission) and trees in the reference data
411 not detected (omission). Commission/Omission statistics and the overall detection accuracy are used to
412 quantify the detection results. The calculation of the commission error, omission error and overall
413 accuracy is based on a conventional method of error matrix assessment (Girard, 2003), as shown by Eq.
414 (20)-(22):

$$Commission\ error = \frac{N_{det} - N_{cor}}{N_{det}} \times 100\% \quad (20)$$

$$Omission\ error = \frac{N_{ref} - N_{cor}}{N_{ref}} \times 100\% \quad (21)$$

$$Overall\ quality = \frac{N_{cor}}{N_{cor} + (N_{det} - N_{cor}) + (N_{ref} - N_{cor})} \times 100\% \quad (22)$$

415 where N_{cor} is the number of correctly detected trees, N_{det} is the total number of detected trees by the
416 algorithm, and N_{ref} is the number of reference trees.

417 **4 Results**

418 **4.1 Parameter Estimation Results**

419 **Table 1** displays the parameters estimated for the energy functions of the proposed model. We then
420 performed experiments with the estimated parameters on real and simulated forest plots to test the
421 robustness of the model.

422 The parameter μ_s is the threshold in the symmetric function used to penalize tree crowns with high
423 asymmetric ratios. In a forest in which most tree crowns are of regular circular shapes, the value of μ_s can
424 be set relatively smaller to more effectively penalize crowns with asymmetric ratios that exceed this
425 threshold. The threshold μ_a works conversely. Because a larger area ratio indicates a more circular
426 shaped crown, it must be set to a larger value to better penalize tree crowns of a non-circular shape.

427 Parameter μ_o in the overlap function is set to penalize an overlapping situation that exceeds a certain
428 degree, which works similarly to the μ_s parameter. The greater the degree of crown overlap in a forest
429 plot, the larger the μ_o value should be set.

430 The results shown in **Table 1** support this reasoning for parameter setting in which the more the tree
431 crowns in the plot are of symmetric circular shape, the smaller the estimated value of μ_s , whereas the
432 larger the value of μ_a . This reasoning is more explicitly evidenced by the simulated forest plots in which

433 the shape irregularity of the tree crowns increases with the increasing degree of canopy overlap from
434 separated to overlapping, which in turn causes an increase in the value of μ_s from 0.32 to 0.45 and the
435 value of μ_o from 0.08 to 0.40, whereas the value of μ_a decreases correspondingly from 0.82 to 0.72. This
436 result also confirms the rationality of our proposed method for parameter estimation. We also notice that
437 the smaller the overlap degree of a plot, the smaller the estimated λ in the sigmoid function, which
438 indicates a better “threshold” behavior of the associated energy function. This relationship is well in line
439 with the assumption that the simpler the plot situation, the easier the *true* tree crowns and the *false* tree
440 crowns can be distinguished.

441 From the estimation results of the real and simulated forest plots, we also conclude that the degrees of
442 crown overlap of the real forest plots are between the touch and overlap situations in the simulated forest
443 plots. This condition can be observed from the ranges of the estimated values of μ_s and μ_o of the real
444 forest plots, which are between the parameters estimated for the touch and overlap simulated forest plots.

445 *****Approximate position of Table 1*****

446 **4.2 Detection Results of Real Forest Plots**

447 We first applied the proposed model with the estimated parameters to the ALS data of the three real forest
448 plots. The detection results of local maxima filtering with a variable window size (also referred to as LM)
449 and the proposed model are illustrated in **Figure 10**, which shows a good visual assessment of the
450 performances of the two methods.

451 The LM results are displayed in the first row (**Figure 10(a)-(c)**). In these images, the red circles with blue
452 crosses in the center represent the corrected detected tree crowns, whereas the green and cyan circles
453 represent the commission and omission errors, respectively. **Figure 10** clearly shows that the LM method
454 is prone to produce commission errors in those coniferous forest plots. This problem is particularly noted
455 in plot 1 and plot 3 in which numerous false treetops occur on the edge of tree crowns because of the
456 branching structure of the pine tree species growing in those plots. Plot 2 is a forest with relatively sparser

457 trees, and commission errors primarily occur near the plot boundaries caused by incomplete crown
458 segments and a lack of reference data.

459 The corresponding images in the second row (**Figure 10(d)-(f)**) show the detection results using the
460 proposed model. As can be easily interpreted, most green circles were successfully removed, indicating
461 that the proposed model could effectively reduce the commission errors. We noticed that a small number
462 of yellow dot line circles appear, which indicate trees over-pruned by the proposed model. From the three
463 images, we can observe that the omission errors produced by the proposed model are primarily trees with
464 small crowns and are severely overlapped by their neighboring larger trees. We also noticed that many
465 commission errors occur at the edge of the plots where crowns are shown incomplete or the reference data
466 are missing.

467 **Table 2** depicts the detailed quantitative assessment of the detection results of the LM and the proposed
468 model. There is an obvious improvement in the results of the proposed model over the LM method on
469 which it is based. The commission errors of the three forest plots significantly decreased, with the largest
470 extend in plot 1, decreasing from 36.2% to 10.3%, whereas the omission errors before and after the
471 application of the proposed model remain at similar levels. On average, the overall detection accuracy
472 increased by approximately 15%, comparing results of the proposed model with those of the LM method.

473 *****Approximate position of Figure 10*****

474 *****Approximate position of Table 2*****

475 **4.3 Detection Results of Simulated Forest Plots**

476 The proposed model with the estimated parameters applied to the simulated forest plots exhibited similar
477 detection results to those of the real forest plots. The proposed model significantly reduced the
478 commission errors resulting from the LM method in the three simulated forest plots. **Figure 11** shows a
479 clear contrast in the detection results of the LM and the proposed model.

480 Similarly, by comparing the corresponding images in **Figure 11(a)-(c)** and **Figure 11(d)-(f)**, it can be
481 observed that nearly all of the green circles (commission errors) in the LM detection results were removed
482 by the proposed model in the three simulated forest plots. Meanwhile, there is only a negligible increase
483 in the number of yellow dot line circles (omission errors). On average, the proposed model increases the
484 overall detection accuracy by approximately 10% compared with the LM method in all of the cases.

485 **Table 3** gives the exact detection results of the LM method and the proposed model on the three
486 simulated plots. It is interesting to examine the influence of the crown overlap degree on the single tree
487 detection results of the LM method. The overall detection accuracy decreases by approximately 10%
488 across the three simulated forest plots with an increasing degree of crown overlap from separated to
489 overlapping. This result is primarily because of the increase in the number of omission errors with the
490 increase in the crown overlap. Trees growing by taller trees are more likely to be missed in the LM
491 detection when crowns are more overlapped. However, the commission errors are less affected by the
492 degree of crown overlap, which remains at a similar level for the three forest plots.

493 *****Approximate position of Figure 11*****

494 *****Approximate position of Table 3*****

495 **4.4 Optimization Process**

496 **Figure 12** presents the statistics associated with the optimization process, using a simulated forest plot
497 with a touching crown as an example. The plots are at the same abscissa scale to simplify the observation
498 of the optimization process. The iteration index is consistently represented on this axis. In all of the
499 experiments, the temperature decrease coefficient α is set to 0.98, and the temperature is updated every
500 500 iterations. For a plot with approximately 200 trees, it takes approximately $1.2e + 5$ iterations for the
501 energy to converge, which is significantly fewer than the total number of configurations ($2^{200} \approx 1.6e +$
502 60) in the entire configuration space. The program takes approximately 3 hours to run in Matlab on a
503 processor with a 2.83 GHz frequency.

504 The first plot (**Figure 12(a)**) shows the evolution of the temperature in accordance with a geometric
505 cooling scheme, as described in Section 3.6. **Figure 12(b)** represents the acceptance rate associated with
506 the “birth-and-death” kernel. The move acceptance rates are high at the beginning of the process and tend
507 to progressively decrease and stabilize to 0. Finally, **Figure 12(c)** plots the global energy. Variations are
508 the highest during the first iterations, and the energy slowly decreases. The decrease becomes faster as the
509 iterations progress and tends to converge slowly to its minimum.

510 *****Approximate position of Figure 12*****

511 **5 Discussion**

512 In this study, we present a hybrid framework to improve the performance of single tree detection from
513 ALS data by taking advantage of low-level image processing techniques and a high-level probabilistic
514 model. The proposed model is applied on the ALS data of real and simulated coniferous forest plots. The
515 results show the feasibility of our approach, and the detection quality is superior to that obtained by the
516 local maxima filtering based method.

517 The proposed method has been proven to be effective in reduce the commission errors that are introduced
518 by LM in all coniferous forest plots. The LM approach requires a *priori* knowledge of the relationship
519 between the tree height and the crown size, and the detection accuracy can be significantly influenced by
520 the specification of the relationship. In many cases, this relationship is either hard to obtain or different
521 from study to study because it depends on certain factors, such as tree species, tree age, tree density,
522 crown overlapping, and species composition of the forest plot. Moreover, Falkowski et al. (2006) noted
523 that the relationship between the tree height and the crown size can be weak under certain forest
524 conditions, which is coherent with our case. In this case, when a relationship is designated between the
525 tree height and the crown size, the parameters set for the LM are simply a trade-off between commission
526 and omission errors. We suggested a relative small window size for the LM to over-extract initial
527 ‘treetops’ at the first stage, and the embedded probabilistic model showed its powerfulness in excluding

528 the false treetops from the final configuration through stochastic inference by considering the spatial
529 layouts and geometric characteristics of the trees in the forest plots.

530 Simulation of forest plots and ALS data provide a valuable tool to examine the performance of tree
531 detection methods under the influence of stem densities and degrees of crown overlap. The detection
532 results evidence the higher the stem density, the more likely the tree crowns are overlapped in the plot,
533 causing smaller trees growing nearby larger trees not easily be detected. The results obtained are coherent
534 with those reported in other studies that denser plots give less accuracy results than sparse plots. The
535 simulated data also provides a fully controlled environment to observe the behavior of the estimated
536 parameters in the designed energy functions with respect to the factor of crown overlap. The increase in
537 crown overlap results in more asymmetric crowns in CHM, which are noted by the estimated parameters
538 and further validate the rationality of the parameter estimation method we proposed. The simulation in
539 our study is intended to test our proposed model under certain key forest variables, i.e., the tree density
540 and crown overlap in our case. Additional sophisticated simulations of forest structure and ALS returns
541 can be found in Morsdorf et al. (2009) and Disney et al. (2010).

542 The detection of single trees from remote sensing data using marked point processes was first performed
543 by Andersen et al. (2002) in an attempt to directly detect trees of a coniferous plot from ALS point clouds
544 using the marked point process in a Bayesian framework. The results have indicated that the algorithm is
545 generally successful in identifying structures associated with individual tree crowns within the forest plot
546 but appears to be sensitive to complex point cloud data. Perrin (2005, 2006) has employed marked point
547 processes to detect tree crowns from CIR aerial imageries of plantations, which leads to a continuous
548 search space for the tree objects, in contrast to the proposed method.

549 The stochastic model we proposed is the first to integrate low-level image processing techniques and a
550 high-level probabilistic model into a hybrid framework for single tree detection. The model assembles
551 marked point processes in terms of object modeling and energy formulation. However, in the model, the

552 parameters of the tree objects are directly derived from low-level representations of LiDAR images
553 produced by traditional image processing techniques rather than random sampling in classical marked
554 point processes. Thus, the model generates a constrained discrete configuration space, in which we
555 sample for the global optimum that contains the final set of detected trees. In this manner, the
556 computation cost is significantly reduced, and the optimization process can be significantly accelerated.

557 The design of proper energy terms is an important issue we attempt to address due to the different types
558 of data we used and the specific manner in which we constructed a configuration. The models used to
559 detect tree crowns in aerial imageries (Perrin et al., 2005, 2006) make use of the distinctive pixel values
560 between the illuminated area near the center of the tree crowns and that of the backgrounds or valleys
561 between the crowns. The contrast between the tree crowns and the background, or treetop areas and
562 valleys between them, can be exaggerated by shadows and stretched spectral or radiometric
563 characteristics in the optical images. However, the elevation differences between those parts in the CHM
564 images are much milder and complex to model than the contrasts in optical imageries. This fact is also the
565 reason we chose a Gibbs energy to measure the morphological characteristics of the tree objects in a
566 configuration, other than a Bayesian framework to model height distributions, considering the complexity
567 required to design a height model valid for all of the trees of various heights and crown forms in the forest
568 area.

569 Parameter estimation is another challenging task in most stochastic models. In this study, we proposed a
570 Monte Carlo-based method to estimate certain key parameters in our model. The Monte Carlo simulation
571 was used to generate random configurations and to create a sufficient number of samples of true and false
572 tree crowns, which enabled the modeling of feature distributions of true and false tree crowns to estimate
573 thresholds in the energy terms. The experimental results on all of the datasets, especially the simulated
574 ones, suggested that the parameter estimation method works reasonably well.

575 The proposed method has certain inherited drawbacks detecting trees from the rasterized canopy height
576 model, which is incapable of finding suppressed trees under dominant crowns (Hyypä et al., 2012). The
577 method is designed to detect trees in the dominant layers in the coniferous forest plots of interest.
578 Exploiting 3D information from the ALS point cloud to detect small trees in the lower forest layer is a
579 possible direction to overcome this disadvantage (Ferraz et al., 2012; Reitberger et al., 2009). Another
580 limitation of the method is that it is unable to recover the omission error produced by local maxima
581 filtering on which it is based. Because tree positions are constrained within the pre-extracted local
582 maxima, the model experienced a reduced ability in the classical marked point process to sample the
583 configuration space more thoroughly. However, experimental results on real and simulated forest plots
584 still suggest that the proposed model is a good compromise regarding complexity, efficiency and
585 accuracy.

586 **6 Conclusions and Future Studies**

587 We propose a hybrid framework to detect single trees from ALS data by combining the low-level image
588 processing techniques of LM and MCWS with a high-level probabilistic model. More specifically, in this
589 model, tree crowns in an ALS recovered CHM are modeled as objects and are considered as a
590 configuration of circles. The probabilistic model enables the consideration of the geometric characteristics
591 and the pair-wise interactions of objects in the configuration. The LM and MCWS are employed to
592 produce a low-level representation of the image, which provides a constrained configuration space for the
593 probabilistic model to sample for the optimal configuration. We also propose a Monte Carlo-based
594 method to estimate important parameters in the proposed model. The model is proven effective when
595 applied to real and simulated coniferous forest plots. The results show that the proposed model has a
596 distinct improvement in the detection quality over the traditional local maxima filtering based approach
597 by approximately 10% on all of the datasets.

598 Future studies should involve a further examination of the optimization methods. An important benefit we
599 gained from our proposed model is that the configuration space is significantly reduced by incorporating
600 features extracted from the CHM image through low-level image processing techniques. However, there
601 remains a significant requirement to accelerate the optimization process. A prior-guided MCMC or a
602 steepest gradient descent algorithm are possibilities we will examine to accelerate the search for the
603 optimal configuration within the discrete configuration space. Second, post-processing will be introduced
604 to recover omission errors from the detection results. Although the proposed model was proven effective
605 in reducing commission errors, the tree positions are constrained in the predetermined set of the local
606 maxima extracted by local maxima filtering. It is possible to recover a portion of the omitted trees from
607 the detected results because those missed crowns will result in more geometrically irregular segments.
608 Finally, automated segmentation of forest stands into homogenous areas with similar forest conditions
609 can be introduced to help train parameters of the proposed model of representative regions and make the
610 model applicable to larger areas. We will also further test the proposed model on more datasets of
611 different forest types and conditions.

612 **Acknowledgements**

613 This research was supported by a grant for a project entitled “Automated Change Detection of 3D
614 Landscape Objects for Powerline Corridor Mapping by Integrating Airborne LiDAR and Multiple-
615 viewing Digital Cameras” funded by Ontario Centres of Excellence and GeoDigital International Inc. and
616 the Natural Sciences and Engineering Research Council of Canada (NSERC). The authors would like to
617 thank the anonymous reviewers for their valuable comments and suggestions.

618 **References**

619 Andersen, H.-E., Reutebuch, S.E., Schreuder, G.F., 2002. Bayesian object recognition for the analysis of
620 complex forest scenes in airborne laser scanner data, *International Archives of Photogrammetry and*
621 *Remote Sensing*, pp. 35-41.

- 622 Bortolot, Z.J., Wynne, R.H., 2005. Estimating forest biomass using small footprint LiDAR data: An
623 individual tree-based approach that incorporates training data. *ISPRS Journal of Photogrammetry and*
624 *Remote Sensing* 59, 342-360.
- 625 Brandtberg, T., 2007. Classifying individual tree species under leaf-off and leaf-on conditions using
626 airborne lidar. *ISPRS Journal of Photogrammetry and Remote Sensing* 61, 325-340.
- 627 Brandtberg, T., Walter, F., 1998. Automated delineation of individual tree crowns in high spatial
628 resolution aerial images by multiple-scale analysis. *Machine Vision and Applications* 11, 64-73.
- 629 Brandtberg, T., Warner, T.A., Landenberger, R.E., McGraw, J.B., 2003. Detection and analysis of
630 individual leaf-off tree crowns in small footprint, high sampling density lidar data from the eastern
631 deciduous forest in North America. *Remote Sens. Environ.* 85, 290-303.
- 632 Breidenbach, J., Næsset, E., Lien, V., Gobakken, T., Solberg, S., 2010. Prediction of species specific
633 forest inventory attributes using a nonparametric semi-individual tree crown approach based on fused
634 airborne laser scanning and multispectral data. *Remote Sens. Environ.* 114, 911-924.
- 635 Chen, Q., Baldocchi, D., Gong, P., Kelly, M., 2006. Isolating individual trees in a savanna woodland
636 using small footprint lidar data. *Photogrammetric Engineering and Remote Sensing* 72, 923-932.
- 637 Descombes, X., Pechersky, E., 2006. Tree Crown Extraction using a Three State Markov Random Field.
- 638 Descombes, X., Zerubia, J., 2002. Marked point process in image analysis. *IEEE Signal Processing*
639 *Magazine* 19, 77-84.
- 640 Disney, M.I., Kalogirou, V., Lewis, P., Prieto-Blanco, A., Hancock, S., Pfeifer, M., 2010. Simulating the
641 impact of discrete-return lidar system and survey characteristics over young conifer and broadleaf forests.
642 *Remote Sens. Environ.* 114, 1546-1560.
- 643 Dubayah, R.O., Drake, J.B., 2000. Lidar Remote Sensing for Forestry Applications. *J Forest* 98, 44-46.
- 644 Erikson, M., 2003. Segmentation of individual tree crowns in colour aerial photographs using region
645 growing supported by fuzzy rules. *Canadian Journal of Forest Research* 33, 1557-1563.

- 646 Falkowski, M.J., Smith, A.M.S., Hudak, A.T., Gessler, P.E., Vierling, L.A., Crookston, N.L., 2006.
647 Automated estimation of individual conifer tree height and crown diameter via two-dimensional spatial
648 wavelet analysis of lidar data. *Can J Remote Sens* 32, 153-161.
- 649 Ferraz, A., Bretar, F., Jacquemoud, S., Gonçalves, G., Pereira, L., Tomé, M., Soares, P., 2012. 3-D
650 mapping of a multi-layered Mediterranean forest using ALS data. *Remote Sens. Environ.* 121, 210-223.
- 651 Gebreslasie, M.T., Ahmed, F.B., van Aardt, J.A.N., Blakeway, F., 2011. Individual tree detection based
652 on variable and fixed window size local maxima filtering applied to IKONOS imagery for even-aged
653 Eucalyptus plantation forests. *International Journal of Remote Sensing* 32, 4141-4154.
- 654 Girard, M.C., 2003. *Processing of Remote Sensing Data*. Taylor & Francis, London.
- 655 Gleason, C.J., Im, J., 2012. A Fusion Approach for Tree Crown Delineation from Lidar Data.
656 *Photogrammetric Engineering and Remote Sensing* 78, 679-692.
- 657 Gonzalez, R.C., Woods, R.E., 2008. *Digital image processing*, 3rd ed. Pearson/Prentice Hall, Upper
658 Saddle River, NJ.
- 659 Gougeon, F.A., 1995. A crown-following approach to the automatic delineation of individual tree crowns
660 in high spatial resolution aerial images. *Can J Remote Sens* 21, 274-284.
- 661 Hastings, W.K., 1970. Monte Carlo sampling methods using Markov chains and their applications.
662 *Biometrika* 57, 97-109.
- 663 Heinzl, J., Koch, B., 2011. Exploring full-waveform LiDAR parameters for tree species classification.
664 *International Journal of Applied Earth Observation and Geoinformation* 13, 152-160.
- 665 Heurich, M., 2008. Automatic recognition and measurement of single trees based on data from airborne
666 laser scanning over the richly structured natural forests of the Bavarian Forest National Park. *Forest
667 Ecology and Management* 255, 2416-2433.
- 668 Hyypä, J., Hyypä, H., Leckie, D., Gougeon, F., Yu, X., Maltamo, M., 2008. Review of methods of
669 small-footprint airborne laser scanning for extracting forest inventory data in boreal forests. *International
670 Journal of Remote Sensing* 29, 1339-1366.

- 671 Hyypä, J., Inkinen, M., 1999. Detecting and Estimating Attributes for Single Trees Using Laser Scanner.
672 The Photogrammetric Journal of Finland 16, 27-42.
- 673 Hyypä, J., Kelle, O., Lehtikainen, M., Inkinen, M., 2001. A segmentation-based method to retrieve stem
674 volume estimates from 3-D tree height models produced by laser scanners. Geoscience and Remote
675 Sensing, IEEE Transactions on 39, 969-975.
- 676 Hyypä, J., Yu, X., Hyypä, H., Vastaranta, M., Holopainen, M., Kukko, A., Kaartinen, H., Jaakkola, A.,
677 Vaaja, M., Koskinen, J., Alho, P., 2012. Advances in forest inventory using airborne laser scanning.
678 Remote Sensing 4, 1190-1207.
- 679 Kaartinen, H., Hyypä, J., Yu, X., Vastaranta, M., Hyypä, H., Kukko, A., Holopainen, M., Heipke, C.,
680 Hirschmugl, M., Morsdorf, F., Næsset, E., Pitkänen, J., Popescu, S., Solberg, S., Wolf, B.M., Wu, J.C.,
681 2012. An international comparison of individual tree detection and extraction using airborne laser
682 scanning. Remote Sensing 4, 950-974.
- 683 Korpela, I., Dahlin, B., Schäfer, H., Bruun, E., Haapaniemi, F., Honkasalo, J., Ilvesniemi, S., Kuutti, V.,
684 Linkosalmi, M., Mustonen, J., 2007. Single-tree forest inventory using lidar and aerial images for 3D
685 treetop positioning, species recognition, height and crown width estimation, International Archives of
686 Photogrammetry and Remote Sensing and spatial information sciences, pp. 227-233.
- 687 Lacoste, C., Descombes, X., Zerubia, J., 2005. Point processes for unsupervised line network extraction
688 in remote sensing. Pattern Analysis and Machine Intelligence, IEEE Transactions on 27, 1568-1579.
- 689 Lafarge, F., Descombes, X., Zerubia, J., Pierrot-Deseilligny, M., 2008. Automatic building extraction
690 from DEMs using an object approach and application to the 3D-city modeling. ISPRS Journal of
691 Photogrammetry and Remote Sensing 63, 365-381.
- 692 Leckie, D., Gougeon, F., Hill, D., Quinn, R., Armstrong, L., Shreenan, R., 2003. Combined high-density
693 lidar and multispectral imagery for individual tree crown analysis. Can J Remote Sens 29, 633-649.

- 694 Maltamo, M., Malinen, J., Packaln, P., Suvanto, A., Kangas, J., 2006. Nonparametric estimation of stem
695 volume using airborne laser scanning, aerial photography, and stand-register data. *Can J Forest Res* 36,
696 426-436.
- 697 Means, J.E., Acker, S.A., Fitt, B.J., Renslow, M., Emerson, L., Hendrix, C.J., 2000. Predicting forest
698 stand characteristics with airborne scanning lidar. *Photogrammetric Engineering and Remote Sensing* 66,
699 1367-1371.
- 700 Metropolis, N., Rosenbluth, A.W., Rosenbluth, M.N., Teller, A.H., Teller, E., 1953. Equation of State
701 Calculations by Fast Computing Machines. *The Journal of Chemical Physics* 21, 1087-1092.
- 702 Meyer, F., Beucher, S., 1990. Morphological segmentation. *Journal of visual communication and image*
703 *representation* 1, 21-46.
- 704 Morsdorf, F., Nichol, C., Malthus, T., Woodhouse, I.H., 2009. Assessing forest structural and
705 physiological information content of multi-spectral LiDAR waveforms by radiative transfer modelling.
706 *Remote Sens. Environ.* 113, 2152-2163.
- 707 Næsset, E., 1997. Determination of mean tree height of forest stands using airborne laser scanner data.
708 *ISPRS Journal of Photogrammetry and Remote Sensing* 52, 49-56.
- 709 Naesset, E., 2002. Predicting forest stand characteristics with airborne scanning laser using a practical
710 two-stage procedure and field data. *Remote Sens. Environ.* 80, 88-99.
- 711 Naesset, E., 2004. Practical large-scale forest stand inventory using a small-footprint airborne scanning
712 laser. *Scand J Forest Res* 19, 164-179.
- 713 Naesset, E., Gobakken, T., Holmgren, J., Hyypä, H., Hyypä, J., Maltamo, M., Nilsson, M., Olsson, H.,
714 Persson, A., Soderman, U., 2004. Laser scanning of forest resources: The Nordic experience. *Scand J*
715 *Forest Res* 19, 482-499.
- 716 Orka, H.O., Naesset, E., Bollandsas, O.M., 2009. Classifying species of individual trees by intensity and
717 structure features derived from airborne laser scanner data. *Remote Sens. Environ.* 113, 1163-1174.

- 718 Ortner, M., Descombes, X., Zerubia, J., 2008. A marked point process of rectangles and segments for
719 automatic analysis of digital elevation models. *Pattern Analysis and Machine Intelligence, IEEE*
720 *Transactions on* 30, 105-119.
- 721 Perrin, G., Descombes, X., Zerubia, J., 2005. Point processes in forestry: an application to tree crown
722 detection. INRIA.
- 723 Perrin, G., Descombes, X., Zerubia, J., 2006. A non-Bayesian model for tree crown extraction using
724 marked point processes.
- 725 Pollock, R., 1996. The automatic recognition of individual trees in aerial images of forests based on a
726 synthetic tree crown image model, *Computer Science*. University of British Columbia.
- 727 Popescu, S.C., Wynne, R.H., Nelson, R.F., 2002. Estimating plot-level tree heights with lidar: local
728 filtering with a canopy-height based variable window size. *Comput Electron Agr* 37, 71-95.
- 729 Pyysalo, U., Hyyppa, H., 2002. Reconstructing tree crowns from laser scanner data for feature extraction.
730 *International Archives of Photogrammetry and Remote Sensing and spatial information sciences* 34,
731 218-221.
- 732 Reitberger, J., Schnörr, C., Krzystek, P., Stilla, U., 2009. 3D segmentation of single trees exploiting full
733 waveform LIDAR data. *ISPRS Journal of Photogrammetry and Remote Sensing* 64, 561-574.
- 734 Solberg, S., Naesset, E., Bollandsas, O.M., 2006. Single tree segmentation using airborne laser scanner
735 data in a structurally heterogeneous spruce forest. *Photogrammetric Engineering and Remote Sensing* 72,
736 1369-1378.
- 737 Sonka, M., Hlavac, V., Boyle, R., 2008. *Image processing, analysis, and machine vision*, 3rd ed.
738 Thompson Learning, Toronto.
- 739 Suratno, A., Seielstad, C., Queen, L., 2009. Tree species identification in mixed coniferous forest using
740 airborne laser scanning. *ISPRS Journal of Photogrammetry and Remote Sensing* 64, 683-693.

- 741 Tomppo, E., Nilsson, M., Rosengren, M., Aalto, P., Kennedy, P., 2002. Simultaneous use of Landsat-TM
742 and IRS-1C WiFS data in estimating large area tree stem volume and aboveground biomass. *Remote*
743 *Sens. Environ.* 82, 156-171.
- 744 Tournaire, O., Brédif, M., Boldo, D., Durupt, M., 2010. An efficient stochastic approach for building
745 footprint extraction from digital elevation models. *ISPRS Journal of Photogrammetry and Remote*
746 *Sensing* 65, 317-327.
- 747 Tournaire, O., Paparoditis, N., 2009. A geometric stochastic approach based on marked point processes
748 for road mark detection from high resolution aerial images. *ISPRS Journal of Photogrammetry and*
749 *Remote Sensing* 64, 621-631.
- 750 Van Lieshout, M.N.M., 2000. Markov point process and their applications. Imperial College Press,
751 London.
- 752 Van Lieshout, M.N.M., 2009. Applications of Stochastic Geometry in Image Analysis, in: Kendall, W.S.,
753 Molchanov, I. (Eds.), *New Perspectives in Stochastic Geometry*. Oxford University Press, pp. 427-450.
- 754 Vastaranta, M., Holopainen, M., Yu, X., Haapanen, R., Melkas, T., Hyypä, J., Hyypä, H., 2011a.
755 Individual tree detection and area-based approach in retrieval of forest inventory characteristics from low-
756 pulse airborne laser scanning data. *Photogram. J. Fin* 22, 1-13.
- 757 Vastaranta, M., Holopainen, M., Yu, X.W., Hyypä, J., Makinen, A., Rasinmaki, J., Melkas, T.,
758 Kaartinen, H., Hyypä, H., 2011b. Effects of Individual Tree Detection Error Sources on Forest
759 Management Planning Calculations. *Remote Sensing* 3, 1614-1626.
- 760 Vastaranta, M., Kankare, V., Holopainen, M., Yu, X., Hyypä, J., Hyypä, H., 2012. Combination of
761 individual tree detection and area-based approach in imputation of forest variables using airborne laser
762 data. *ISPRS Journal of Photogrammetry and Remote Sensing* 67, 73-79.
- 763 Vauhkonen, J., Ene, L., Gupta, S., Heinzl, J., Holmgren, J., Pitkanen, J., Solberg, S., Wang, Y.S.,
764 Weinacker, H., Hauglin, K.M., Lien, V., Packalen, P., Gobakken, T., Koch, B., Naesset, E., Tokola, T.,

- 765 Maltamo, M., 2012. Comparative testing of single-tree detection algorithms under different types of
766 forest. *Forestry* 85, 27-40.
- 767 Wang, L., Gong, P., Biging, G.S., 2004. Individual tree-crown delineation and treetop detection in high-
768 spatial-resolution aerial imagery. *Photogrammetric Engineering and Remote Sensing* 70, 351-358.
- 769 White, J., Wulder, M., Vastaranta, M., Coops, N., Pitt, D., Woods, M., 2013. The Utility of Image-Based
770 Point Clouds for Forest Inventory: A Comparison with Airborne Laser Scanning. *Forests* 4, 518-536.
- 771 Wulder, M., 1998. Optical remote-sensing techniques for the assessment of forest inventory and
772 biophysical parameters. *Progress in physical geography* 22, 449-476.
- 773 Wulder, M., Niemann, K.O., Goodenough, D.G., 2000. Local maximum filtering for the extraction of tree
774 locations and basal area from high spatial resolution imagery. *Remote Sens. Environ.* 73, 103-114.
- 775 Xie, Y., Sha, Z., Yu, M., 2008. Remote sensing imagery in vegetation mapping: a review. *Journal of Plant*
776 *Ecology* 1, 9-23.
- 777 Yao, W., Krzystek, P., Heurich, M., 2012. Tree species classification and estimation of stem volume and
778 DBH based on single tree extraction by exploiting airborne full-waveform LiDAR data. *Remote Sens.*
779 *Environ.* 123, 368-380.
- 780 Yu, X.W., Hyypä, J., Holopainen, M., Vastaranta, M., 2010. Comparison of Area-Based and Individual
781 Tree-Based Methods for Predicting Plot-Level Forest Attributes. *Remote Sensing* 2, 1481-1495.
- 782 Yu, X.W., Hyypä, J., Kaartinen, H., Maltamo, M., 2004. Automatic detection of harvested trees and
783 determination of forest growth using airborne laser scanning. *Remote Sens. Environ.* 90, 451-462.
- 784 Zhang, J., Sohn, G., 2010. A comparison of single tree detection algorithm using simulated and real
785 samples, *Silvilaser 2010, the 10th International Conference on LiDAR Applications for Assessing Forest*
786 *Ecosystems, Freiburg, Germany*, pp. 480-488.

787 Table 1: Parameter estimation results of the proposed model for all of the forest plots.

		Parameter Estimation					
		Real Forest Plots			Simulated Forest Plots		
		Plot 1	Plot 2	Plot3	Separate	Touch	Overlap
Symmetric Function	μ_s	0.43	0.39	0.45	0.32	0.37	0.45
	λ_s	0.10	0.11	0.13	0.06	0.08	0.15
Area Ratio Function	μ_a	0.69	0.68	0.67	0.82	0.76	0.72
	λ_a	-0.07	-0.07	-0.11	-0.03	-0.06	-0.14
Overlap Function	μ_o	0.28	0.32	0.38	0.08	0.26	0.40
	λ_o	0.04	0.05	0.05	0.01	0.03	0.05

788

789 Table 2: Detection results of the proposed model with estimated parameters compared with local maxima
790 filtering (LM) on the real coniferous forest plots.

	Detected Trees	Correct		Commission		Omission		Overall Accuracy
		No.	%	No.	%	No.	%	
<i>Plot 1 - 120 trees</i>								
LM	185	118	63.8%	67	36.2%	2	1.7%	63.1%
Proposed Model	126	113	89.7%	13	10.3%	7	5.8%	85.0%
<i>Plot 2 - 40 trees</i>								
LM	51	38	74.5%	13	25.5%	2	5.0%	71.7%
Proposed Model	41	38	92.7%	3	7.3%	2	5.0%	88.4%
<i>Plot 3 - 122 trees</i>								
LM	141	115	81.6%	26	18.4%	7	5.7%	77.7%
Proposed Model	123	112	91.1%	11	8.9%	10	8.2%	84.2%

791

792 Table 3: Detection results of the proposed model with estimated parameters compared with local maxima
793 filtering (LM) on the simulated forest plots.

	Detected Trees	Correct		Commission		Omission		Overall Accuracy
		No.	%	No.	%	No.	%	
<i>Separate Plot - 186 trees</i>								
LM	213	184	86.4%	29	13.6%	2	1.1%	85.6%
Proposed Model	182	181	99.5%	1	0.5%	5	2.7%	96.8%
<i>Touching Plot - 234 trees</i>								
LM	252	218	86.5%	34	13.5%	16	6.8%	81.3%
Proposed Model	216	215	99.5%	1	0.5%	19	8.1%	91.5%
<i>Overlapping Plot - 261 trees</i>								
LM	256	226	88.3%	30	11.7%	35	13.4%	77.6%
Proposed Model	221	221	100.0%	0	0.0%	40	15.3%	84.7%

794

795 Figure 1: (a) Location of the study area in the Province of Ontario, Canada; (b) a photo and (c) ortho view of the
796 ALS data of a forest plot in the study area rendered by height.

797 Figure 2: (a)-(c) Point process simulated forest plots with different degrees of crown overlap: (a) plot with
798 separated crowns; (b) plot with tree crowns slightly touching each other; (c) plot with overlapping crowns. (d)-
799 (f) the corresponding ALS point clouds of the three forest plots generated.

800 Figure 3: Flow chart of the proposed method.

801 Figure 4: An example showing the configuration construction from a CHM. (a) a subset of local maxima. Local
802 maxima are shown as red crosses; (b) a marked-controlled watershed segmentation of the CHM using local
803 maxima in (a) as the marker function; (c) the configuration constructed from the local maxima. Radii of the tree
804 crowns are extracted from the corresponding segments in (b).

805 Figure 5: Asymmetric ratio calculation for (a) symmetric and (b) asymmetric tree crowns.

806 Figure 6: Area ratio calculation for tree objects with (a) symmetric and (b) asymmetric tree crowns.

807 Figure 7: Overlap ratio calculation of overlapping tree crowns.

808 Figure 8: Plots of the sigmoid function $F(x) = 1/(1 + \exp - (x - \mu)/\lambda) - 1$ with respect to different values
809 of μ and λ . In the left plot, λ is set to 0.2 for all three curves. In the right plot, μ is set to 0.5 for all three curves.

810 Figure 9: Likelihood distributions, posterior probability and fitted sigmoid functions for the asymmetric ratio,
811 area ratio and overlap ratio. Row 1: Likelihood models of those ratios for the reference group; Row 2: Likelihood
812 models of those ratios for the error group; Row 3: Posterior probabilities (red lines) for those ratios for the error
813 group and the fitted sigmoid functions (blue dashed lines).

814 Figure 10: Detection results of the proposed model with estimated parameters compared with traditional local
815 maxima filtering on real coniferous forest plots. (a)-(c) show the local maxima filtering results; (d)-(f) show the
816 detection result of the proposed model using the corresponding local maxima filtering detection as the initial
817 configuration. (the green circles with triangles in the center represent the commission errors; the cyan dot line

818 circles represent the omission errors resulting from the LM; the yellow circles represent the omission errors
819 produced by the proposed model.)

820 Figure 11: Detection results of the proposed model with estimated parameters compared with local maxima
821 filtering on simulated forests. (a)-(c) show the local maxima filtering detection on the three simulated forest plots;
822 (d)-(f) show the proposed model detection results using the corresponding local maxima filtering detection as
823 the initial configuration. (the green circles with triangles in the center represent the commission errors; the cyan
824 dot line circles represent the omission errors resulting from the LM; the yellow circles represent the omission
825 errors produced by the proposed model.)

826 Figure 12: Statistics associated with the optimization process of the simulated forest plot with touching crowns:
827 (a) Temperature; (b) Acceptance rate; (c) Global energy.

828