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Approximate Nash Region of the Gaussian Interference Channel with Noisy Output Feedback
Victor Quintero, Samir M. Perlaza, Jean-Marie Gorce, and H. Vincent Poor

Abstract—In this paper, an achievable $\eta$-Nash equilibrium ($\eta$-NE) region for the two-user Gaussian interference channel with noisy channel-output feedback is presented for all $\eta \geq 1$. This result is obtained in the scenario in which each transmitter-receiver pair chooses its own transmit-receive configuration in order to maximize its own individual information transmission rate. At an $\eta$-NE, any unilateral deviation by either of the pairs does not increase the corresponding individual rate by more than $\eta$ bits per channel use.

Index Terms—Gaussian Interference Channel, Noisy channel-output feedback, $\eta$-Nash equilibrium region.

I. INTRODUCTION

The interference channel (IC) is one of the simplest yet insightful multi-user channels in network information theory. An important class of ICs is the two-user Gaussian interference channel (GIC) in which there exist two point-to-point links subject to mutual interference and independent Gaussian noise sources. In this model, each output signal is a noisy version of the sum of the two transmitted signals affected by the corresponding channel gains. The analysis of this channel can be made considering two general scenarios: (1) a centralized scenario in which the entire network is controlled by a central entity that configures both transmitter-receiver pairs; and (2) a decentralized scenario in which each transmitter-receiver pair autonomously configures its transmission-reception parameters. In the former, the fundamental limits are characterized by the capacity region, which is approximated to within a fixed number of bits in [1] for the case without feedback; in [2] for the case with perfect channel-output feedback; and in [3] and [4] for the case with noisy channel-output feedback. In the latter, the fundamental limits are characterized by the $\eta$-Nash equilibrium ($\eta$-NE) region. The $\eta$-NE of the GIC is approximated in the cases without feedback and with perfect channel-output feedback in [5] and [6], respectively.

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In this paper the $\eta$-NE region of the GIC is studied assuming that there exists a noisy feedback link from each receiver to its corresponding transmitter. The $\eta$-NE region is approximated by two regions for all $\eta \geq 1$: a region for which an equilibrium transmit-receive configuration is presented for each of the information rate pairs (an achievable region); and a region for which any information rate pair that is outside of this region cannot be an $\eta$-NE (impossibility region). The focus of this paper is on the achievable region.

The results presented in this paper are a generalization of the results presented in [5] and [6], and they are obtained thanks to the analysis of linear deterministic approximations in [7] and [8].

II. DECENTRALIZED GAUSSIAN INTERFERENCE CHANNELS WITH NOISY CHANNEL-OUTPUT FEEDBACK

Consider the two-user decentralized Gaussian interference channel with noisy channel-output feedback (D-GIC-NOF) depicted in Figure 1. Transmitter $i$, with $i \in \{1, 2\}$, communicates with receiver $i$ subject to the interference produced by transmitter $j$, with $j \in \{1, 2\} \setminus \{i\}$. There are two independent and uniformly distributed messages, $W_i \in \mathcal{W}_i$, with $\mathcal{W}_i = \{1, 2, \ldots, 2^{N_i R_i}\}$, where $N_i$ denotes the fixed block-length in channel uses and $R_i$ the information transmission rate in bits per channel use. At each block, transmitter $i$ sends the codeword $X_i = (X_{i,1}, X_{i,2}, \ldots, X_{i,N_i})^T \in \mathcal{C}_i \subseteq \mathbb{R}^{N_i}$, where $\mathcal{C}_i$ is the codebook of transmitter $i$. The channel coefficient from transmitter $j$ to receiver $i$ is denoted by $h_{ij}$; the channel coefficient from transmitter $i$ to receiver $i$ is denoted by $h_{ii}$; and the channel coefficient from channel-output $i$ to transmitter $i$ is denoted by $\overline{h}_{ii}$. All channel coefficients are assumed to be non-negative real numbers. At a given channel
use \( n \in \{1, 2, \ldots, N\} \), with
\[
N = \max(N_1, N_2),
\]
the channel output at receiver \( i \) is denoted by \( \vec{Y}_{i,n} \). During channel use \( n \), the input-output relation of the channel model is given by
\[
\vec{Y}_{i,n} = h_{ii}X_{i,n} + h_{ij}X_{j,n} + \vec{Z}_{i,n},
\]
where \( X_{i,n} = 0 \) for all \( n \) such that \( N \geq n > N_i \) and \( \vec{Z}_{i,n} \) is a real Gaussian random variable with zero mean and unit variance that represents the noise in the input of receiver \( i \). Let \( d > 0 \) be the finite feedback delay measured in channel uses. At the end of channel use \( n \), transmitter \( i \) observes \( \vec{Y}_{i,n} \), which consists of a scaled and noisy version of \( \vec{Y}_{i,n-d} \). More specifically,
\[
\vec{Y}_{i,n} = \left[ \sum_{n=1}^{N_i} Y_{i,n} \right] / \left( h_{ii} \vec{Y}_{i,n-d} + \vec{Z}_{i,n} \right),
\]
for \( n \in \{1, 2, \ldots, d\} \)
\[
\vec{Z}_{i,n} = \left[ \sum_{n=1}^{N_i} Z_{i,n} \right] / \left( h_{ii} \vec{Y}_{i,n-d} + \vec{Z}_{i,n} \right),
\]
for \( n \in \{d+1, d+2, \ldots, N\} \).

Within this context, a rate pair \((R_1, R_2) \in \mathbb{R}_+^2\) is said to be achievable if it complies with the following definition.

**Definition 1 (Achievable Rate Pairs):** A rate pair \((R_1, R_2) \in \mathbb{R}_+^2\) is achievable if there exists at least one pair of configurations \((s_1, s_2)\) such that the decoding bit error probabilities \(p_1(s_1, s_2)\) and \(p_2(s_1, s_2)\) can be made arbitrarily small by letting the block-lengths \(N_1\) and \(N_2\) grow to infinity.

The aim of transmitter \( i \) is to autonomously choose its transmit-receive configuration \(s_i\) in order to maximize its achievable rate \(R_i\). Note that the rate achieved by transmitter-receiver \( i \) depends on both configurations \(s_1\) and \(s_2\) due to mutual interference. This reveals the competitive interaction between both links in the decentralized interference channel.

The fundamental limits of the two-user D-GIC-NOF in Figure 1 can be described by six parameters: \(\text{SNR}_i, \text{SNR}_{ij}, \text{INR}_{ij}\), and \(\text{INR}_{ij}\) with \(i \in \{1, 2\}\) and \(j \in \{1, 2\} \setminus \{i\}\), which are defined as follows:

\[
\text{SNR}_i \triangleq h_{ii}^2,
\]
\[
\text{INR}_{ij} \triangleq h_{ij}^2 + h_{ij}^2 + h_{ij}^2 + 1.
\]

The analysis presented in this paper focuses exclusively on the case in which \(\text{INR}_{ij} > 1\) for all \(i \in \{1, 2\}\) and \(j \in \{1, 2\} \setminus \{i\}\). The reason for exclusively considering this case follows from the fact that when \(\text{INR}_{ij} \leq 1\), the transmitter-receiver pair \(i\) is impaired mainly by noise instead of interference. In this case, feedback does not bring a significant rate improvement. Denote by \(C\) the capacity region of the two-user GIC-NOF with fixed parameters \(\text{SNR}_1, \text{SNR}_2, \text{INR}_{ij}, \text{SNR}_{ij}\), and \(\text{SNR}_{ij}\). The achievable region \(C\) in [4, Theorem 2] and the converse region \(\overline{C}\) in [4, Theorem 3] approximate the capacity region \(C\) to within 4.1 bits [4].

### III. Game Formulation

The competitive interaction between the two transmitter-receiver pairs in the interference channel can be modeled by the following game in normal-form:

\[
G = (\mathcal{K}, \{A_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}}).
\]

The set \(\mathcal{K} = \{1, 2\}\) is the set of players, that is, the set of transmitter-receiver pairs. The sets \(A_1\) and \(A_2\) are the sets of actions of players 1 and 2, respectively. An action of a player \(i \in \mathcal{K}\), which is denoted by \(s_i \in A_i\), is basically its transmit-receive configuration as described above. The utility function of player \(i\) is \(u_i : A_1 \times A_2 \rightarrow \mathbb{R}_+\) and it is defined as the information rate of transmitter \(i\),

\[
u_i(s_1, s_2) = \begin{cases} R_i, & \text{if } p_i(s_1, s_2) < \epsilon \\ 0, & \text{otherwise} \end{cases}
\]

where \(\epsilon > 0\) is an arbitrarily small number. This game formulation for the case without feedback was first proposed in [9] and [10].

A class of transmit-receive configurations that are particularly important in the analysis of this game is referred to as
the set of $\eta$-Nash equilibria ($\eta$-NE), with $\eta > 0$. This type of configurations satisfy the following definition.

**Definition 2 ($\eta$-Nash equilibrium):** Given a positive real $\eta$, an action profile $(s_i^*, s_j^*)$ is an $\eta$-Nash equilibrium (NE) in the game $G = (K, \{A_k\}_{k \in K}, \{u_k\}_{k \in K})$, if for all $i \in K$ and for all $s_i \in A_i$, it follows that

$$u_i(s_i, s_j^*) \leq u_i(s_i^*, s_j^*) + \eta. \quad (13)$$

Let $(s_1^*, s_2^*)$ be an $\eta$-Nash equilibrium action profile. Then, none of the transmitters can increase its own transmission rate more than $\eta$ bits per channel use by changing its own transmit-receive configuration and keeping the average bit error probability arbitrarily close to zero. Note that for $\eta$ sufficiently large, from Definition 2, any pair of configurations can be an $\eta$-NE. Alternatively, for $\eta = 0$, the definition of Nash equilibrium is obtained [11]. In this case, if a pair of configurations is a Nash equilibrium ($\eta = 0$), then each individual configuration is optimal with respect to each other. Hence, the interest is to describe the set of all possible $\eta$-NE rate pairs $(R_1, R_2)$ of the game in (11) with the smallest $\eta$ for which there exists at least one equilibrium configuration pair.

The set of rate pairs that can be achieved at an $\eta$-NE is known as the $\eta$-Nash equilibrium ($\eta$-NE) region.

**Definition 3 ($\eta$-NE Region):** Let $\eta > 0$ be fixed. An achievable rate pair $(R_1, R_2)$ is said to be in the $\eta$-NE region of the game $G = (K, \{A_k\}_{k \in K}, \{u_k\}_{k \in K})$ if there exists a pair $(s_1^*, s_2^*) \in A_1 \times A_2$ that is an $\eta$-NE and the following holds:

$$u_1(s_1^*, s_2^*) = R_1 \quad \text{and} \quad u_2(s_1^*, s_2^*) = R_2. \quad (14)$$

The $\eta$-NE regions of the two-user GIC with and without perfect channel-output feedback have been approximated to within a constant number of bits in [5] and [6], respectively. The next section introduces a generalization of these results.

**IV. MAIN RESULTS**

**A. Achievable $\eta$-Nash Equilibrium Region**

Let the $\eta$-NE region (Definition 3) of the D-GIC-NOF be denoted by $\mathcal{N}_\eta$. This section introduces a region $\mathcal{N}^\prime_\eta \subseteq \mathcal{N}_\eta$ that is achievable using a coding scheme that combines rate splitting [12], common randomness [5], [6], block Markov superposition coding [13] and backward decoding [14]. In the following, this coding scheme is referred to as randomized Han-Kobayashi scheme with lossy channel-output feedback (RHK-NOF). This coding scheme is presented in [8] and uses the same techniques of the schemes in [5] and [6]. Therefore, the focus of this section is on the results rather than the description of the scheme. A motivated reader is referred to [15]. The RHK-NOF is proved to be an $\eta$-NE action profile with $\eta \geq 1$. That is, any unilateral deviation from the RHK-NOF by any of the transmitter-receiver pairs might lead to an individual rate improvement which is at most one bit per channel use. The description of the achievable $\eta$-Nash region $\mathcal{N}^\prime_\eta$ is presented using the constants $\alpha_{1,i}$: the functions $a_{2,i} : [0, 1] \to \mathbb{R}_+$, $a_{3,i} : [0, 1]^{i} \to \mathbb{R}_+$, with $i \in \{3, \ldots, 6\}$; and $a_{7,i} : [0, 1]^{i} \to \mathbb{R}_+$, which are defined as follows, for all $i \in \{1, 2\}$, with $j \in \{1, 2\} \setminus \{i\}$:

$$a_{1,i} := \frac{1}{2} \log \left( 2 + \frac{\tilde{\text{SNR}}_i}{\text{INR}_{j,i}} \right) - \frac{1}{2}, \quad (15a)$$

$$a_{2,i} (\rho) := \frac{1}{2} \log \left( b_{1,i} (\rho) + 1 \right) - \frac{1}{2}, \quad (15b)$$

$$a_{3,i} (\rho, \mu) := \frac{1}{2} \log \left( \frac{\tilde{\text{SNR}}_i \left( b_{2,i} (\rho) + 2 \right) + b_{1,i} (1) + 1}{\tilde{\text{SNR}}_i \left( (1-\mu) b_{2,i} (\rho) + 2 \right) + b_{1,i} (1) + 1} \right). \quad (15c)$$
Theorem 2 of a two-user D-GIC-POF with parameters $SNR_1 = 24$ dB, $SNR_2 = 18$ dB, $INR_{12} = 48$ dB, $INR_{21} = 30$ dB, $SNR_1 \in \{-100, 18, 50\}$ dB, $SNR_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$. Note that in this case, the feedback parameter $\overrightarrow{SNR}_2$ does not have an effect on the achievable $\eta$-NE region $\mathcal{N}_\eta$ and the achievable capacity region $\mathcal{C}$ ([4, Theorem 2]). This is due to the fact that when one transmitter-receiver pair is in low interference regime (LIR) and the other transmitter-receiver pair is in high interference regime (HIR), feedback is useless on the transmitter-receiver pair in HIR [15, [16].

Figure 3 shows the achievable region $\mathcal{C}$ in [4, Theorem 2] of a two-user centralized GIC-NOF and the achievable $\eta$-NE region $\mathcal{N}_\eta$ in Theorem 1 of a two-user D-GIC-NOF with parameters $SNR_1 = 24$ dB, $SNR_2 = 18$ dB, $INR_{12} = 16$ dB, $INR_{21} = 10$ dB, $SNR_1 \in \{-100, 18, 50\}$ dB, $SNR_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$. Figure 4 shows the achievable region $\mathcal{C}$ in [4, Theorem 2] of a two-user centralized GIC-NOF and the achievable $\eta$-NE region $\mathcal{N}_\eta$ in Theorem 1 of a two-user D-GIC-NOF with parameters $SNR_1 = 24$ dB, $SNR_2 = 18$ dB, $INR_{12} = 48$ dB, $INR_{21} = 30$ dB, $SNR_1 \in \{-100, 18, 50\}$ dB, $SNR_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$. In this case, the achievable $\eta$-NE region $\mathcal{N}_\eta$ in Theorem 1 and achievable region $\mathcal{C}$ on the capacity region [4, Theorem 2] are almost identical, which implies that in the cases in which $SNR_i < INR_{ij}$, for both $i \in \{1, 2\}$, with $j \in \{1, 2\} \setminus \{i\}$, the achievable $\eta$-NE region is almost the same as the achievable capacity region in the centralized case studied in [4]. At low values of $\overrightarrow{SNR}_1$ and $\overrightarrow{SNR}_2$, the achievable $\eta$-NE region approaches the rectangular region reported in [5] for the case of the two-user decentralized GIC (D-GIC). Alternatively, for high values of $\overrightarrow{SNR}_1$ and $\overrightarrow{SNR}_2$, the achievable $\eta$-NE region approaches the region reported in [6] for the case of the two-user decentralized GIC with perfect channel-output feedback (D-GIC-POF). These observations are formalized by the following corollaries.

Denote by $\mathcal{N}_{\eta, PF}$ the achievable $\eta$-NE region of the two-user D-GIC-POF presented in [6]. The region $\mathcal{N}_{\eta, PF}$ can be obtained as a special case of Theorem 1 as shown by the following corollary.

**Corollary 1 (\(\eta\)-NE Region with Perfect Output Feedback):** Let $\mathcal{N}_{\eta, PF}$ denote the achievable $\eta$-NE region of the two-user D-GIC-POF with fixed parameters $SNR_i$ and $INR_{ij}$, with $i \in \{1, 2\}$ and $j \in \{1, 2\} \setminus \{i\}$. Then, the following holds:

$$\mathcal{N}_{\eta, PF} = \lim_{\overrightarrow{SNR}_i \to \infty} \mathcal{N}_{\eta}$$

(18)

Denote by $\mathcal{N}_{\eta, PF}$ the achievable $\eta$-NE region of the two-user D-GIC presented in [5]. The region $\mathcal{N}_{\eta, PF}$ can be obtained as a special case of Theorem 1 as shown by the following corollary.

**Corollary 2 (\(\eta\)-NE Region without Output Feedback):** Let $\mathcal{N}_{\eta, PF}$ denote the achievable $\eta$-NE region of the two-user D-GIC, with fixed parameters $SNR_i$ and $INR_{ij}$, with $i \in \{1, 2\}$
Theorem 1: Let \( \eta \geq 1 \) be fixed. The achievable \( \eta \)-NE region \( \mathcal{N}_\eta \) is given by the closure of all possible achievable rate pairs \( (R_1, R_2) \in \mathcal{C} \) in [4, Theorem 2] that satisfy, for all \( i \in \{1, 2\} \) and \( j \in \{1, 2\} \setminus \{i\} \), the following conditions:

\[
R_i \geq \left( a_{2,i}(\rho) - a_{3,i}(\rho, \mu_j) - a_{4,i}(\rho, \mu_j) - \eta \right)^+,
\]

\[
R_i \leq \min \left( a_{2,i}(\rho) + a_{3,j}(\rho, \mu_i) + a_{5,j}(\rho, \mu_i) - a_{2,j}(\rho) + \eta, \right.
\]

\[
a_{3,i}(\rho, \mu_j) + a_{7,i}(\rho, \mu_1, \mu_2) + 2a_{3,j}(\rho, \mu_i) + a_{5,j}(\rho, \mu_i) - a_{2,j}(\rho) + \eta,
\]

\[
a_{2,i}(\rho) + a_{3,i}(\rho, \mu_j) + 2a_{3,j}(\rho, \mu_i) + a_{5,j}(\rho, \mu_i) + a_{7,j}(\rho, \mu_1, \mu_2) - 2a_{2,j}(\rho) + 2\eta),
\]

\[
R_1 + R_2 \leq a_{1,i} + a_{3,i}(\rho, \mu_j) + a_{7,i}(\rho, \mu_1, \mu_2) + a_{2,j}(\rho) + a_{3,j}(\rho, \mu_1) - a_{2,i}(\rho) + \eta,
\]

for all \( (\rho, \mu_1, \mu_2) \in \left[0, 1 - \max \left( \frac{1}{\text{INR}_{12}^2}, \frac{1}{\text{INR}_{21}^2} \right) \right] \times [0, 1] \times [0, 1] \).

B. Impossibility Region

This section introduces an impossibility region, denoted by \( \mathcal{N}_\eta \). That is, \( \mathcal{N}_\eta \supseteq \mathcal{N}_\eta \). More specifically, any rate pair \( (R_1, R_2) \in \mathcal{N}_\eta \) is not an \( \eta \)-NE. This region is described in terms of the convex region \( \mathcal{B}_{G-IC-NOF} \). Here, for the case of the two-user D-GIC-NOF, the region \( \mathcal{B}_{G-IC-NOF} \) is given by the closure of the rate pairs \( (R_1, R_2) \in \mathbb{R}^2_+ \) that satisfy for all \( i \in \{1, 2\} \), with \( j \in \{1, 2\} \setminus \{i\} \):

\[
\mathcal{B}_{G-IC-NOF} = \left\{ (R_1, R_2) \in \mathbb{R}^2_+ : R_i \geq L_i, \right. \]

\[
\text{for all } i \in \mathcal{K} = \{1, 2\} \},
\]

where,

\[
L_i \triangleq \left( \frac{1}{2} \log \left( 1 + \text{SNR}_i \right) - \eta \right)^+.
\]

Note that \( L_i \) is the rate achieved by the transmitter-receiver pair \( i \) when it saturates the power constraint in (5) and treats interference as noise. Following this notation, the impossibility region of the two-user GIC-NOF, i.e., \( \mathcal{N}_\eta \), can be described as follows.

Theorem 2: Let \( \eta \geq 1 \) be fixed. The impossibility region \( \mathcal{N}_\eta \) of the two-user D-GIC-NOF is given by the closure of all possible non-negative rate pairs \( (R_1, R_2) \in \mathcal{C} \cap \mathcal{B}_{G-IC-NOF} \) for all \( \rho \in [0, 1] \).

The impossibility region in Theorem 2 has been first presented in [6] and it is very loose in this case. A better impossibility region is presented in [15].

V. Conclusions

In this paper, an achievable \( \eta \)-Nash equilibrium (\( \eta \)-NE) region for the two-user Gaussian interference channel with noisy channel-output feedback has been presented for all \( \eta \geq 1 \). This result generalizes the existing achievable regions of the \( \eta \)-NE for the the cases without feedback and with perfect channel-output feedback.

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