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HTML ABSTRACT + LINKS

Comments on the mean flow averaged model

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The dynamic behavior of convective motion in a confined porous medium saturated by a pure fluid under the action of mechanical vibration is studied. A redefinition of vibrational Rayleigh number is proposed from which we distinguish the domain of validity of the mean flow. The weakly nonlinear stability analysis performed demonstrates, contrary to published results, that the bifurcation is of supercritical nature and the subcritical branch does not exist. It is emphasized that, in order to find the thermal behavior of the system for the onset of convection, we should separate the vibrational effect from the thermal effect involving the temperature difference. © *2004 American Institute of Physics*. [DOI: 10.1063/1.1810771]

I. INTRODUCTION

It is found theoretically and experimentally (Hirata *et al.*,[1] Shu *et al.*,[2] Charrier *et al.*,[3] Gershuni and Lyubimov,[4] Rogers and Schatz[5] and Zavarykin *et al.*[6]) that interaction of mechanical vibrations with nonhomogeneity in the density field gives rise to a new driving force which may set up the mean flow. We are interested in the influence of vertical mechanical vibration, which is characterized by $b \sin \hat{\omega} t \mathbf{k}$, on the onset of convection in a rectangular porous cavity of height H and length L saturated by a pure fluid. Our preliminary research showed that, under vertical vibration parallel to the temperature gradient, the stabilizing effect would be obtained (quite similar to the effect found out by Gershuni and Lyubimov[4] for a pure fluid medium). This was embodied in an increase of the critical temperature difference ΔT. It should be noted that the problem depends on two nondimensional parameters, namely, the filtration thermal Rayleigh number ($\mathrm{Ra}=g\beta_\theta KH\Delta T/\nu a$) and vibrational Rayleigh number $\mathrm{Ra}_v=\mathrm{Ra}^2R^2A/2(A^2\hat{\omega}^2+1)$. K is the permeability, ν the viscosity, a the thermal diffusivity, $R=b\hat{\omega}^2/g$, and $A=\mathrm{Da}\,M/\epsilon\,\mathrm{Pr}$ of the order 10^{-6} in classical porous media. $\mathrm{Da}=K/H^2$ is the Darcy number, M the volumetric specific heat ratio, and ϵ the porosity.

To perform an experiment under earth-surface conditions (Bardan,[7] Bardan and Mojtabi[8]), we should split the vibrational Rayleigh number into two parts: one part which depends on the temperature difference and the other part which depends on vibrational parameters. The separation of vibrational and thermal parameters in the averaged approach was proposed in the monographs of Gershuni and Zhukovitski.[9] Historically, the independent use of the vibrational Rayleigh number was proposed in the context of experiments under weightlessness. However, the improper use of vibrational Rayleigh number in the case of simultaneous driving mechanisms gives erroneous results, which is the subject of this paper.

In our previous analysis (Bardan and Mojtabi[10]), we kept the vibrational Rayleigh number fixed and searched for the critical values of Rayleigh number. Unfortunately, the analysis suffers from two major shortcomings, which are as follows.

(1) In an experiment, we cannot fix the vibrational Rayleigh number and change the thermal Rayleigh number in order to find the effect of vibration on the onset of convection; both Ra and Ra_v depend on ΔT.

(2) The confined porous cavity heated from below, undergoing the mechanical vibration, constitutes a dynamic system in which the resulting convective motion may have a harmonic or subharmonic response. So, our formulation should show that it is valid only under high-frequency and small amplitude vibration; which indeed is characterized by a harmonic response.

This is why we replace Ra_v by $P_v\mathrm{Ra}^2$ from which the limit of the mechanical parameter is found, beyond which the harmonic response ceases to exist.

We underline that, by using the modified vibrational parameter P_v (which does not depend on ΔT), the results of the stability analysis (linear and weakly nonlinear) changes drastically. Only under P_v formulation, may we find the validity domain of the mean flow model and its limitations; it is not a question of parameter definition.

II. MEAN FLOW EQUATIONS

The equations of the mean motion given by Bardan and Mojtabi[10] are rewritten by setting $\mathrm{Ra}_v=P_v\mathrm{Ra}^2$. $P_v=R^2A/2(A^2\hat{\omega}^2+1)$ is a vibrational parameter that does not depend on ΔT,

$$\nabla \cdot \mathbf{U} = 0, \tag{1}$$

$$A\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} = -\nabla P + \mathrm{Ra}\, T\mathbf{k} + P_v\mathrm{Ra}^2\mathbf{W_T}\cdot\nabla(T\mathbf{k}), \tag{2}$$

$$\frac{\partial T}{\partial t} + (\mathbf{U}\cdot\nabla)T = \nabla^2 T, \tag{3}$$

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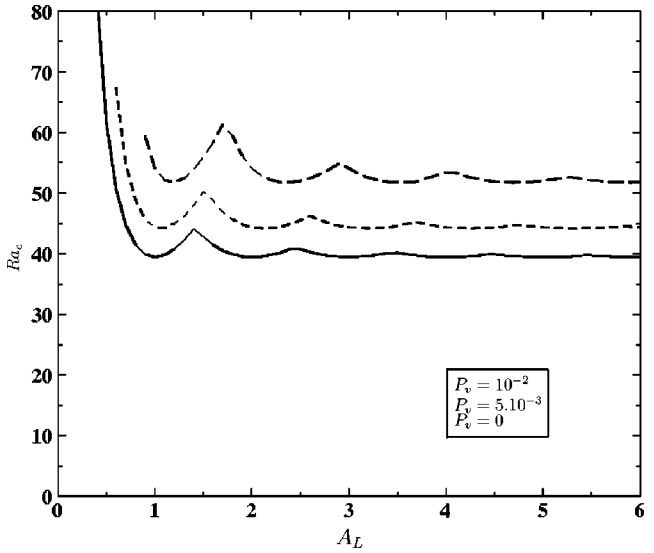


FIG. 1. Critical Rayleigh number Ra_c of the first primary bifurcation point is the aspect ratio A_L. In the Ra_v formulation, $\mathrm{Ra}_c \to \infty$ when $\mathrm{Ra}_v \to \infty$, Bardan and Mojtabi (Ref. 10). In the P_v formulation, there exists a limit value P_{v_l} which cannot be exceeded.

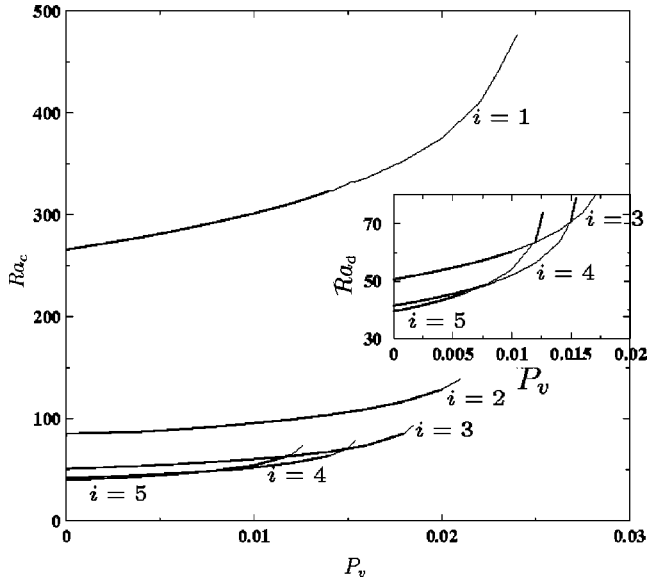


FIG. 2. Critical Rayleigh numbers corresponding to the synchronous modes $i=5$, $i=4$, $i=3$, $i=2$, and $i=1$ plotted vs P_v for $A_l=5$. Inset: Enlargement for $P_v \in [0, 0.002]$.

$$\boldsymbol{\nabla} \wedge T\mathbf{k} = \boldsymbol{\nabla} \wedge \mathbf{W_T}, \tag{4}$$

$$\boldsymbol{\nabla} \cdot \mathbf{W_T} = 0. \tag{5}$$

The boundary conditions are in accordance with the physical statement of the problems (12)–(15) and (6),

$$\mathbf{W_T} \cdot \mathbf{n} = 0 \tag{6}$$

on $\partial\Omega$ ($\mathbf{n}$ is the outside normal vector).

III. LINEAR STABILITY

The linear stability analysis shows that the conductive state becomes unstable at the critical Rayleigh number value Ra_c given by

$$\mathrm{Ra}_c = \left(\frac{i^2 + j^2 A_L^2}{2i^2 P_v}\right)\left(1 - \sqrt{1 - 4\pi^2 P_v \frac{i^2 + j^2 A_L^2}{A_L^2}}\right), \tag{7}$$

where $A_L = L/H$ is the aspect ratio. The number of cells of the emerging convective flow is linked with the values i and j, i.e., i is the number of cells along the x axis and j along the z axis. We may conclude from this equation that there is a limit value P_{v_l} for the vibrational parameter:

$$P_{v_l} = \frac{1}{4\pi^2\left[\left(\frac{i}{A_L}\right)^2 + j^2\right]}. \tag{8}$$

Dealing with the first primary bifurcation points, the minimum value of the critical Rayleigh number is obtained for $j=1$.

The critical Rayleigh number does not exist when $P_v > P_{v_l}$. Figures 1 and 2 show how P_{v_l} depends on A_L and i.

In the limit case of an infinite layer, we have $A_L \approx i \to \infty$ and a finite value of P_{v_l} still exists. This limit will also concern the case studied in Gershuni *et al.*[11]

A scale analysis is given below to demonstrate that the limit in P_v comes from the validity domain of the time-averaged formulation.

IV. SCALE ANALYSIS

Noting $\omega = \hat{\omega} H^2/a$, the governing equations for a Boussinesq fluid in a saturated porous medium under vertical vibration may be written as

$$\boldsymbol{\nabla} \cdot \mathbf{u} = 0, \tag{9}$$

$$A\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} = -\boldsymbol{\nabla} p + \mathrm{Ra}\,\theta\mathbf{k} + R\,\mathrm{Ra}\,\theta \sin(\omega t)\mathbf{k}, \tag{10}$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{u} \cdot \boldsymbol{\nabla})\theta = \boldsymbol{\nabla}^2 \theta \tag{11}$$

with the boundary conditions

$$\mathbf{u} \cdot \mathbf{n} = 0 \text{ on } \partial\Omega, \tag{12}$$

$$\theta = 1 \text{ for } z = 0, \quad \forall x, \tag{13}$$

$$\theta = 0 \text{ for } z = 1, \quad \forall x, \tag{14}$$

$$\frac{\partial \theta}{\partial x} = 0 \text{ for } x = 0 \text{ and } A_L, \quad \forall z. \tag{15}$$

In the limiting case of high frequency and small amplitude of vibration with respect to some references, we may adopt the time-averaged formulation. In this section, these references are defined and complete the explanation given in Gershuni and Lyubimov.[4] If the fields are decomposed into two parts, one with slow evolution with time and the other with fast evolution with time, for a given function f, the temporal average over a period τ of vibration is defined as $F = \int_{t-\pi/2}^{t+\pi/2} f(s)ds$ and we may write

$$\mathbf{u} = \mathbf{U}(t) + \mathbf{u}'(\omega t), \quad \theta = T(t) + \theta'(\omega t), \quad p = P(t) + p'(\omega t). \tag{16}$$

By substituting (16) in the system (9)–(11) and using the fact that the average of oscillating fields over a period is zero, we find two coupled systems of equations, one governing the rapid time evolution and the other governing the slow time evolution (averaged system). The closure for the averaged system comes from a relation between these two systems. In order to resolve the closure issue, the oscillating momentum equation is considered:

$$A\frac{\partial \mathbf{u}'}{\partial t} + \mathbf{u}' = -\nabla p' + \mathrm{Ra}\,\theta'\mathbf{k} - R\,\mathrm{Ra}\,T\sin(\omega t)\mathbf{k} - R\,\mathrm{Ra}\,\theta'\sin(\omega t)\mathbf{k} + R\,\mathrm{Ra}\,\overline{\theta'\sin(\omega t)}\mathbf{k}. \tag{17}$$

If we consider $T \gg \theta'$ and $T \approx 1$, under high frequency one can write: unsteady term ≈buoyancy term (containing T), which in nondimensional form implies that

$$O(u') \approx \frac{R\,\mathrm{Ra}}{A\omega}. \tag{18}$$

Moreover, the order of unsteady term is bigger than the order of viscous term, which results in

$$O(A\omega) \gg 1. \tag{19}$$

The oscillating energy equation gives the oscillating temperature scale:

$$\frac{\partial \theta'}{\partial t} + \mathbf{U}\cdot\nabla\theta' + \mathbf{u}'\cdot\nabla\theta' + \mathbf{u}'\cdot\nabla T - \overline{\mathbf{u}'\cdot\nabla\theta'} = \nabla^2\theta'. \tag{20}$$

Unsteady term and convective term (containing T) are of the same order:

$$O(\theta') \approx \frac{R\,\mathrm{Ra}}{A\omega^2}. \tag{21}$$

In addition, the order of unsteady term is bigger than the order of diffusion term:

$$O(\omega) \gg 1. \tag{22}$$

Reconsidering the first assumption $T \gg \theta'$ and replacing the corresponding scales, we may write

$$O(R) \ll O\left(\frac{A\omega^2}{\mathrm{Ra}}\right) \ll 1. \tag{23}$$

It follows that there exists $P_{v_l} \approx O(A^3\omega^4/\mathrm{Ra}^2)$. The physical interpretation of P_{v_l} is related to the harmonic and subharmonic response of the system. For $P_v > P_{v_l}$, there is only subharmonic response.

Figure 3 represents the harmonic or subharmonic domain. Beyond the stars the time-averaged formulation is valid.

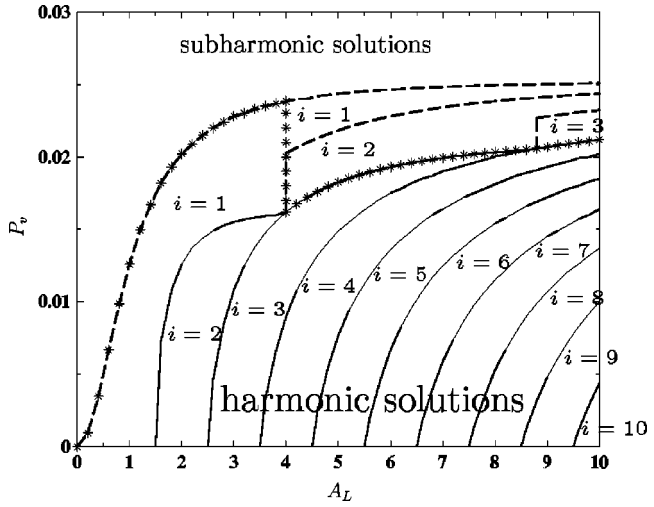


FIG. 3. For a chosen pair (A_L-P_v), the figure indicates the number i of cells on the emerging branch of the synchronous solution. For a fixed A_L value, the crossover values of P_v between the synchronous and subharmonic solutions are given by the stars. For all P_v under the stars, the first primary bifurcation gives rise to synchronous solutions.

V. NONLINEAR ANALYSIS

The amplitude equation (24) is established in the P_v formulation following the same method as in Bardan and Mojtabi,[10]

$$a\frac{\partial K}{\partial t^{(2)}} = bK(\mu + cK^2) \tag{24}$$

with $\mu = (\mathrm{Ra} - \mathrm{Ra}_c)/\varepsilon^2$ the bifurcation parameter,

$$a = \frac{\pi^2(i^2 + j^2A_L^2)^2[A_L^2 + \pi A(i^2 + j^2A_L^2)]}{4i^2A_L^3}, \tag{25}$$

$$b = -\frac{\pi^2(i^2 + j^2A_L^2)[2\pi^2(i^2 + j^2A_L^2)^2 - i^2A_L^2\mathrm{Ra}_c]}{4i^2A_L^3\mathrm{Ra}_c}, \tag{26}$$

$$c = \frac{\pi^6(i^2 + j^2A_L^2)^4[2\pi^2(i^2 + j^2A_L^2)^2 - i^2A_L^2\mathrm{Ra}_c]}{32i^4A_L^7}. \tag{27}$$

When $c > 0$ (respectively, $c < 0$), the bifurcation is supercritical and stable (respectively, subcritical and unstable) as $-b\mu/a < 0$ (respectively, $-b\mu/a > 0$). Whatever the mode considered and whatever the aspect ratio, c, which has a positive value for $P_v = 0$, remains positive as $P_v < P_{v_l}$. In the P_v formulation, the existence of P_{v_l} does not allow the bifurcation to become subcritical and unstable.

VI. CONCLUSION

The most important findings under the limiting case of high-frequency and small amplitude of vibration are summarized as follows.

(1) In order to interpret the results of the onset of convection correctly, we must separate the vibrational parameter from the thermal parameter involving ΔT.

(2) The stability diagram ($\mathrm{Ra}_c - P_v$) reveals that for each mode, there exists a limiting value of P_v which provides the maximum limit for achieving the stabilizing effect in the harmonic mode.

(3) The weakly nonlinear stability analysis shows that bifurcations remain supercritical in the domain of validity of the mean flow formulation.

We can say that, under gravity conditions, some published results in the Ra_v formulation could lead to erroneous conclusions as the results deal with harmonic solutions which cease to exist. By using the P_v formulation, the existence of a harmonic solution is guaranteed.

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