Simulation of non linear transient elastography: finite element model for the propagation of shear waves in soft tissues

W Ye, A. Bel-Brunon, S Catheline, A. Combescure, M Rochette

5th International Conference on Computational and Mathematical Biomedical Engineering - CMBE2017
10–12 April 2017, United States
P. Nithiarasu, A.M. Robertson (Eds.)

SIMULATION OF NON LINEAR TRANSIENT ELASTOGRAPHY: FINITE ELEMENT MODEL FOR THE PROPAGATION OF SHEAR WAVES IN SOFT TISSUES

W. Ye [1,3], A. Bel-Brunon[1], S. Catheline[2], A. Combescure[1], and M. Rochette[3]
[1]LaMCoS, INSA-Lyon, Universit de Lyon, CNRS UMR 5259, France,
wenfeng.ye,aline.bel-brunon,alain.combescure@insa-lyon.fr
[2]INSERM LabTAU Unit 1032, Universit de Lyon, France, stefan.catheline@inserm.fr
[3]ANSYS France, Montigny-le-Bretonneux, France, michel.rochette@ansys.com

SUMMARY

Weakly nonlinear viscoelastic Landau's theory, widely used in acoustical physic, is introduced into a Finite Element formulation to model the nonlinear behaviour of finite amplitude shear waves in soft solids, typically, in biological tissues. Numerical models for plane waves are developed and compared to transient elastography experiments. A good agreement is achieved as we observe the generation of odd harmonics. Simulation results are confronted to an existing analytical model; we show that the numerical model is an extension of the analytical formulation and helps identifying non-linear parameters in a wider range of experimental conditions.

Key words: *nonlinear shear wave; elastography; hyperelasticity; finite element*

1 INTRODUCTION

Transient elastography is a medical imaging technique which characterizes the elastic properties of biological tissues *in vivo* by observing shear wave propagation. These mechanical properties provide useful clinical information for diagnostic process. Assuming that the soft tissue is homogeneous, isotropic and linear elastic, the shear modulus μ can be estimated by $\mu = \rho c_s^2$ where ρ is the mass density and c_s is the shear wave speed. Although the measurement could be affected by multiple factors, such as boundary conditions, frequencies or geometries, this technique has been clinically proven to be able to evaluate many diseases, such as cancers and liver fibrosis. Lately, experimental efforts have been made to determine the nonlinear properties of biological soft tissues [1, 2, 3]. At the same time, theoretical work is necessary to support experimental results. In the work of [4, 5, 6], analytical models are established but the solutions can be achieved only under simple cases. Thanks to its possibility to represent real geometries and its flexibility, Finite Element Method (FEM) could be a performing tool for analyzing shear wave propagation in elastography.

In this work, the weakly nonlinear elastic theory of the third- (forth-) order Landau's law is developed in the Finite Element formulation; viscosity is considered by Voigt model in finite strain domain. Secondly, the FE simulations are carried out to model the nonlinear dynamic response of nearly-incompressible soft tissues and comparison with experimental results [2] is performed.

2 MATERIAL AND METHODS

2.1 Material modeling

In seismology and physic acoustics, non-linear elasticity is usually modeled using Landau's weakly nonlinear theory [7]. Assuming that W can be approximated by a series expansion, at the third order and in the decoupled form, we have [8]:

Density	$\rho\ (kg/m^3)$	1000
Shear modulus	$\mu\ (kPa)$	4.4
Third-order elastic param.	$A\ (kPa)$	41.1
Fourth-order elastic param.	$D\ (kPa)$	0
Bulk modulus	$K\ (kPa)$	10^4
Shear viscosity	$\eta\ (kPa\ s)$	0.6×10^{-3}
Bulk viscosity	$\xi\ (kPa\ s)$	0.5×10^{-5}

Table 1: Material parameters

$$W = W^{dev} + W^{vol} \text{ with } W^{dev} = \mu I_2 + \frac{A}{3} I_3 + D I_2^2 \tag{1}$$

with $I_k = tr(\mathbf{E}^k)$ for $k = 1, 2, 3$, μ, A and D represent the shear moduli at the second, third and fourth order. They all have the same order of magnitude (kPa) in soft solids, and they play a key role in nonlinear shear wave propagation. In the following, we will consider the nonlinear parameter $\gamma = \mu + \frac{A}{3} + D$.

Viscosity also needs to be taken into account. Indeed in finite amplitude wave propagation, nonlinear elasticity generates higher harmonics of the fundamental frequency. On the other hand, the absorption of the medium decreases the amplitude of deformation which limits the generation of these harmonics. As a result, nonlinear wave propagation cannot be modeled without considering viscosity. In nonlinear acoustic, the viscoelasticity of soft solids is often described by the Voigt model [9]. In finite strain the formulation becomes [4]:

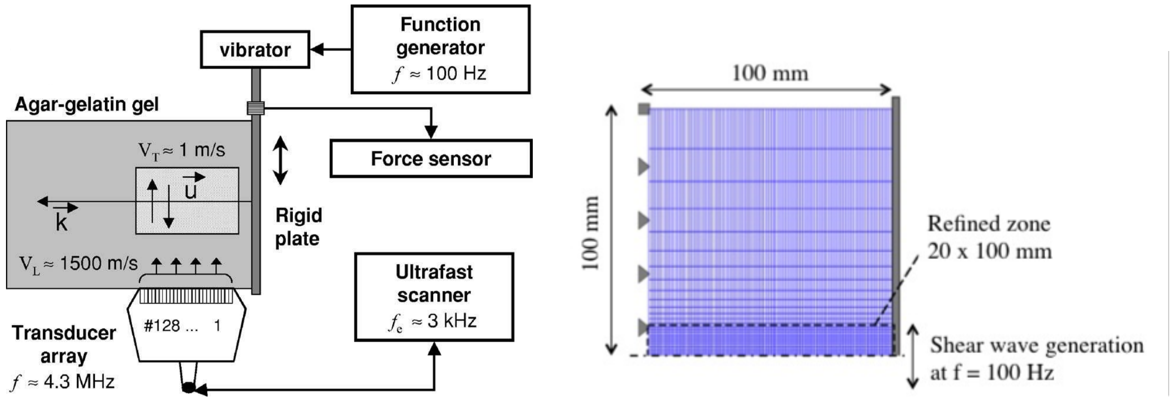
$$\mathbf{S}^{visco} = 2\eta \dot{\mathbf{E}}^{dev} + \xi \dot{\mathbf{E}}^{vol} \tag{2}$$

where η and ξ are the shear and bulk viscosity coefficients, respectively, $\mathbf{S}^{visco}$ the PK2 stress tensor and $\dot{\mathbf{E}}^{dev}$ is given by [10]:

$$\dot{\mathbf{E}}^{vol} = \frac{1}{3}(\dot{\mathbf{E}} : \mathbf{C}^{-1})\mathbf{C}, \;\; \dot{\mathbf{E}}^{dev} = \dot{\mathbf{E}} - \dot{\mathbf{E}}^{vol} \tag{3}$$

2.2 Numerical simulation of plane waves

In the following, all simulations are carried out by our in-house Finite Element codes implemented in *Fortran*. Landau's hyperelastic model and Voigt viscous model are implemented in the code. Bi-linear quadrangular elements combined with selective integration strategy are used to handle the volumetric locking and the hourglass effect at the same time. For these fast dynamic (wave propagation) simulations, a classical explicit time integration scheme is used. However, the time step will be largely limited by the big value of the bulk modulus K. We consider the propagation of plane shear wave into a soft solid, the experiment is described in [2]. The model is formulated in plane strain, it contains 4949 nodes and 4800 elements. To generate plane waves, a vertical displacement is prescribed (smoothed harmonic excitation, frequency f) on the rigid plate at the right side of the phantom. The simulation time is chosen to establish nonlinear effect and avoid reflections. The time step is $\Delta t = 0.5 \times 10^{-6}\ s$ to keep the stability of the explicit integration scheme. The material parameters are chosen to be close to the values reported in the experiments, see Tab.1. Note that these parameters lead to a volumetric wave speed $c_l = 100\ m/s$, and the shear wave speed $c_s = 2.1\ m/s$ in infinitesimal deformation. The realistic bulk modulus K (order of GPa) or realistic volumetric wave speed ($c_l \simeq 1500\ m/s$) is not ensured. In this paper, we use the assumption that in a homogeneous medium the volumetric wave does not play an important role to keep the time step reasonable. Besides, it should be noted that only the nonlinear coefficient γ is given in [2], there is so far no effective way to discriminate the third and fourth order shear moduli A and D separately. Herein, we set $D = 0$ to determine the value of A.



(a) Experimental set-up (b) Numerical model (half)

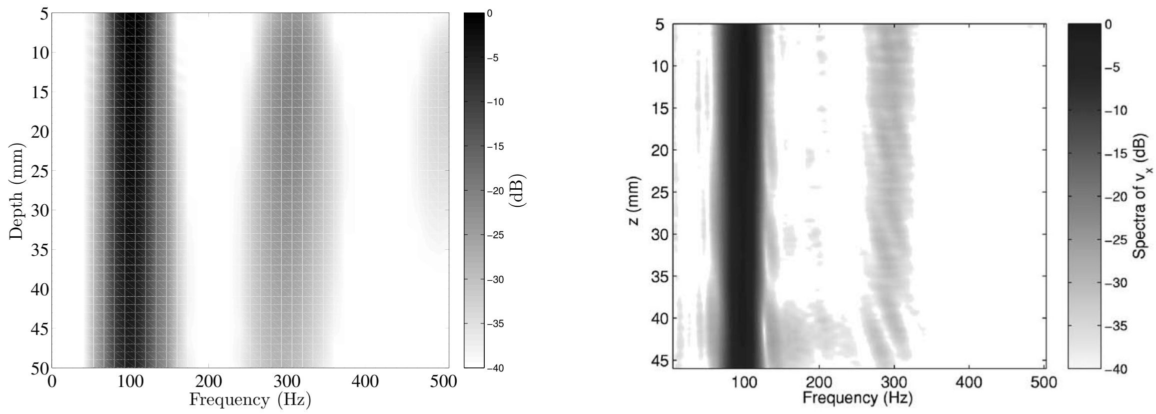
Figure 1: Nonlinear plane shear wave experiment [2] and the corresponding model.

2.3 Analytical model for harmonics amplitude

Zabolotskaya *et al.* [4] have proposed an analytical formulation for fundamental and first odd harmonic amplitude as a function of the propagation distance. This expression is valid as long as Goldberg number Γ, a dimensionless coefficient function of γ characterizing the relative importance of nonlinearity and viscosity, remains small. Besides this model does not give the expression of the other odd harmonic components. We therefore compare the harmonic amplitudes obtained by the plane wave numerical model with the ones determined by the analytical model, for different excitation amplitudes, excitation frequencies, material non-linear elastic parameters and viscosity parameters.

3 RESULTS AND CONCLUSIONS

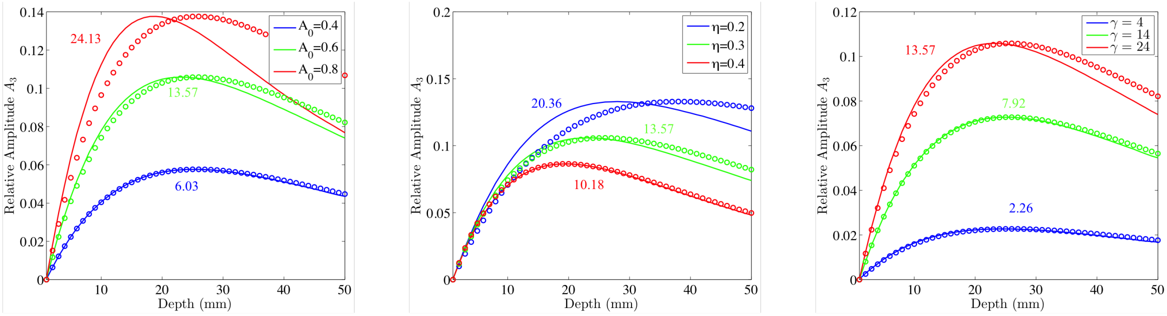
The results are displayed in terms of shear wave speed spectrum at the distance of propagation from 5 to 50 mm (Fig.2) and compared to experimental results reported in [2]. It can be seen that the two results are very similar, they both exhibit the odd harmonic at $3f$ which penetrates the whole measured zone. The experimental results looks more precise in spectrum, this is because that the experiment has a longer duration which leads to a better Fourier transform.



(a) Simulation results (b) Experimental results

Figure 2: Spectra of the vertical velocity *v.s.* propagation depth z

The comparison with the analytical model (Fig.3) shows that the numerical model extends the harmonics amplitude modeling to larger Goldberg numbers Γ than the analytical form, that is to say to larger application range. The comparison indicates that the analytical solution is accurate when Γ is smaller than about 13. By measuring the harmonics amplitude in a plane wave experiment, one can evaluate Γ using the analytical or numerical relationship between Γ and the harmonics amplitude, then determine γ, which gives a first equation linking A and D. However a second type of test is necessary to discriminate A and D.



(a) Varying amplitude

(b) Varying viscosity

(c) Varying elasticity ($\gamma = \mu + \frac{A}{3} + D$)

Figure 3: Evolution of the third harmonic component along the propagation distance for varying parameters. Lines: FEM simulations; circles: analytical model; number: Goldberg number.

FEM simulations of the nonlinear shear wave propagation by using Landau's viscoelastic law are presented. In plane wave, the numerical results show a good qualitative agreement with the experimental work [2]. It is also consistent with the existing analytical model and extends its application. Consequently, Landau's hyperelastic model combined with Voigt model can be further used in the numerical study of the nonlinear shear wave propagation in soft solids. It has already been applied to non-plane shear waves. This FEM model has a broad perspective in numerical simulations of complex nonlinear wave phenomena, such as real geometry, heterogeneity, diffraction and focalisation effect.

REFERENCES

[1] S. Catheline, J.L. Gennisson, M. Tanter, M. Fink. Observation of shock transverse waves in elastic media. *Physical review letters*, 91(16):164301, 2003.

[2] X. Jacob, S. Catheline, J.L.. Gennisson, C. Barrière, D. Royer, M. Fink. Nonlinear shear wave interaction in soft solids. *The Journal of the Acoustical Society of America*, 122(4):1917–1926, 2007.

[3] Y. Jiang, G.Y. Li, L.X. Qian, X.D. Hu, D. Liu, S. Liang, Y. Cao. Characterization of the nonlinear elastic properties of soft tissues using the supersonic shear imaging (ssi) technique: inverse method, ex vivo and in vivo experiments. *Medical image analysis*, 20(1):97–111, 2015.

[4] E.A. Zabolotskaya, M.F. Hamilton, Y.A. Ilinskii, G.D. Meegan. Modeling of nonlinear shear waves in soft solids. *The Journal of the Acoustical Society of America*, 116(5):2807–2813, 2004.

[5] M.S. Wochner, M.F. Hamilton, Y.A. Ilinskii, E.A. Zabolotskaya. Cubic nonlinearity in shear wave beams with different polarizations. *The Journal of the Acoustical Society of America*, 123(5):2488–2495, 2008.

[6] M. Destrade, M.D. Gilchrist, G. Saccomandi. Third-and fourth-order constants of incompressible soft solids and the acousto-elastic effect. *The Journal of the Acoustical Society of America*, 127(5):2759–2763, 2010.

[7] L.D. Landau, E.M. Lifshitz. *Theory of Elasticity*. Pergamon, New York, 1986.

[8] M.F. Hamilton, Y.A. Ilinskii, E.A. Zabolotskaya. Separation of compressibility and shear deformation in the elastic energy density (l). *Acoustical Society of America Journal*, 116:41–44, 2004.

[9] S. Catheline, J.L. Gennisson, G. Delon, M. Fink, R. Sinkus, S. Abouelkaram, J. Culioli. Measurement of viscoelastic properties of homogeneous soft solid using transient elastography: an inverse problem approach. *The Journal of the Acoustical Society of America*, 116(6):3734–3741, 2004.

[10] T. Belytschko, W.K. Liu, B. Moran, K. Elkhodary. *Nonlinear finite elements for continua and structures*. John Wiley & Sons, 2013.