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► To cite this version:

Julien Waeytens, Sara Sadr. Computer-aided placement of air quality sensors using adjoint framework and sensor features to localize indoor source emission. *Building and Environment*, 2018, 144, pp.184-193. 10.1016/j.buildenv.2018.08.012 . hal-01862664

HAL Id: hal-01862664

<https://hal.science/hal-01862664>

Submitted on 27 Aug 2018

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1 Computer-aided placement of air quality sensors using
2 adjoint framework and sensor features to localize indoor
3 source emission

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7 **Abstract**

8 With the improvement in sensor technologies, air quality is increasingly be-
9 ing monitored. Two major factors in obtaining relevant information are
10 the optimal placement and the number of air quality sensors. Moreover,
11 in cases of poor air quality, the information of the pollution level given by
12 the deployed sensors is not sufficient. An advanced understanding of the
13 data is required to precisely identify the source pollution and thus propose
14 effective solutions. In this article, a virtual testing strategy based on com-
15 putational fluid dynamics (CFD) is presented for the optimal placement of
16 indoor air quality sensors. We determine the placement of sensors in view
17 of localizing the maximum of sources emitting on the indoor environment
18 surfaces. Therefore, an adjoint framework is used to obtain the observable
19 region associated with a given sensor position. The proposed method takes
20 into account technical sensor features, such as the limit of detection (LOD).
21 Two applications are studied: a simple 2D case and a real 3D room. In
22 these examples, we first show that reducing the LOD of the sensors by one
23 order of magnitude can increase the observable area by more than 50%.

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Then, we note that one-fourth of the potential sensor placements observe almost nothing and that 80% of the potential sensor placements have an observable area two times smaller than the optimal sensor position determined by the proposed CFD-based strategy.

Keywords: sensor placement, computational fluid dynamics, adjoint problem, source emission, sensor detection limit, indoor air quality

1. Introduction

According to a survey conducted in 2015 by the French Ministry of Ecological Transition, air pollution is the second environmental concern of French people, just after climate change. As people spend approximately 80% of their time in indoor environments, increasing attention has been focused on indoor air quality (IAQ). Volatile organic compounds (VOCs) are characteristic chemical species present in indoor environments. Several studies have shown that the concentration of VOCs can be higher in indoor locations, such as early childhood education facilities [1], schools [2], universities [3], office buildings [4] and homes [5], compared to the concentrations outside. As reported in [6], VOCs in indoor environments can come from the outdoor air via ventilation and from indoor sources. There are a wide range of indoor sources, *e.g.* combustion, smoking, building materials, office machines, furnishings, paints, termiticides and cleaning products. As permanent and occasional exposure, even at low VOC levels, has an impact on human health [7], it is important to monitor indoor air quality and to precisely localize sources to propose an appropriate action plan to improve air quality. The monitoring of air quality is facilitated by the improvement in sensor technologies, notably nanotechnologies. Hence, the gas sensors become cheaper, smaller, more sensitive, less energy-consuming, etc... To

46 get more details on low-cost sensors for air quality purposes, the reader can
47 refer to the review article [8]. The localization of VOC sources can also
48 be useful for the preservation of cultural heritage, notably artwork, and for
49 structural health monitoring purposes. In most regions of France, the pres-
50 ence of woodborers, such as termites, has harmful effects on the safety of
51 structures. The VOC chemical signature of termites can be used for their
52 early detection and localization, which will provide the ability to limit the
53 use of termiticides and to preserve the structure.

54

55 To efficiently monitor air quality, the number of sensors and their po-
56 sitioning are crucial. In most measurement campaigns, the gas sensors are
57 placed in an empirical way. For example, in a room, an air quality sensor is
58 usually positioned at the breathing zone height or approximately $0.5m$ from
59 the ceiling in the middle of the room. Unfortunately, this placement does
60 not take into account the characteristics of the room, *i.e.* the geometry and
61 the ventilation. As a consequence, bad sensor placement may lead to the
62 nondetection of some sources. To well-position gas sensors, we can take ad-
63 vantage of numerical simulations derived from physical models. In indoor air
64 quality applications, the gas concentration can be predicted using multizone
65 [9; 10; 11; 12] and CFD [9; 13; 14] models. Multizone techniques, which pro-
66 vide the time evolution of the averaged concentration in each zone as output,
67 are easy to use and run on a standard laptop. Nevertheless, they consider
68 strong hypotheses, such as a well-mixed concentration. With the ongoing
69 improvement of computers and numerical methods, CFD approaches ap-
70 pear to be promising for the prediction of indoor air quality and for optimal
71 sensor placement. In fact, CFD provides a fine description of the spatial
72 concentration in the indoor environment, but the computations are time

73 consuming. A good compromise to study the indoor air quality of an entire
 74 building would be to couple multizone and CFD models, as proposed in [15].
 75 To the best of the authors' knowledge, few publications have addressed the
 76 optimal placement of gas sensors for IAQ applications. The design of an op-
 77 timal sensor network, *i.e.* the number and positioning of sensors, has been
 78 studied in greater depth in terms of chemical and biological warfare (CBW)
 79 and transmission of infectious diseases (TID). The sensor positions are cho-
 80 sen to early detect and localize indoor contamination. Different methods
 81 aim to maximize the coverage area of sensors and to minimize the response
 82 time for various sets of release scenarios. In [16], the sensor coverage area is
 83 evaluated using CFD and an adjoint advection-diffusion equation, whereas
 84 physical model-free approaches based on a dynamical systems approach are
 85 preferred in [17]. Note that the adjoint framework is a useful numerical tool
 86 for various applications. First, it provides, at a low computational cost,
 87 the functional gradient and the Hessian matrix involved in inverse calcula-
 88 tions to update the parameters of fluid mechanics models [18; 19] and to
 89 reconstruct the concentration fields [20; 21; 22]. Additionally, it is used in
 90 sensitivity analyses to study the influence of physical model parameters on
 91 a quantity of interest [23; 24]. The adjoint framework is also considered
 92 for estimating the modeling or discretization error on a quantity of interest
 93 [25; 26; 27].

94

95 Once the positions of the sensors are fixed, knowledge of the concentra-
 96 tion given by the deployed sensors is not sufficient for proposing efficient
 97 solutions for indoor air quality improvement or for localizing woodborers.
 98 One needs to localize and to quantify the source emissions. To achieve this
 99 purpose, two families of methods can be found in the literature, *i.e.* data-

100 driven methods and physical model-based methods. Direct measurements
 101 of the source emissions on different surfaces of the environment (furniture,
 102 wall, floor, door, etc.) can be planned using innovative sensors, such as fibers
 103 placed in a specific device for on-site emission control [28; 9]. This method
 104 enables accurate in situ quantification of the source emissions for building
 105 materials and furniture, but it requires a large number of sensor devices.
 106 Another data-driven method to evaluate source emissions is indirect mea-
 107 surements. In contrast to the previous methods, the air quality sensors are
 108 placed in the room volume and not directly on a surface. Databases of the
 109 chemical signatures of sources and *a priori* information of the studied envi-
 110 ronment collected via questionnaire, including the type and the age of the
 111 building materials, renovations, cleaning products and ventilation, are com-
 112 monly considered in these methods. Finally, the sensor outputs associated
 113 with various chemical compounds are analyzed via statistical tools, such
 114 as proper component analysis and linear regression, to identify the source
 115 emissions [4; 5; 29; 30]. In practice, the chemical compounds emitted by
 116 some items in the studied environment may not be referenced in a database.
 117 Consequently, these methods may only approximately identify the sources.
 118 Physical model-based approaches via inverse modeling techniques can also
 119 be valuable for the localization and the quantification of source emissions.
 120 In general, inverse problems that couple model and sensor outputs are not
 121 well-posed in the sense of Hadamard, *i.e* the existence, uniqueness and non-
 122 high sensitivity of the solution to the sensor outputs. To address this issue,
 123 a sufficient number of well-positioned sensors is required, and regularization
 124 must be considered in the mathematical formulation of the inverse problem.
 125 In deterministic settings, Tikhonov regularization is commonly considered
 126 and consists of adding penalization terms to the data misfit functional, as

discussed in [15; 31] for convective-diffusive transport source inversion. In probabilistic inversion formalism, notably Bayesian model updating, which was applied in [32] for CO₂ regional source estimations, the model parameter probability distributions are interesting on two counts. They ensure the problem regularization and provide a confidence interval on the identified source emissions. Nevertheless, probabilistic inversions can be much more time consuming than deterministic ones. Finally, the adjoint framework, previously mentioned for the optimal placement of sensors, can also be used for source localization, as shown in [33; 15].

In the present article, we propose a virtual testing strategy, taking into account the specificities of the indoor environment (geometry and ventilation) via CFD and gas sensor features (limit of detection), to efficiently select the number and positions of sensors to localize indoor sources. We define the “optimal sensor placement” as the combination of gas sensors that maximizes the coverage area. The authors showed in previous works [21] that the sensor observable area can be computed at a reasonable cost using the adjoint framework. Herein, we emphasize that the coverage area can be increased not only by adding sensors but also by using sensors with a lower limit of detection. The rest of this article is organized as follows. In Section 2.1, a physical direct model to predict the gas dispersion is presented. Then, we define the adjoint equations in Section 2.2 and introduce a new adjoint-based criterion integrating sensor features to evaluate the observable area of potential sensor positions in Section 2.3. An overview of the optimal sensor placement strategy is given in Section 2.4, and it is applied to a 2D case and a real 3D room in the last section.

153 2. Materials & Methods

154 2.1. Simulation of pollutant propagation - Direct problem

155 To predict the dispersion of gas, advection-diffusion-reaction models are
 156 commonly used [9; 13; 14]. As a first step, we consider non-reactive gases,
 157 *i.e.* reaction phenomena are not modeled. Hence, the cartography of the
 158 gas concentration in a two- or three-dimensional space domain Ω is obtained
 159 from the advection-diffusion model. Four types of boundaries can be dis-
 160 tinguished. A boundary presenting a known prescribed concentration C_p
 161 is denoted $\partial_p\Omega$. Potential pollution emissions, to be precisely located by
 162 the optimal placement of gas sensors, are on the boundary $\partial_u\Omega$, whereas a
 163 boundary that does not present source emission is $\partial_n\Omega$. Lastly, $\partial_o\Omega$ denotes
 164 the outgoing flow boundary.

165
 166 The pollutant concentration $C(\mathbf{x}, t)$ in the domain $\Omega \subset \mathbb{R}^n$, $n \in \{2, 3\}$
 167 can be obtained by solving the unsteady advection-diffusion model, which
 168 is also called the “direct problem”,

$$\left\{ \begin{array}{l} \frac{\partial C}{\partial t}(\mathbf{x}, t) + \mathbf{v}(\mathbf{x}, t) \cdot \nabla C(\mathbf{x}, t) - \nu(\mathbf{x}, t) \Delta C(\mathbf{x}, t) = 0 \quad \text{in } \Omega \times [0, T] \\ C(\mathbf{x}, t) = C_p(\mathbf{x}, t) \quad \text{on } \partial_p\Omega \times [0, T] \\ C(\mathbf{x}, t) = C_u(\mathbf{x}, t) \quad \text{on } \partial_u\Omega \times [0, T] \\ \nabla C(\mathbf{x}, t) \cdot \mathbf{n} = 0 \quad \text{on } \partial_n\Omega \times [0, T] \\ \nabla C(\mathbf{x}, t) \cdot \mathbf{n} = 0 \quad \text{on } \partial_o\Omega \times [0, T] \\ C(\mathbf{x}, t = 0) = C_0(\mathbf{x}) \quad \text{in } \Omega \end{array} \right. \quad (1)$$

169 In Eq. (1), $\mathbf{v}$ is the flow velocity, ν denotes the diffusion parameter, which is
 170 the sum of the molecular and turbulent diffusion, and $\mathbf{n}$ denotes the outside

171 normal vector to the surface.

172

173 When the flow and the source emission can be considered stationary with
 174 respect to the monitoring time, the concentration field $C(\mathbf{x})$ can be obtained
 175 at a lower computation cost using a steady advection-diffusion model

$$\begin{cases} \mathbf{v}(\mathbf{x}) \cdot \nabla C(\mathbf{x}) - \nu(\mathbf{x}) \Delta C(\mathbf{x}) = 0 & \text{in } \Omega \\ C(\mathbf{x}) = C_p(\mathbf{x}) & \text{on } \partial_p \Omega \\ C(\mathbf{x}) = C_u(\mathbf{x}) & \text{on } \partial_u \Omega \\ \nabla C(\mathbf{x}) \cdot \mathbf{n} = 0 & \text{on } \partial_n \Omega \\ \nabla C(\mathbf{x}) \cdot \mathbf{n} = 0 & \text{on } \partial_o \Omega. \end{cases} \quad (2)$$

176 For example, Eq. (2) can be used to model the dispersion of moisture or
 177 woodborers emissions during a measurement campaign under mastered air
 178 flow conditions, *e.g.* when the indoor occupants have left. In the following,
 179 we limit our study to stationary cases.

180 2.2. Sensitivity area of a gas sensor - Adjoint problem

181 Physically, the solution of the adjoint problem corresponds to a sensitiv-
 182 ity function in terms of a quantity of interest. Hence, to obtain the sensor
 183 observable area, we choose the gas concentration at the sensor location $\mathbf{x}_s$
 184 as the quantity of interest. It is given by

$$J = \int_{\Omega} f_s(\mathbf{x} - \mathbf{x}_s) C(\mathbf{x}) d\Omega \quad (3)$$

185 where f_s is a space function to extract the gas concentration at the sensor
 186 location $\mathbf{x}_s$. In practice, we can take:

$$f_s(\mathbf{x} - \mathbf{x}_s) = \begin{cases} 1/|\Omega_s| & \text{for } \mathbf{x} \in \Omega_s \\ 0 & \text{elsewhere.} \end{cases} \quad (4)$$

187 The domain Ω_s is a sphere of radius R_s centered at the sensor location $\mathbf{x}_s$.

188

189 From the quantity of interest, we introduce the adjoint problem (5) and
190 compute its numerical solution $\tilde{C}$.

$$\left\{ \begin{array}{l} -\mathbf{v}(\mathbf{x}) \cdot \nabla \tilde{C}(\mathbf{x}) - \nu(\mathbf{x}) \Delta \tilde{C}(\mathbf{x}) = f_s(\mathbf{x} - \mathbf{x}_s) \quad \text{in } \Omega \\ \tilde{C}(\mathbf{x}) = 0 \quad \text{on } \partial_p \Omega \\ \tilde{C}(\mathbf{x}) = 0 \quad \text{on } \partial_u \Omega \\ \nabla \tilde{C}(\mathbf{x}) \cdot \mathbf{n} = 0 \quad \text{on } \partial_n \Omega \\ \nu \nabla \tilde{C}(\mathbf{x}) \cdot \mathbf{n} + \mathbf{v}(\mathbf{x}) \cdot \mathbf{n} \tilde{C}(\mathbf{x}) = 0 \quad \text{on } \partial_o \Omega \end{array} \right. \quad (5)$$

191 Note that the adjoint problem (5) is a backward-advection-diffusion problem
192 with a source emission located at the sensor position. This adjoint problem
193 can be solved with the same CFD software as that used for the direct prob-
194 lem. For greater detail on the derivation of the adjoint problem, the reader
195 can refer to [21].

196 2.3. Computation of sensor observable area - A new adjoint-based criterion

197 After defining the adjoint problem, we propose an adjoint-based criterion
198 (6) that takes into account the sensor features, *i.e.*, the LOD of the gas
199 sensor, in view of obtaining the sensor observable area.

$$|\nabla J| \frac{A_s S}{dI_m} > 1 \quad (6)$$

200 where:

- 201 • J (resp., ∇J) is the functional (resp., functional gradient) associated
202 with the gas concentration at the sensor location $\mathbf{x}_s$ defined in Eq. (3)
- 203 • A_s is the minimum source area expected to be localized

204 • S is the order of magnitude of the source emission

205 • dI_m is the limit of detection of the gas sensor

206 The sensitivity of the gas concentration at the sensor location $\mathbf{x}_s$ to the
 207 surface source emissions, which corresponds to the functional gradient ∇J ,
 208 can be evaluated using the adjoint framework. Following the method in [21],
 209 we can show that:

$$\nabla J(\mathbf{x}) = \nu(\mathbf{x}) \nabla \tilde{C}(\mathbf{x}) \cdot \mathbf{n} \quad (7)$$

210 where $\mathbf{n}$ denotes the unit outer normal vector along the surface.

211
 212 In summary, the observable area of a gas sensor located at a given posi-
 213 tion $\mathbf{x}_s$ can be numerically predicted by

$$\boxed{\mathbf{x} \in \partial_u \Omega \text{ such that } |\nu(\mathbf{x}) \nabla \tilde{C}(\mathbf{x}) \cdot \mathbf{n}| \frac{A_s S}{dI_m} > 1.} \quad (8)$$

214 Let us physically interpret the different terms in the proposed criterion
 215 (8). The first part $|\nabla J(\mathbf{x})|$ takes into account the sensor position $\mathbf{x}_s$ and
 216 gives the sensitivity map of the gas sensor output to the surface source emis-
 217 sion. It is numerically obtained from the solution $\tilde{C}$ of the adjoint problem
 218 defined in Eqs. (5). In [21], we proved that a null value of $|\nu(\mathbf{x}) \nabla \tilde{C}(\mathbf{x}) \cdot \mathbf{n}|$
 219 on a boundary $\partial_b \Omega \subset \partial_u \Omega$ implies that potential source emissions on $\partial_b \Omega$
 220 cannot be detected by a sensor at the position $\mathbf{x}_s$.

221 The new contribution in this article concerns the next two terms. The sec-
 222 ond term $A_s \times S$ relies on *a priori* information of the source emissions that
 223 are expected to be detected. If we are interested in emissions on large sur-
 224 faces, such as painted walls, A_s should be approximately a few tens of square
 225 meters. By contrast, if we are interested in emissions on small surfaces, such
 226 as furniture, A_s should be approximately one square meter. A small value

227 of A_s , *i.e.* less than one square meter, can also be useful for the early de-
 228 tection of termites. The order of magnitude of potential emissions is taken
 229 into account with the parameter S . For formaldehyde furniture emission it
 230 can be higher than $1ppm$ [34] whereas it is a hundred times lower for VOCs
 231 emitted by molds [35]. In the proposed criterion (8), the observable area for
 232 a given positioned sensor depends on the product $A_s \times S$. Hence, the higher
 233 this product is, the larger the observable area.
 234 Lastly, the sensor detection limit, depending on technology features, cor-
 235 responds to the third term in the proposed criterion (8). In the Results
 236 Section, we show how the observable area increases as the limit of detection
 237 of the sensors decreases.

238 2.4. Outline of the virtual testing strategy for the optimal placement of air 239 quality sensors

240 This section aims to present the steps in computer-aided sensor place-
 241 ment. The process is summarized in Figure 1.

Figure 1: Architecture of the computer-aided method for the optimal placement of air quality sensors

242 The proposed strategy necessitates a mock-up of the studied environment
 243 and a fine description of the air flow. The air flow map can be obtained from
 244 experiments, empirical models or computational fluid dynamics. Let us em-
 245 phasize that [22] previously pointed out that a rough approximation of the
 246 real flow can lead to a non-representative concentration simulation. Thus,
 247 special attention must be given to obtaining the air flow map; otherwise,
 248 the proposed placement of gas sensors can be incorrect. After defining a
 249 list of n_t potential sensor positions, we solve the adjoint problem (5) asso-
 250 ciated with each sensor position. All the n_t adjoint solutions are stored in
 251 a database. Note that this step is fully parallelizable and is performed only
 252 once in a off-line stage.

253 In the proposed virtual testing strategy, the observable area is computed for
 254 the n_t sensor positions. As shown in the previous section, the observable
 255 area is obtained from the adjoint-based observable criterion (8). In addition
 256 to the adjoint solution, *a priori* information of the sensor technology is also
 257 required, *i.e.* the limit of detection dI_m of the sensor and the source to be
 258 localized, *i.e.* the orders of magnitude of area A_s and level S of the source
 259 emissions. Lastly, the optimal placement corresponds to the one with the
 260 largest observable area. When the number n_m of desired sensors is strictly
 261 greater than one, the optimal placement is performed in a hierarchical man-
 262 ner. We start by optimally placing the first sensor and fix its position; then,
 263 we seek the optimal placement of the second sensor and fix its position,
 264 and so on. As practical outputs for the users, the computer-aided method
 265 provides, as a visualization on the numerical mock-up, the observable area
 266 of each selected sensor position and the coverage area in square meters for
 267 each sensor and for the combination of all n_m sensors.

268 3. Results

269 3.1. Application 1 - 2D simple problem

270 To gain a better understanding, let us first consider a 2D academic prob-
 271 lem (see Figure 2). The 2D domain Ω is a square with 10 m sides, and the
 272 flow $\mathbf{V}$ is uniform. The velocity amplitude (resp. the velocity orientation
 273 angle) is 1 m/s (resp., 27°), and the diffusion parameter ν is $2.2 \times 10^{-2}\text{ m}^2/\text{s}$
 274 **which corresponds to the order of magnitude of the turbulent diffusion**. As
 275 introduced in Section 2.1, $\partial_o\Omega$ denotes the outgoing flow boundary. In this
 276 example, we aim to optimally place gas sensors to localize and quantify
 277 sources coming from the boundary $\partial_u\Omega$. We focus on the detection of a
 278 source emitting on an area greater than 1 m on $\partial_u\Omega$ and whose order of
 279 magnitude of the amplitude is approximately 100 ppm . From this informa-
 280 tion, we take $A_s = 1\text{ m}$ and $S = 100\text{ ppm}$ in the observable criterion (8).
 281 Moreover, in the domain Ω , gas sensors can be placed at a limited number
 282 of positions. The sixteen potential sensor positions are shown in Figure 2.

Figure 2: Geometry of the 2D problem (left) and potential positions of the gas sensor (right)

283 In the followings, firstly the influence of the LOD of the gas sensor on

the observable area is studied for a unique given sensor. Then, the virtual testing strategy is illustrated to optimally place several sensors.

3.1.1. Influence of the LOD on the observable area for a given sensor position

In this section, we consider a given sensor position, that is, Sensor #3 (see Figure 2). The objective is to evaluate its observable area for different LODs of the gas sensor. We use the proposed adjoint-based criterion (8). First, one needs to solve the adjoint problem defined in Eq. (5), which corresponds to a backpropagation of a pollutant emitted at Sensor location #3 (see Figure 3). Then, from the adjoint solution $\tilde{C}$ and the LOD dI_m , we compute the criterion (8) and deduce the observable area associated with the considered sensor. In Figure 3, we present the adjoint field $\tilde{C}$ and the observable area of Sensor #3 for two different LODs: $10ppm$ and $0.1ppm$. For the considered flow, source emissions on the bottom edge cannot be detected by Sensor #3. The observable area is located around the middle of the left edge. Its precise position along the left edge is $4.9m \pm 1.1m$ for an LOD of $10ppm$ and $4.9m \pm 2.2m$ for an LOD of $0.1ppm$.

The evolution of the observable area for a wide range of LODs is presented in Figure 4. As expected, a reduction in the LOD leads to an increase in the observable area. The observable area for Sensor #3 is one and a half times larger (resp. two times larger) when using a gas sensor with a $1ppm$ LOD (resp., $0.1ppm$ LOD) than one with a $10ppm$ LOD. In summary, this study shows that the LOD of the gas sensor has a strong impact on the observable area for detecting source emissions. Consequently, the LOD must be considered in the optimal placement strategies of air quality sensors.

Figure 3: Adjoint problem solution $\tilde{C}$ associated with Sensor #3 and its observable area for an LOD of 10 ppm and 0.1 ppm - Definition of Source Areas A, B, C for numerical validation of the observable criterion

Figure 4: Observable Area of Sensor 3 as a function of the LOD - The observable area for an LOD of 0.1ppm is 100% larger than that with an LOD of 10ppm

310 On the basis of the adjoint-based criterion (8), we were able to eval-
 311 uate the observable area associated with a given sensor position. Let us
 312 numerically validate this observable criterion. Sensor position #3 is still
 313 considered, and three source locations are defined on the left edge of the 2D

domain (see Figure 3). The different sources are 1 meter in length, and their amplitude is $100ppm$. As predicted by the proposed virtual testing strategy, a source in Area A can be detected by the sensor with both LODs ($0.1ppm$ and $10ppm$), a source emitted in Area B can be detected only by the sensor with an LOD of $0.1ppm$, and neither the $0.1ppm$ LOD sensor nor the $10ppm$ LOD sensor can detect a source in Area C. For each source, we simulate the associated gas dispersion by solving the direct advection-diffusion equations (2) and obtain the gas concentration at Sensor position #3. From this concentration, we can verify whether the source is detected by the sensors with an LOD of $0.1ppm$ or $10ppm$. In Table 1, the results show that the sensor observable area is well predicted by the adjoint-based criterion (8). A $100ppm$ source emitted in Area B leads to a gas concentration of $3.27ppm$ at Sensor position #3. This concentration can be detected by the $0.1ppm$ LOD sensor but not by the $10ppm$ LOD sensor. This result is in agreement with the predicted observable area shown in Figure 3.

Source location	Concentration at sensor position #3	Source detected by the sensor ? (Y : Yes, N : No)	
		$0.1ppm$ LOD	$10ppm$ LOD
Area A ($4.9m \pm 0.5m$)	$56.06ppm$	Y	Y
Area B ($3.3m \pm 0.5m$)	$3.27ppm$	Y	N
Area C ($2.2m \pm 0.5m$)	$0.04ppm$	N	N

Table 1: Numerical validation of the adjoint-based observable criterion - Concentration at Sensor position #3 simulated for different source locations and verification of the source detection for a $0.1ppm$ LOD sensor and a $10ppm$ LOD sensor

329 *3.1.2. Optimal placement of gas sensors considering a fixed LOD*

330 The optimal placement of the gas sensors is achieved using the virtual
331 testing strategy presented in Section 2.4. First, we determine the optimal
332 placement of the first sensor. Hence, for each sensor position, the associated
333 adjoint problem is solved and saved in a database. The use of the adjoint
334 solutions and the LOD in criterion (8) enable the evaluation of the observable
335 area of each sensor. Herein, the LOD is fixed to $10ppm$. The sensor position
336 with the largest observable area is selected as the “optimal placement”.
337 The observable area associated with each sensor is summarized in Figure 5.
338 Note that the sum of the observable areas on the bottom and left edges
339 corresponds to the total observable area on the boundary $\partial_u\Omega$. We observe
340 that Sensors #1 to #11 can detect a source emission only on the left edge,
341 whereas Sensors #12 and #14 are able to detect a source emission located
342 in the lower-left corner of the domain involving the left and bottom edges.
343 Among the 16 potential positions, 13 positions have a total observable area
344 between 1 and $2.5m$, and only 3 positions give an observable area larger
345 than $3m$. Sensor #12, which has a total observable area slightly larger than
346 those of Sensor #15 and Sensor #16, is selected as the optimal placement.

347 As mentioned in Figure 1, the placement of several air quality sensors
348 is performed in a hierarchical manner. After finding the optimal placement
349 of the first sensor, we fix this sensor and determine the optimal placement
350 of the second sensor, and so on. Previously, Sensor #12 was determined
351 as the optimal placement for the first sensor. To determine the optimal
352 placement of several sensors, we evaluate the observable areas (see Figure
353 5) and select the sensor combination with the highest total observable area.
354 The combination of Sensors #12 and #16 gives the highest total observable

Figure 5: Optimal placement (red rectangles) in the 2D domain of several sensors of with an LOD of $10ppm$ - Observable Areas - Isovalues of the total observable area are represented as solid lines

area, that is, $6.5m$. We can see that adding Sensor #16 improves the observable area on only the bottom edge, increasing from $2m$ to $5.5m$. The observable area on the left edge is increased by the optimal placement of three sensors, *i.e.*, Sensors #12, #16 and #4. We note that the the total observable area is three times larger for the optimal placement of four sensors (#12, #16, #4, #2) than for the optimal placement of a single sensor #12.

3.1.3. Optimal placement of several sensors considering different LOD

In Section 3.1.1, we showed that the LOD has a significant influence on the observable area for detecting source emissions. As a consequence, two factors can be investigated to improve the observable area: the number of gas sensors and the LOD. In this section, we study the optimal placement

367 of several sensors in the 2D domain when considering sensors with an LOD
 368 of either $10ppm$ or $1ppm$. The results are summarized in Figure 6. We ob-
 369 serve that the optimal positions of gas sensors may differ according to the
 370 LOD. At a $10ppm$ LOD, Sensor #12 is selected as optimal, whereas Sensor
 371 #16 is optimal at a $1ppm$ LOD. Note that Sensor #16 with a $1ppm$ LOD
 372 has a total observable area that is approximately twice the size of that of
 373 Sensor #12 with a $10ppm$ LOD. To reach a total observable area of $9m$, we
 374 can either use two Sensors (#16, #14) with a $1ppm$ LOD or three Sensors
 375 (#12, #16, #4) with a $10ppm$ LOD.

376

Figure 6: Evolution of the observable area as a function of the number of sensors and the LOD ($10ppm$ and $1ppm$)

377 In Figure 7, we present the observable area associated with the optimal
 378 placement of four gas sensors with LODs of either $10ppm$ or $1ppm$. The cov-
 379 erage is disparate on $\partial_u \Omega$ for a $10ppm$ LOD while it is widespread for a $1ppm$

380 LOD. Nevertheless, for both cases, source emissions cannot be detected in
 381 the upper part of the left edge and the right part of the bottom edge due to
 382 the considered flow $\mathbf{V}$ and the distribution of the sixteen potential sensor
 383 positions. Lastly, for the optimal placement with 1ppm LOD gas sensors,
 384 the observable area of some sensors overlaps, notably Sensors #12 and #16.
 385 Thus, a source emitted in the overlapped region would be detected by both
 386 Sensors #12 and #16.

Figure 7: Map of the observable area for the optimal placement of 4 sensors with LODs
 of 10 ppm (left) and of 1 ppm (right)

387 3.2. Application 2 - 3D laboratory room

388 In this section, we illustrate the computer-aided method for the optimal
 389 placement of gas sensors in a real 3D laboratory room, including furniture
 390 and ventilation systems, located at the IFSTTAR research institute. The
 391 dimensions of the room are $5.9\text{m} \times 6.2\text{m} \times 4.2\text{m}$, which correspond to a
 392 volume of 150m^3 . As mentioned in Figure 1, we first need a numerical mock-
 393 up and indoor air flow map (see Figure 8). For that, the incoming flows from
 394 the heating duct, the two ventilation grids and the door were measured using

395 a 1D hot wire anemometer, and the values are reported in Figure 8. Note
 396 that the flow exits only from the extractor hood. From this information
 397 and the numerical mock-up, we simulate the stationary turbulent flow using
 398 the $k - \omega$ SST Reynolds Average Navier-Stokes (RANS) model in the CFD
 399 software “Code_Saturne” [36]. In Figure 8, we can see that the airflow
 400 entering from the contour of the door is highly turbulent in the vicinity
 401 of the door and that a portion of it goes straight in the direction of the
 402 extractor hood. In terms of the incoming flow from the second ventilation
 403 grid, the main portion circulates close to the ground between the wall and
 404 the furniture. Lastly, the velocity flow from the heating duct oriented in
 405 the z-direction impacts the top of the furniture immediately below, which
 406 generates flow recirculation. From the RANS turbulent flow simulation,
 407 let us note that we also evaluate the turbulent diffusion involved in the
 408 diffusion parameter ν . A practical rule of thumb consists to compute the
 409 turbulent diffusion ν_t from the turbulent viscosity μ_t and the density ρ by
 410 $\nu_t = (1/Sc_t)\mu_t/\rho$ where the turbulent Schmidt number Sc_t is taken to 0.7.

Figure 8: Numerical mock-up of the laboratory room and measured incoming flows (left)
 - Flow simulated by CFD software (right)

411 In the laboratory room, the potential sensor positions presented in Fig-

ure 10 are equally distributed every 50cm at three heights above the ground,
 namely, 0.5m , 1m and 1.5m . There are 121 sensor positions per height, for
 a total of 363 potential sensor positions. The potential sensor positions are
 shown in Figure 10. Herein, we aim to select the sensor positions that pro-
 duce the maximum observable on all the lateral surfaces (door face, furniture
 face, extractor hood face, back face). To evaluate the observable area, we
 compute the adjoint concentration $\tilde{C}$ associated with each sensor position
 and store the values in a database. In practice, the adjoint problems (5) are
 solved using the Streamline Upwind Petrov-Galerkin (SUPG) formulation
 [37] in the finite element code “FreeFem++” [38]. In Figure 9, we present
 the adjoint solution associated with Sensor position #86, which is located
 close to the furniture wall 0.5 m above the floor. The adjoint field $\tilde{C}$ being
 a sensitivity function of the concentration at the sensor position to source
 emissions, Figure 9 shows that a gas sensor at position #86 is sensitive to
 sources on a part of the furniture face and of the back face. Thus, a single
 sensor at position #86 may not be able to detect source emissions on the
 extractor hood and door faces.

Figure 9: Simulation of the adjoint concentration $\tilde{C}$ associated with the optimal sensor position #86

To quantify the observable on each lateral face, we use the observable

430 criterion (8), taking into account the sensor features. Herein, we consider
 431 that the source emission to be detected has an amplitude of approximately
 432 $10ppm$ on a surface of approximately $0.25m^2$ and that the limit of detection
 433 of the sensor is $0.01ppm$, *i.e.* $A_s = 0.25m^2$, $S = 10ppm$ and $dI_m = 0.01ppm$
 434 in Eq. (8). The observable criterion (8) involving the ratio $A_s S / dI_m$, the ob-
 435 servability results presented in the next paragraphs will be the same for other
 436 combinations of A_s , S and dI_m , as long as $A_s S / dI_m = 250$. For each poten-
 437 tial sensor position, we compute the observable criterion and determine the
 438 observability on the four lateral walls. The highest observable area, which
 439 is approximately $7m^2$, is obtained for Sensor position #86. Consequently,
 440 Sensor #86 is considered to be the optimal placement of the first sensor.
 441 In Figure 10, we use sectors to represent the sensor positions with “accept-
 442 able observability”, *i.e.* more than half of the maximum $7m^2$ observability.
 443 Note that 15% (64 positions) of all the potential sensor positions satisfy this
 444 criterion. These results highlight the fact that haphazard placement of gas
 445 sensors may make it impossible to detect source emissions.

446 Figure 10 shows that most of the sensors are sensitive to source emissions on
 447 the furniture and back faces. Only 6 sensor positions (resp. 8 sensor posi-
 448 tions) can cover a part of the extractor hood face (resp. the door face). We
 449 can see that the observable areas differ depending on the height of the sensor
 450 because the air flow is highly three-dimensional in an indoor environment.

451 After fixing Sensor #86 as the first optimal sensor position, the proposed
 452 numerical strategy selects Sensors #9, #235 and #268 as the optimal po-
 453 sitions of four sensors to maximize the observable area on the wall faces.
 454 The optimal positions are represented by red circles in Figure 10, and the
 455 associated observability maps are shown in Figure 11. Sensors #86 and #9
 456 are selected to cover both the furniture face and the back face. Neverthe-

less, Sensor #86 is sensitive to source emissions on the upper part of the furniture face, whereas Sensor #9 can detect sources on the lower part. The sensitivity of the gas concentration at Sensor position #9 to sources emitted from the lower part between the furniture and the wall is due to the air flow. Indeed, the air flow from the second ventilation grid passes under the furniture and licks the lower part of the furniture wall (see Figure 8). Then, the main part of the flow goes out from the corner of the furniture and the door faces, where Sensor #9 is located. Sensor #235 provides information for the extractor hood face and covers additional areas on the back face. Finally, the observability on the door face is provided by the fourth sensor, that is, Sensor #268.

Let us consider Sensor positions #86, #9, #235 and #268 and study the influence of the LOD on the observability of the sources. The parameters A_s and S are kept at $0.25m^2$ and $10ppm$. In Figure 11, we show the observable area associated with each sensor for a 10 ppb LOD and for a 2 ppb LOD. As expected, a lower sensor LOD increases the observable area. At a 10 ppb LOD, Sensor #86 can detect sources on a portion of the furniture and back faces. When the LOD is decreased to 2 ppb, sources can be detected on a larger surface of the furniture and back faces and on the door and extractor hood faces, which were not observable at a 10 ppb LOD. Similar results are obtained for Sensor #235. Reducing the LOD to 2 ppb enables the observation of new surfaces, such as the door face. Nevertheless, we can see an exception for Sensor #9. In this case, as the flow is confined between the wall and the furniture, the observable area is slightly increased for a 2 ppb LOD compared to that for a 10 ppb LOD. Finally, we observe that some wall areas are covered by multiple sensors when the LOD is 2 ppb.

In the last paragraph, we propose numerical validation of the observable

484 criterion (8) in the 3D laboratory room. A 10 ppm source positioned on
 485 the door face, as shown in Figure 11, is injected on a surface of $0.25m^2$.
 486 From a direct simulation, we predict the gas dispersion in the laboratory
 487 room and obtain the concentrations at the different sensor positions. The
 488 results are given in Table 2. The concentration is higher than $10ppb$ only at
 489 Sensor position #268. Hence, at a $10ppb$ LOD, the source can be detected
 490 by only Sensor #86. In Figure 11, we can see that this result was correctly
 491 predicted by the proposed observable criterion when considering a $10ppb$
 492 LOD. At a $2ppb$ LOD, the observable criterion indicates that the source
 493 represented in Figure 11 can be detected by all the optimal sensor positions
 494 except Sensor #9. The predicted result is validated by the gas concentration
 495 from the direct simulation reported in Table 2, showing that only the gas
 496 concentration at Sensor #9 is below the $2ppb$ LOD. Therefore, Sensor #9 is
 497 not able to detect the source, completing the validation study.

Sensor position number	Concentration at sensor position	Source detected by the sensor ? (Y : Yes, N : No)	
		$2ppb$ LOD	$10ppb$ LOD
#86	$3ppb$	Y	N
#9	$< 1ppb$	N	N
#235	$3ppb$	Y	N
#268	$49ppb$	Y	Y

Table 2: Numerical validation of the adjoint-based observable criterion in the 3D laboratory room - Concentration at sensor positions #86, #9, #235 and #268 simulated for the source location defined in Figure 11 and verification of the source detection for $2ppb$ LOD sensors and $10ppb$ LOD sensors

498 **4. Conclusions & Prospects**

499 We proposed a CFD-based virtual testing strategy for the optimal place-
500 ment of gas sensors to efficiently localize surface source emissions in indoor
501 air quality assessment. This strategy relies on a criterion that integrates the
502 adjoint framework and sensor features, such as the limit of detection, to eval-
503 uate, at a reasonable computation cost, the coverage area associated with
504 different sensor positions. We considered the “optimal sensor placement”
505 to be the combination of sensors that maximizes the coverage area. In the
506 two studied applications, we showed that many potential sensor positions
507 observe almost nothing and thus are unable to localize sources, which high-
508 lights the importance of using such sensor placement strategies. Then, we
509 emphasized that the coverage area can be increased not only by adding sen-
510 sors but also by using sensors with a lower limit of detection. Hence, when
511 positioning indoor air quality devices, we have to consider both the limit
512 of detection and the number of sensors. Finally, this work can be extended
513 to the localization of sources emitted inside a defined volume, especially for
514 outdoor air quality purposes.

515 **Acknowledgments**

516 This work was supported by the FUI 18 MIMESYS funded by Re-
517 gion Ile-de-France, which involves several partners: EcologicSense, TERA,
518 ETHERA, FLUIDYN, CSTB, ESIEE Paris, and IFSTTAR. We also want to
519 thank our colleagues Erick Merliot for the realization of the numerical mock-
520 up of the studied room and Rachida Chakir for the fruitful discussions about
521 the direct simulations of the air flow and gas dispersion.

522 **References**

- 523 [1] T. Hoang, R. Castorina, F. Gaspar, R. Maddalena, P. Jenkins,
524 Q. Zhang, T. McKone, E. Benfenati, A. Shi, A. Bradman, Voc ex-
525 posures in california early childhood education environments, *Indoor*
526 *Air* 27 (3) (2017) 609–621.
- 527 [2] C. Godwin, S. Batterman, Indoor air quality in michigan schools, *In-*
528 *door Air* 17 (2) (2006) 109–121.
- 529 [3] N. Goodman, A. Wheeler, P. Paevere, P. Selleck, M. Cheng, A. Steine-
530 mann, Indoor volatile organic compounds at an australian university,
531 *Build. and Environ.* 135 (2018) 344 – 351.
- 532 [4] D. Campagnolo, D. E. Saraga, A. Cattaneo, A. Spinazz, C. Mandin,
533 R. Mabilia, E. Perreca, I. Sakellaris, N. Canha, V. G. Mihucz, T. Szigeti,
534 G. Ventura, J. Madureira, E. de Oliveira Fernandes, Y. de Kluizenaar,
535 E. Cornelissen, O. Hnninen, P. Carrer, P. Wolkoff, D. M. Cavallo, J. G.
536 Bartzis, Vocs and aldehydes source identification in european office
537 buildings - the officair study, *Build. and Environ.* 115 (2017) 18 – 24.
- 538 [5] A. Bari, W. Kindzierski, A. Wheeler, M.-E. Héroux, L. Wallace, Source
539 apportionment of indoor and outdoor volatile organic compounds at
540 homes in edmonton, canada, *Build. and Environ.* 90 (2015) 114 – 124.
- 541 [6] Indoor air quality (iaq), pollutants, their sources and concentration
542 levels, *Build. and Environ.* 27 (3) (1992) 339 – 356.
- 543 [7] W. H. Organization, Selected pollutants, Tech. rep., WHO Regional
544 Office for Europe (2010).

- [8] L. Morawska, P. K. Thai, X. Liu, A. Asumadu-Sakyi, G. Ayoko, A. Bartonova, A. Bedini, F. Chai, B. Christensen, M. Dunbabin, J. Gao, G. S. Hagler, R. Jayaratne, P. Kumar, A. K. Lau, P. K. Louie, M. Mazaheri, Z. Ning, N. Motta, B. Mullins, M. M. Rahman, Z. Ristovski, M. Shafiei, D. Tjondronegoro, D. Westerdahl, R. Williams, Applications of low-cost sensing technologies for air quality monitoring and exposure assessment: How far have they gone?, *Environ. Int.* 116 (2018) 286 – 299.
- [9] D. Bourdin, P. Mocho, V. Desauziers, H. Plaisance, Formaldehyde emission behavior of building materials: On-site measurements and modeling approach to predict indoor air pollution, *J. of Hazard. Mater.* 280 (2014) 164 – 173.
- [10] C. Dimitroulopoulou, M. Ashmore, M. Hill, M. Byrne, R. Kinnersley, Indair: A probabilistic model of indoor air pollution in uk homes, *Atmos. Environ.* 40 (33) (2006) 6362 – 6379.
- [11] F. Haghighat, P. Fazio, T. Unny, A predictive stochastic model for indoor air quality, *Build. and Environ.* 23 (3) (1988) 195 – 201.
- [12] W. Nazaroff, C. G., Mathematical modeling of chemically reactive pollutants in indoor air, *Environ. Sci. & Technol.* 20 (9) (1986) 924 – 934.
- [13] W. Yan, Y. Zhang, Y. Sun, D. Li, Experimental and cfd study of unsteady airborne pollutant transport within an aircraft cabin mock-up, *Build. and Environ.* 44 (1) (2009) 34 – 43.
- [14] G. Gan, H. B. Awbi, Numerical simulation of the indoor environment, *Build. and Environ.* 29 (4) (1994) 449 – 459.

- 569 [15] X. Liu, Z. Zhai, Inverse modeling methods for indoor airborne pollutant
570 tracking: literature review and fundamentals, *Indoor Air* 17 (6) (2007)
571 419–438.
- 572 [16] X. Liu, Z. Zhai, Protecting a whole building from critical indoor con-
573 tamination with optimal sensor network design and source identification
574 methods, *Build. and Environ.* 44 (11) (2009) 2276 – 2283.
- 575 [17] A. D. Fontanini, U. Vaidya, B. Ganapathysubramanian, A methodology
576 for optimal placement of sensors in enclosed environments: A dynamical
577 systems approach, *Build. and Environ.* 100 (2016) 145 – 161.
- 578 [18] D. Papadimitriou, K. Giannakoglou, Computation of the hessian matrix
579 in aerodynamic inverse design using continuous adjoint formulations,
580 *Comput. & Fluids* 37 (8) (2008) 1029 – 1039.
- 581 [19] J. Waeytens, P. Chatellier, F. Bourquin, Inverse computational fluid
582 dynamics: influence of discretisation and model errors on flows in water
583 network including junctions, *ASME J. Fluids Eng.* 137 (9) (2015) 17p.
- 584 [20] H. Elbern, H. Schmidt, O. Talagrand, A. Ebel, 4d-variational data as-
585 simulation with an adjoint air quality model for emission analysis, *J. of*
586 *Environ. Model. and Soft.* 15 (2000) 539–548.
- 587 [21] J. Waeytens, P. Chatellier, F. Bourquin, Sensitivity of inverse
588 advection-diffusion-reaction to sensor and control: a low computational
589 cost tool, *Comput. and Math. with Appl.* 6 (66) (2013) 1082–1103.
- 590 [22] J. Waeytens, P. Chatellier, F. Bourquin, Impacts of discretisation er-
591 ror, flow modeling error and measurement noise on inverse transport-

592 diffusion-reaction in a t-junction, *ASME J. Fluids Eng.* 139 (5) (2017)
593 10p.

594 [23] J. Sykes, J. Wilson, R. Andrews, Sensitivity analysis for steady state
595 groundwater flow using adjoint operators, *Water Resour. Res.* 3 (1985)
596 359–371.

597 [24] F. Kauker, T. Kaminski, M. Karcher, M. Dowdall, J. Brown, A. Hos-
598 seini, P. Strand, Model analysis of worst place scenarios for nuclear
599 accidents in the northern marine environment, *J. of Environ. Model.*
600 and Soft. 77 (2016) 13–18.

601 [25] R. Becker, R. Rannacher, An optimal control approach to a posteriori
602 error estimation in finite elements methods, *Acta Numerica*, Cambridge
603 Press 10 (2001) 1–102.

604 [26] J. T. Oden, S. Prudhomme, Estimation of modeling error in computa-
605 tional mechanics, *J. Comput. Phys.* 182 (2002) 496–515.

606 [27] J. Waeytens, L. Chamoin, P. Ladevèze, Guaranteed error bounds on
607 pointwise quantities of interest for transient viscodynamics problems,
608 *Comput. Mech.* 49 (3) (2012) 291–307.

609 [28] V. Desauziers, D. Bourdin, P. Mocho, H. Plaisance, Innovative tools
610 and modeling methodology for impact prediction and assessment of the
611 contribution of materials on indoor air quality, *Herit. Sci.* 3 (1) (2015)
612 28.

613 [29] C. Wang, X. Yang, J. Guan, Z. Li, K. Gao, Source apportionment of
614 volatile organic compounds (vocs) in aircraft cabins, *Build. and Envi-*
615 *ron.* 81 (2014) 1 – 6.

- 616 [30] B. Clarisse, A. Laurent, N. Seta, Y. L. Moullec, A. E. Hasnaoui, I. Mo-
617 mas, Indoor aldehydes: measurement of contamination levels and iden-
618 tification of their determinants in paris dwellings, *Environ. Res.* 92 (3)
619 (2003) 245 – 253.
- 620 [31] V. Akcelik, G. Biros, O. Ghattas, K. R. Long, B. van Bloemen Waan-
621 ders, A variational finite element method for source inversion for con-
622 vectivediffusive transport, *Finite Elem. in Anal. and Des.* 39 (8) (2003)
623 683 – 705.
- 624 [32] K. Gurney, R. Law, A. Denning, P. Rayner, D. Baker, P. Bousquet,
625 L. Bruhwiler, Y. Chen, P. Ciais, S. Fan, I. Fung, M. Gloor, M. Heimann,
626 K. Higuchi, J. John, T. Maki, S. Maksyutov, K. Masarie, P. Peylin,
627 M. Prather, B. Pak, J. Randerson, J. Sarmiento, S. Taguchi, T. Taka-
628 hashi, C. Yuen, Towards robust regional estimates of CO₂ sources and
629 sinks using atmospheric transport models, *Nature* 415 (6872) (2002)
630 626–630.
- 631 [33] X. Liu, Z. Zhai, Location identification for indoor instantaneous point
632 contaminant source by probability-based inverse Computational Fluid
633 Dynamics modeling, *Indoor Air* 18 (1) (2008) 2–11.
- 634 [34] W. Liang, X. Yang, Indoor formaldehyde in real buildings: Emission
635 source identification, overall emission rate estimation, concentration in-
636 crease and decay patterns, *Build. and Environ.* 69 (2013) 114 – 120.
- 637 [35] A.-L. Pasanen, A. Korpi, J.-P. Kasanen, P. Pasanen, Critical aspects
638 on the significance of microbial volatile metabolites as indoor air pol-
639 lutants, *Environ. Int.* 24 (7) (1998) 703 – 712.

- 640 [36] F. Archambeau, N. Méchitoua, M. Sakiz, Code Saturne: A Finite Vol-
641 ume Code for the computation of turbulent incompressible flows - In-
642 dustrial Applications, Int. J. on Finite Vol. 1 (1).
- 643 [37] T. J. Hughes, M. Mallet, M. Akira, A new finite element formulation
644 for computational fluid dynamics: Ii. beyond supg, Comput. Meth. in
645 Appl. Mech. and Eng. 54 (3) (1986) 341 – 355.
- 646 [38] F. Hecht, New development in freefem++, J. Numer. Math. 20 (3-4)
647 (2012) 251–265.

Figure 10: Potential positions of gas sensors at three levels $h = 0.5m$, $h = 1m$ and $h = 1.5m$ and observability representation by wall surface for sensors with an observable area more than half of the highest observability achieved by Sensor #86 - Optimal sensor positions are shown as white circles

Figure 11: Map of the observable areas associated with gas sensors #86, #9, #235 and #268 at two different LODs (10 ppb and 2 ppb). The total observable areas are indicated in parentheses - Definition of a $0.5m \times 0.5m$ source (white square) for the numerical validation of the observability criterion