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Zeger-Jan Kock[1] and Birgit Pepin[2]

[1]Eindhoven University of Technology / Fontys University of Applied Sciences, z.kock@fontys.nl; [2]Eindhoven University of Technology, The Netherlands

In this study we have investigated the resources used by first year engineering students in a technical university in the Netherlands, for their learning of Calculus and Linear Algebra. Using a case study approach we have focused on how the resources and their use (a) differed from upper secondary school as compared to university, and (b) differed between the two university courses. The results indicate that, in terms of (a) students built on secondary school experiences and emulated these into their university courses, where some subsequently experienced difficulties. In terms of (b), we argue that the course organization and the alignment of curriculum materials with the learning goals had an impact on the students' choice and use of resources. Human resources played an important but varying role.

Keywords: Student use of resources, Case study, Transition from school to university, Calculus, Linear Algebra.

INTRODUCTION

At university level a large diversity of resources is currently made available for students learning mathematics. These include traditional curriculum resources (e.g. readers, textbooks); digital (curriculum) resources (e.g. YouTube, websites, apps); and also human resources (e.g. drop-in clinics run by tutors; setups for peer groups). The ways in which university mathematics teachers interact with various resources has been investigated by Gueudet (2017), for example, and several studies have been conducted related to university students and their use of particular resources to learn mathematics (Anastasakis, Robinson, & Lerman, 2017; Biza, Giraldo, Hochmuth, Khakbaz, & Rasmussen, 2016; Inglis, Palipana, Trenholm, & Ward, 2011). However, relatively little is known about how students of mathematics in their first year of university "cope with" the plethora of available resources available to them, and how they organise and coordinate them for their learning.

Using a case study approach, we have studied the resources in a Calculus (CS) and a Linear Algebra (LA) course, and their use by students, in the context of a first year Bachelor College programme at a technical university in the Netherlands. Moreover, we have investigated, retrospectively, which resources were used by the students, and how, in upper secondary school (as compared to university). Hence, we propose the following research question:

Which kinds of resources are used by the students, and how, in first year university Calculus and Linear Algebra courses, and how do these practices compare to students' experiences at upper secondary school?

In this paper, first, we briefly outline selected insights from the relevant literature, including our theoretical frame of "resources" (and their use) as a lens to develop a better understanding of students' mathematics learning. Second, we describe our chosen methodology and data collection strategies, before we present our findings and discuss our results in the third section. Fourth, we present our conclusions and outline implications for the practice of university mathematics learning and teaching.

THEORETICAL FRAMEWORKS

Transition from secondary to tertiary education

In terms of mathematics learning, the transition from secondary school to university is challenging for many students (Pepin, 2014), as discontinuities exist between secondary and tertiary mathematics education. The literature reports numerous differences between studying mathematics at school as compared to university. It is said that in comparison to secondary education, at university: (a) the mathematical content is introduced at a higher speed; (b) more mathematical autonomy is expected; (c) the levels of generalization and abstraction are higher; (d) the approach is more formal with an increased emphasis on proof; and (e) the institutional cultures at the two institutions (secondary school, university) are different (Artigue, 2016; Gueudet, 2008). The ways the content is made available to students also differ between secondary and tertiary education (e.g. Corriveau & Bednarz, 2017). University students have to autonomously manage the various resources to learn mathematics, and it is argued that secondary school does not prepare them well for this task (Williams, Black, Davis, Pepin, & Wake, 2011). Thus, it can be expected that first year university students have to find new ways of working with the resources they have access to, and that are proposed to them in their courses.

Student use of resources

The use of resources by students has been the subject of relatively little research. Selected studies (e.g. Anastasakis, et al., 2017) indicate that students, in their selection of resources, have been predominantly motivated by the goal to be successful in examinations (and to obtain high grades). The authors of this study made an inventory of the resources used by students when studying for mathematics modules, and explicitly related these to their learning goals. The most widely used resources were those that the university provided for the students, and their own notes. The use of particular resources, for example mathematics textbooks, was specifically linked to the study of worked examples, which were said to help students to prepare for examinations; albeit this often lead to emphasise the surface aspects of the examples (Biza, et al., 2016). In their review study Biza, et al. (2016) identified several limitations of tertiary mathematics textbooks, in particular the emphasis on formal aspects of mathematics, at the cost of opportunities to develop intuitive meanings and understandings. Relating the use of particular resources to examination grades, a study by Inglis, et al. (2011) found that students who attended lectures or used the university's mathematics support centres had higher grades than students who often watched online lectures. The authors suggest that students might need explicit guidance on how to combine the use of various resources into an effective learning strategy. Before this guidance can be given, or be reified in a blended learning environment, more in-depth information on the actual use of resources by students is needed.

The lens of resources

In this study we use the notion of “re-source/s” that students have access to and interact with in/for their learning. We assume that the ways university students learn mathematics is influenced/shaped by their use of the various resources at their disposal. By “use of resources” we denote, for example, which resources students choose (amongst the many on offer) and for what purpose (e.g. revision); the ways they align them (e.g. first lecture then checking the textbook, etc.); which ones seem central to achieve particular learning goals (e.g. for weekly course work, examinations, for their engineering topic area). However, we do not address the specific learning of CS and LA, that is how students interact with particular (e.g. cognitive) resources to learn particular topic areas in CS and/or in LA.

Gueudet and Pepin (in press) have defined student resources as anything likely to re-source (“to source again or differently”) students’ mathematical practice, leaning on Adler’s (2000) definition of mathematics “re-sources” (in Adler’s case used by teachers). In this study we distinguish between (1) material resources, and (2) human resources. (1) For material resources a further distinction has been made between (a) curriculum resources (those resources proposed to students and aligned with the course curriculum), and general resources (which students might find/access randomly on the web). Curriculum resources are developed, proposed and used by teachers and students for the learning (and teaching) of the course mathematics, inside and outside the classroom (Pepin & Gueudet, 2014). They can include text resources, such as textbooks, readers, websites and computer software, but also feedback on written work. General resources are the non-curricular material resources mobilized by students, such as general websites (e.g. Wikipedia, YouTube). (2) In terms of human resources we refer to formal or casual human interactions, such as conversations with friends, peers or tutors.

METHOD

Context

The study took place at a university of technology in the Netherlands, with a student body of approximately 13000 engineering students. The university offers 15 bachelor courses related to technology and engineering.

We selected two first year courses in the first term of the 2016-2017 academic year: Calculus (CS); and Linear Algebra (LA). We purposefully chose these courses, as they were different in size and target group: the CS course was obligatory for all first year engineering students, approximately 2000 students, whereas the LA course was targeted at “applied mathematics and physics” engineering students only, approximately 130 students. The CS course was organized by the mathematics department, differentiated at three levels (A, B, and C), according to perceived level of difficulty and with varying level of emphasis on formal aspects of mathematics (e.g. proof).

In CS, six hours of lectures were organised each week, and one hour of tutorials in groups of eight students. In the course catalogue, and this was supported by lecturers, the aim of CS was to give engineering students a "basis" to be able to "calculate correctly". It appeared that the aims of the CS course were to provide students with a basic set of mathematical/computational tools they could subsequently use in their engineering studies and in their future work as engineers.

In LA, four hours of lectures were organised each week, and four hours of tutorials, in groups of approximately 30 students. As in CS, the LA learning aims were described as the acquisition of mathematical skills. Moreover, aims of the course were to help students develop the skills and realize the importance of correct mathematical communication, including writing formal proofs. Completing a mathematical writing assignment was part of the course requirements to reach this aim. It appeared that the purpose of LA was to prepare students for higher mathematics (used in the mathematics and physics courses).

Participants

In total, 24 students participated in the study: 18 CS students (involved in nine different engineering programs and all taking the B level CS course); 1 CS student who dropped out of university; 5 LA students (all studying for the 'applied mathematics' engineering course). In terms of background, of the interviewed CS students 15 came from secondary schools in the Netherlands, three came from other educational systems. For the Dutch students the CS content was partly familiar, in particular for those who took "strong mathematics" courses (Wiskunde D) at secondary school. Four of the five interviewed LA students came from secondary schools in the Netherlands, one student had attended secondary school in Belgium.

Data collection strategies

Data collection strategies included the following:

(1) Student interviews: The CS students were interviewed in four focus groups, and one individual interview. During the interviews students were asked to make a drawing of the resources they used for their mathematics course (Schematic Representation of Resource System, SRRS - Pepin, Xu, Trouche, & Wang, 2017). These helped the interviewer to understand the ways the resources were used, and for which purpose. The LA students were interviewed in two groups of two, and one individual interview.

(2) Documents/curriculum resources: Relevant curriculum materials and documents (digital and text materials) were collected and analyzed. These materials were provided by the university for the students (e.g. examples of examinations, LA syllabus, LA

study guide, LA assignments, CS study guide, the CS textbook, course summaries in the university's course catalogue, video clips, videos of the lectures).

(3) Teacher interviews: Interviews with two CS lecturers and one CS tutor were conducted, as well as one LA lecturer and one LA tutor.

For analysis, the interviews were transcribed and interview quotations were coded using ATLAS-ti software. The codes were based on our knowledge from the literature concerning the different curriculum resources and their use. In the next step of the analysis the findings from CS and LA were compared, and subsequently these with those from upper secondary school.

RESULTS

Resources at secondary school

In terms of curriculum materials/material resources the textbook was an important resource for most secondary school students, and so were the graphical calculator (also used in examinations) "to quickly plot graphs" (interview reference: CSS01), and past examination papers for revision and practice to prepare for the national examinations. The textbook was seen as the main source of exercises, which were done in class or at home. Regarding homework one student remarked:

> At school I didn't do my homework. There was homework but yeah, if you worked on it during the class … you would get halfway and then at home I was like oh, I get it. I don't have to do the remaining exercises (LAS01).

Online general resources (e.g. YouTube; Kahn academy) were hardly mentioned in relation to secondary school.

In terms of human resources the teacher and classmates were mentioned as an important support for secondary school students. Interestingly, teachers' explanations of the mathematical concepts were not important for all students: in some schools students apparently worked largely independently (with the textbook) and the teacher was only occasionally consulted.

In short, to learn the mathematics and pass the examinations, students reported that it was sufficient to follow the teachers' explanations, do (all) the exercises in the textbook, and practice with the past examination papers. There were few resources, and the ones provided could be straightforwardly accessed and used for solving the problems posed.

Resources for CS

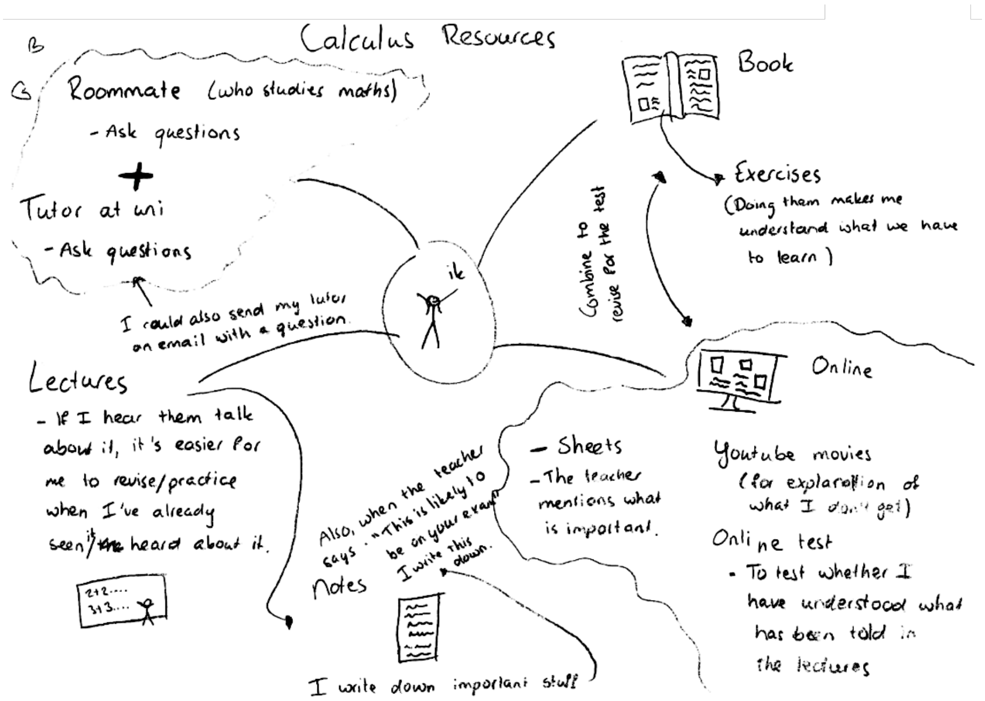
Figure 1 shows the typical resources used by a CS student. The lecture appeared to be an important starting point for many students, albeit not for all. They provided an orientation on the subject ("it's easier for me to revise/practice when I have already seen/heard about it"; Figure 1), and to some extent an enculturation into the world of mathematical concepts and their usages. A lecturer said:

I do the historical aspect too. Some things have been known for 3000 years, so you have to know that too. So I also do applications and add historical things (..) I just want them to be excited about the subject (CSLI03).

CS was supported by an almost 1100 page general CS textbook (authored by an "external" author) chosen from commonly used university CS textbooks. It contained the essential theory and part of the homework exercises (explained by someone "outside" the students' environment), but its content was not specifically aligned with the lectures and the final examinations. Curriculum resources, such as the textbook, were mainly used for exercises, whereas lecture notes (by the teacher) seemed important for knowing about the content to be learnt. Students' own notes were often used for orientation ("I write down important stuff; also when the teacher says 'this is likely to be on your exam', I write this down."; Figure 1). Selected students used the textbook for additional/different purposes: to read the theory in the textbook to prepare in advance for the lecture.

At the same time digital resources, such as general online videos (e.g. YouTube; Kahn academy) and video-recorded lectures (whole lectures or clips of particular moments), were said to provide additional explanations. The course's online tests and the weekly coursework were used to check whether one had a good (basic) understanding of the content. Moreover, students also mentioned human resources, such as the lecturer/tutor and the roommate "to ask questions" (Figure 1). Friendship groups were important for many students, and for some they were their first line of support (before the tutor or peers in the tutorial/s), to work with on the coursework or to consult about difficult theory or exercises.

Figure 1. Purpose and coordination of resources by a CS student



Resources for LA

In terms of material resources LA was supported by a 200 page course-specific reader, authored by the lecturer and developed and improved over the years. It contained the essential theory and the exercises required to prepare for the examinations - this was “the backbone” of the course, according to the lecturer and the tutors. Other resources, such as the lectures, the homework/coursework exercises, the lecture notes and the videos with worked examples were all aligned with the reader, to become a comprehensive and complete set of resources for the students. The importance of using these resources to individually make sense of the mathematical content, and doing lots of practice exercises “at home”, was emphasised by a student:

> But now you have like a huge amount of homework and then you also have workgroups where you can work on it, but then you don't get very far. (…) And if you don't do it at home, you just won't get it and you won't make your tests really good. So you really have to do a lot at home (LAS01).

As in the CS course, the lectures appeared a starting point for many students; they provided an orientation on the subject, and an enculturation into the world of mathematics (e.g. when mathematical proof was explained as an essential mathematical thinking process). Moreover, in terms of human resources, students relied on peer groups (e.g. they collaboratively solved problems during the weekly tutorials), and on the tutors to provide help- this was an important support in the LA course. Tutors were considered more approachable than the lecturers, although students were generally positive about the possibilities to ask questions to lecturers.

Comparing resources and their use

School – university: Whilst selected resources (e.g. textbooks, past examination papers) appeared to be part of the “staple diet” for every student at university or school level, at university students tended to use more, and more varied resources than at upper secondary school, including online lecture videos, video clips (of “difficult” notions), online texts. In addition, selected resources, such as lecture notes, were not mentioned in the secondary school context, where the theory would be taught by the teacher who aligned his/her lessons with the book. Some of the additional resources, such as video lectures, teachers’ lecture notes, readers on specific topics, and online tests, were part of the curriculum resources made available by the university. Other resources, such as online applets and videos, were identified by the students themselves.

In terms of human resources there were also differences: at secondary school for most students the teacher provided practically all of the necessary guidance for learning the mathematical topic. At university the lecturer provided the theory, an overview of what was important for their learning of the topic area (and also for the examinations), and selected worked examples. The practical guidance, i.e. how to solve particular mathematical problems, was mainly provided in the tutorials and the exercises/coursework accompanying the lectures. Hence, students had to find their own learning/peer groups and supports for learning, as on their own it was not possible to manage the amount of work and the pace it was taught. This situation was exacerbated in the CS course, as only one hour of tutorial group work was offered, and students had to work collaboratively outside this hour for completion of their tasks. Hence, many CS students organized and coordinated their own support to work with their peers on the coursework, or to consult about difficult theory or exercises.

The students reported that, compared to secondary school, at university: (a) the pace was faster, (b) the content was more difficult to understand, and (c) the mathematical content was offered in larger steps/sections. The interview data suggest that the role and importance of resources changed as a result of this, as students needed more support structures and feedback on their work. This was particularly pertinent with one (autistic) student, who had dropped out of university. He claimed that he had done all possible CS textbook exercises and interim tests – a practice he had succeeded with at school, but he could not make sense of the questions when he sat the final examination. He was lost in the immensity of resources on offer, which he could not possibly all trial out and use for his learning. And he clearly missed the guidance and support given by his schoolteacher, practices which had provided him with confidence for his learning, and success.

CS- LA: Amongst the university curriculum resources, the student usages of the LA reader and CS textbook differed. To come to understand the topic/s, most LA students reported reading the reader, or the lecture notes, which were aligned with the reader. CS students mentioned the textbook as one of their resources, mainly used for worked examples and exercises. In CS, lecture notes and online resources were considered practically as important as the textbook, as part of the provided resource system. This can be understood in the light of the fact that the LA reader was very different, in relation to the course, to the CS textbook: the LA reader was a “book” prepared by the lecturer to align with his lecture, hence further lecture notes or online resources became secondary/ complementary. The reader contained all information for students to pass the examinations, and all other resources were related to/in line with the reader. In contrast, the CS book was only a backup for the lecture notes (which provided the essential notions to learn and study for the examinations), and students were only expected to “dip into” it for clarification, explanation and/or further exercises. Hence, the textbook did not provide a succinct support for CS students (e.g. to pass the CS examinations).

An important difference between CS and LA was due to the different organization of the tutor hours: in LA- 4 tutor hours/week, tutor groups of ca. 30 students/ tutor; in CS- 1 tutor hour/week, tutor groups of ca. 9 students/tutor. This meant that CS students had to work on practically all of the problems by themselves, as there was less support from the tutor. The fact that a wider variety of resources (used) was reported by the CS students, can in part be understood in the light of the different course organisation: in the LA case they adhere to the resources provided by the lecturer; in the CS case the students had to identify and organize their own support (e.g. online resources, friendship groups).

CONCLUSION

The results of this study have shown that the students built on secondary school experiences and they took these as default positions into their courses. However, learning mathematics at university was for most students different from learning mathematics at secondary school. At secondary school the resources (text book, past examinations, teacher) were well aligned and the teacher provided guidance and support. At university more difficult content had to be understood in a shorter period of time; and students had to identify and coordinate the relevant resources, and organise their own support system (including human support such as friendship learning groups), in particular in CS.

When comparing the two courses, the results indicate that (a) the course organization and (b) the provisions and organisation of the curriculum materials (in line with the learning goals) had an impact on which resources students used, and how they used them. In the LA course, with aligned curriculum resources, four weekly tutorial hours and group work, the use of resources largely corresponded to the intentions of use by the university teachers. In the CS course resources were not clearly aligned (although selected resources were recommended); it seemed that students were provided with a "bag of tools" to choose from. Moreover, students had only one weekly tutorial hour (plus six hours of lecture). This meant that students had (a) to identify which were the relevant resources for their individual needs, and (b) to find and navigate their own path through these resources, in order to work efficiently (with regards to examinations) and effectively (with regards to the learning of the mathematics). In both courses human resource, such as lecturers and tutors, peers and friends, played important albeit changing roles for orientation and help seeking.

The results of this exploratory study indicate that in particular large courses, such as CS (> 2000 students), could become better manageable for the students, if they were supported and coached in their resource choice and organisation/management, so that they can cater for their individual needs and preferences. This finding was less visible in a smaller course, where less resources were on offer, and where resources and resource use were more prescriptive and well aligned with the learning goals (e.g. the LA course). However, when particular educational reforms are implemented (e.g. towards more blended learning, with an abundance of digital learning tools on offer), students need to be supported in their "use" of these resources, in particular at transition from school to university. Course designers would also need to take this into consideration.

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