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*This paper presents the a posteriori analysis of a study and research path (SRP) on comparing reality and forecasts of the number of users of certain social networks, which appears as a teaching and learning proposal for mathematical modelling. We analyse the main elements of the SRP that have been experienced with a first-year course at university in management sciences degrees in two consecutive courses, 2015/16 and 2016/17. We focus our analysis on two essential dialectics for mathematical modelling to be developed: the questions-answers and the media-milieu dialectics. In particular, we take empirical results from the two successive implementation of the SRP to outline through which mechanism these two dialectics could be prompted.*

*Keywords: Mathematical modelling, study and research paths, dialectics, questions-answers, media-milieu.*

## INTRODUCTION: THE SRP AS TEACHING PROPOSAL FOR MATHEMATICAL MODELLING

The starting point of the research is the problem of inquiring into the conditions that can help, and constraints that hinder, that mathematical modelling can be integrated and developed in the teaching and learning of mathematics into current educational systems, in particular, at university level. Researchers and practitioners agree that teaching should not be focused only on the formal transmission of knowledge, but also should provide students of the tools for enquiring into the study of real phenomena and integrate mathematics as an essential modelling tool. This change requires moving from a more traditional pedagogical paradigm of transmission of knowledge, which mostly focuses on introducing students to already built mathematical knowledge, to a paradigm of inquiry where the solving of problematic questions leads learning processes and motivates the study of new knowledge.

In the particular case of the research on modelling and their applications and on inquiry-based approaches some big steps have been made showing how, under certain suitable conditions in different educational levels and curricular frames, modelling activities may be successfully put into practice (Artigue & Blomhøj, 2013). However, although school institutions and researchers agreed that modelling

should play an important role for a change towards a new pedagogical paradigm, the real situation in school and university is not satisfactory (Stillman et al. 2013) and the dissemination and long-term survival of these teaching proposals based on modelling follows as a big challenge for mathematics education (Galbraith 2007, Burkhardt 2006).

> In the case of applications and modelling a shared excitement unites many who have enthused about early experiences in the field, for example when students unleash latent power that for whatever reason had remained fettered in their previous mathematical life. However this very exhilaration can work against further progress, both individually, and particularly at a system level, by creating a sense of adequate achievement that obscures the reality that there is so much more to do.

In our research, developed in the framework of the anthropological theory of the didactic, we focus on the use of the study and research paths (SRP) as epistemological and didactic model (Chevallard, 2015; Winslow et al., 2013; Serrano et al., 2013) to face the problem of moving towards a functional teaching of mathematics and, particularly, where mathematics are conceived as a modelling tool for the study of problematic questions. According to Barquero and Bosch (2015), the starting point of an SRP should be a lively question of real interest for the community of study (students and teacher/s). The study of $Q_0$, called the *generating question*, evolves and opens many other *derived questions* $Q_1$, $Q_2$,…, $Q_n$. The continuous looking for answers to $Q_0$ (and to its derivative questions) is the main purpose of the study and an end in itself. As a result, the study of $Q_0$ and its derived questions $Q_i$ leads to successive temporary answers $A_i$ that can be helpful in elaborating a final response $R^{\heartsuit}$ to $Q_0$. These first characteristics can be associated to the first level of analysis of the SRP that we here consider, it consists in the dialectics establishing between the questions posed and the likely answers appearing (*questions-answers dialectic*) which also provide the basic structure of an SRP to be implemented and to be enriched after each implementation. This first layer refers to the evolution of questions to be faced and the necessary knowledge to be used. Another central dimension for an SRP is the *media-milieu dialectics*, which constitutes the second level of analysis. As described in the aforementioned investigations, the implementation of an SRP can only be carried out if the students have some pre-established responses accessible through the different means of communication and diffusion (that is, the *media*), to elaborate the consecutive provisional answers $A_i$. These *media* are any source of information, such as: textbooks, treatises, research articles, class notes, or the teacher acting as main media. However, the answers provided are constructions that have been elaborated to provide answers to questions that are different to the ones that may be put forward throughout the mathematical modelling process. Thus they have to be re-constructed according to the new needs. Other types of *milieus* will therefore be necessary to test the validity and appropriateness of these answers. This second level of analysis put attention to the evolution of the students' milieu.

With this aim, we present an analysis of a particular SRP about the evolution of users of certain social networks that we will analyse in term of these two central dialectics and, more concretely, focusing on two critical questions:

(1) How to enhance dialectics between posing questions and looking for answer as engine of the modelling process? How to transfer to students the responsibilities of posing questions and looking for answers? (2) What *milieu* is necessary for students to facilitate a rich development of modelling? How a richer media-milieu dialectics can be developed?

## DIDACTIC ANALYSIS OF A MODELLING PROCESS: THE CASE OF AN SRP ABOUT THE EVOLUTION OF THE NUMBER OF FACEBOOK USERS

We focus on analysing the case of an SRP on *Comparing forecasts against reality in the case of Facebook users' evolution*. The first time it was experienced was during the winter term of the academic year 2015-16 with first-year students of Business Administration Degree and of Innovation Management (BAIM), all from the 'Escola Superior de Ciències Socials i de l'Empresa-Tecnocampus', Pompeu Fabra University (see Barquero, Monreal, Ruíz-Munzón & Serrano, 2017). During the academic year 2016-17 it has been implemented again in the same university degree. The SRP has run in a modelling workshop that was optional activity for students during these last two academic years. In this paper we analyse and compare both implementations by using two central dialectics: the questions-answers and the media-milieu dialectics.

The initial situation starts from real news about a research performed by Princeton in 2014, in which it was predicted that Facebook would lose the 80% of its users before 2017. Hence, the generating question $Q_0$ presented to students is about: *Can these forecasts be true? How can we model and fit real data about Facebook users' evolution to provide our forecast the short- and long-term evolution of the social network? How can we validate the conclusions of Princeton?* The experimentation was structured in three interconnected phases linked to the generating question $Q_0$, building up the a priori design of the SRP, then reflected in the design of the c-book unit. A first phase that focuses on the open research of real data about Facebook users, a second one focused on finding mathematical models (mainly based on elementary functions) that may provide a good fitting to real data, and a third one about the use of these models to forecast the behaviour of the social network in short, medium- and long-term in terms of number of users and about how to decide about best and most reliable model.

Previously, during the first term (4 ECTS of the subject) students had been getting familiar with the main properties of some groups of functions (polynomial, rational, irrational, exponential and logarithmic functions) as well as with basic topics on differential calculus and its applications to the study of the monotony and optimization of one real variable functions. Actually, before starting the first session

we asked the students to answer a test on some of the mathematical tools that mainly make the workshop up, as indentifying some types of elementary functions or the concept of fitting model in certain scatter plots.

In the first experimentation 27 students, working in 'consultant teams' of 3-4 people, got the order from MS2 Consulting ('Mathematical Solutions Squared') previously described as $Q_0$ and they were asked to deliver a final report by the end of their work as an oral presentation as response to the MS2 request. The implementation combined face-to-face sessions in the teaching device called 'Math modelling workshop' (in a total of six 90-minutes weekly sessions) for the miss-in-common of the junior consultant teams' partial reports, with work out of the classroom. For the second experimentation 12 students (18 students started the workshop, but they left it in the second session due to external matters) worked also in teams of 3-5 people. This time we opened a Moodle virtual classroom to provide the students the teaching aid of the workshop, as well as some communication and collaborative tools (forum, a different wiki for each phase, etc.) to write their progress and pose their new questions. The generating question $Q_0$ was presented in a small dossier, next to the initial subquestions of each of the SRP phases ($Q_1$, $Q_2$ and $Q_3$). The workshop run over seven face-to-face 90-minutes sessions before the final session, in which students should present their conclusions in an oral presentation in front of an external committee with representatives from MS2 Consulting.

Next we sketch in the case of the two implementations how the different dialectics were prompted by both: (a) the design of the unit (by its initial design but also by the different changes introduced according to students' requirements: new questions and answers not envisioned, new media required, etc.) and (b) the didactic gestures and devices to manage its implementation.

**Integrating the dialectics of questions-answers as engine of the SRP**

The *a priori* design of the SRP was basically the same in both implementations, structured in three interconnected phases linked to $Q_0$, which guided the design of the workshop throughout its implementation. A first phase focuses on the open search of data about Facebook users; a second one focused on mathematical models (mainly based on elementary functions) that might provide a good fit to Facebook users data; and a third part focused on the use of these models to provide short-, medium- and long-term forecasts about the number of users of Facebook and on how to decide on the best and most reliable model. Figure 1 (and the explanation below) shows the link between different questions ($Q_i$) that were planned as likely to appear in the real implementation of the SRP and some expected answers (*Ai*) from the working teams. The only difference of the second design with respect to the first one was motivated for the context in which $Q_0$ was presented originally: the predictions made by Princeton were supposed to happen in 2017, and this year was present tense for the students of the second experimentation. Hence, we decided to make the same

questions, but giving freedom to students of focusing in any other social network students were interested in.

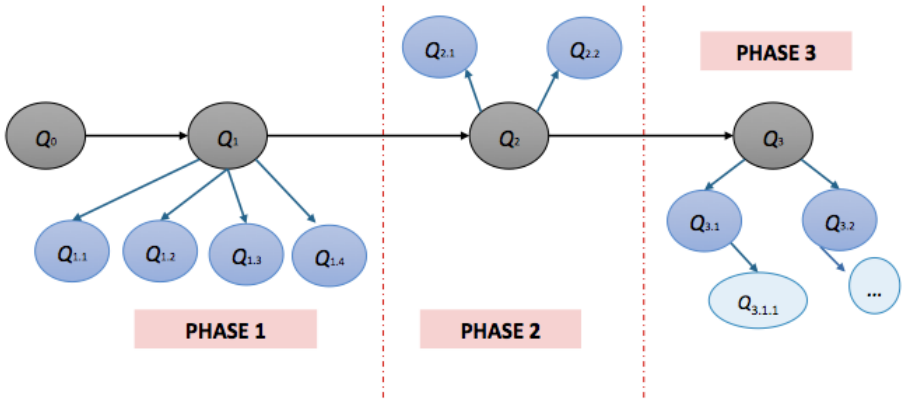


**Figure 1: Tree of questions and answers of the different phases of the SRP**

$Q_1$: Which data sets about the users of the social network are better to consider in our research? → $A_1$: Each group look for the data to be used and shared; the whole community agree on the terminology (year, period, units, etc.) and on the dependent and independent variables to take into account.

$Q_{1.1}$: Which time intervals may be considered? $Q_{1.2}$: How can data be well-organized? $Q_{1.3}$: How to organise and visualise data? $Q_{1.4}$: What can we say about the growth tendency of the data analysed?

$Q_2$: Which mathematical models provide the best fitting of data about the network users? → $A_2$: Each consultant group is asked to propose and justify three mathematical models fitting real data.

$Q_{2.1}$: Which models (based on elementary functions: linear, parabolic, exponential, etc.) may fit the data? $Q_{2.2}$: How can the coefficients of the model be determined?

$Q_3$: How can we decide about the 'best' fitting model? Can we use this model to predict the future evolution of users? → $A_3$: The teams need to create tools to justify why a mathematical model/s is/are the 'best' with respect to: (a) fitting data and (b) forecasting the evolution of users.

$Q_{3.1}$: How can we compare the error committed between reality and forecasts provided by models? $Q_{3.2}$: Can be the same model used for the short- and long-term forecasts?

Let us now comment the main features of the *a posteriori* analysis of the experimentations, referring here to the questions-answers dialectics level.

Regarding the first phase, we should remark the ease with which the students found real data about the evolution of the social net. The students mainly found the information by means of a graphical representation. This fact strongly determined their analysis, since they mainly focused in the graphical analysis growth tendency of the data, but not in their numerical versant, making $Q_{1.4}$ being treated before the

other ones (it was considered that students would have data in table format before having graphs). With respect to the first experimentation, the fact that many groups found the same data triggered an intense debate and interchange of ideas among them, which took us to consider a brainstorming session about the previous hypothesis in the classroom, and as a consequence, the duration of the first phase was extended from 3 to 4 sessions. Due to the wealth of answers collected during the brainstorming we asked the students to deliver a first report in a poster format, synthesizing their findings, conclusions and new questions made by them. In the second experimentation the fact that students could choose a social network implied a disruption with the usual *topos* of the students in the process of study, since they were responsible on the delimitation of the field of study. They noticed about the difficulty of finding reliable data of some of their choices (Snapchat, Instagram, Twitter…), so finally only Facebook and Instagram were object of study, and not only the number of users with respect to the time, but also other variables that could have a relation. Another question that raised here was the role of the intervals of time of the data obtained, and how to work when data are not regularly spaced in time. These questions enriched the *a priori* design of the SRP. The presentation of the first phase was done on the third session, and there had not been interchange of ideas with other groups during the first phase. Furthermore, we asked students to present their plan of work: the questions that they wanted to deal with, when and how. This showed that each group had planned the next steps in many different ways and with many different variables. Nevertheless, the lack of time and our interest in the study of one real variable function made us proposed the students to use only the variables "Time" and "Users".

Let us focus now on the second phase. In both experimentations the analysis of the different proposals made arise a non-expected aspect: the use of piecewise functions. Then the expected answer to $Q_2$ about the consideration of models based elementary functions (linear, quadratic, exponential, etc.) was extended. In the case of the first experimentation, since many groups worked finally with very similar data on the worldwide evolution of FB users, we took two new decisions: (a) give each team a second set of different data, corresponding to different geographical areas, in order to contrast their hypothesis and extend their study; and (b) ask for more than one fitting model for each data set. This was not necessary in the second experimentation, since each group had different data sets. Besides, in both workshops new questions and answers appeared at this stage with respect to the change of tendency of the fitting models, in accordance to a particular action or to decisions of the corresponding social network (IPO, new rival social nets, purchases of the company, new developments, etc.), which determined the moments of change of tendency. Furthermore, in the second experimentation we let the students choose a software for representing their data and the functions. This made question $Q_{1.2}$ emerge again, since they needed to adapt their data to the different software used. Just one group decided to use *Geogebra*, so they were provided the applets we used in the previous

experimentations (Barquero et al. 2017). Instead, another one decided to make interpolation in order to find functions fitting their data, so they used *Symbolab* and added some questions about how to solve non-linear systems of equations. The third group worked with *Excel* for representing a scatter plot, and used linear and non-linear regression. This motivated a big change in the SRP, since question $Q_{3.1}$ emerged naturally in the exposition of their findings at the beginning of the third phase (since they have used the R-squared of their model given by the software). This gave birth to an interesting discussion on different ways of measuring the error, and the professors had to present this question as a central matter.

Concerning the third and last phase, in both experimentations we only had two face-to-face sessions of the workshop, but were not enough for a rich development of $Q_3$. Although this time constraint, in the first implementation of the SRP there were some applets designed and made available for students to help on the simulation of models and its contrast to real data. It helped students to delve into $Q_3$, but not many new questions appeared from this work. With respect to question $Q_{3.2}$ only one group dared with long term forecasts to give a date for the moment in which the users of the chosen social net would start decreasing. Both implementations finished with a final presentation of their modelling work and conclusions to an external committee.

Before finishing, we should remark that in both implementations the common discussions, presentations and brainstorming session became the main device for students to formulate and organize new questions, debate answers and contrast them.

**The progressive enrichment of the milieu: the media-milieu dialectics**

Since we have the first layer of analysis of the SRP in terms of the arborescence of the questions-answers, it is important to ask when, where and how questions can arise and answers can be developed. It is at this new level when there may appear the different elements taking part of the milieu, composed of varied elements: questions, temporary answers, pre-existing answers in or out school, means to validate answers, experimental data, etc., accessible through different kind of media (textbook, lectures, website resources, etc.). The relation among these elements can be analysed through the *media-milieu dialectics*. The constant dialectics between the search for data (for instance, real data about users of social networks, or about the company changes) and pre-existing answers (ways to organise data, common models to fit population evaluation, elementary functions, tools to control error, etc.) that exist in different media available for students (web resources, contents of Mathematics course, answers from lecturers from other courses...) and the creation of the appropriate means (milieu) to integrate (or refuse) them has been central in our SRP. Let us stress the importance of some of them.

In the first phase of the SRP, it was important to some groups the topics worked in another course called ‘Introduction to digital communities’ (running in parallel to the workshop) who helped on providing a general sense and functionality to $Q_0$ and to

show how the students could look for real data and some techniques to organise them. All these elements took part of the *media* accessible to students, at the time it enriches students' *milieu* mainly composed at this stage of the data sets that each team chose to work with (shared and debated early with the whole class in the first implementation, and before the second phase in the second implementation; even strongly in this case, since the variety of data found was higher and let them make comparisons between different social networks). All these elements helped them to prepare a first report with the first temporary answer $A_1$ (a poster format given in the first implementation, and a face-to-face presentation in the second). Here we should remark the importance of making their plans explicit (especially in face-to-face sessions) to construct a common frame to be the source of new questions, as well as and to integrate in their milieu new concepts about modelling, and ideas of other groups that could help them. It is in the second phase in which we find more differences. In the first experimentation, the a priori design contained some *Geogebra* applets proposed to help students to explore different models based on elementary functions ($Q_2$). These applets provided the main *media* for students to visualize data jointly with model simulation, and also took part of their *milieu* as main tools for contrasting, comparing and deciding on the 'best' models to choose. Nevertheless, there were other tools not planned in the *a priori* design (as piecewise functions, or Gaussian functions, most of them part of their milieu, since they had been introduced in previous courses) but provided by designing new applets. In the second experimentation only one group used these applets, so their path followed was more similar to the first ones; but two groups decided to use other software mentioned above (that they could know from Statistics or other subjects), which made the main difference with the first implementation: meanwhile the first applet seemed to drive students to apply only a trial-error method, tools like interpolation or regression made students arise an earlier answer. Here again the common forum stated as a face-to-face session motivated an enrichment of the student's milieu. Regarding the third phase, there were several important questions that were not addressed properly, such as $Q_{3.1}$ about the way to measure the differences between data and forecasts, but here there is a main difference between both implementations. In the first one the students assumed and uncritically used the milieu made available through the design of an applet, a sort of *black box* to get immediate answers. Instead, in the second one students had to construct their own tool for measuring the error, and one group made it with *Excel*. Just one group could answer $Q_{3.1}$ but the answer was totally produced by them, so they could communicate it to the rest of the class, extending the appropriate milieu to other groups.

## FINAL REFLECTIONS

First of all, we should mention that students are not in general motivated to validate their results after a work of research, since a lecturer will finally do it. In this workshop students were responsible to validate or justify every decision they made

by the end of each phase. And this is the main reason why other questions arise and contribute to enrich the a priori design of the SRP.

In this paper we focus on the case of an SRP on comparing forecasts against reality in the case of the evolution of the number of users of certain social networks to show the use of two dialectics: the one of the questions-answers and of the media-milieu, corresponding to two of the three complementary level of didactic analysis of teaching and learning processes (Chevallard, 2008). Besides their analytic use, they suppose a productive framework to enrich teaching and learning practices, in particular, on modelling.

In what concerns to the *questions-answers dialectics*, the generating question $Q_0$ about the controversy of the article by Princeton was adopted by the students with a great interest from the very beginning and, up to the end of the process, was kept alive. From the two presented implementations we can underline very important conditions that were created. First, the flexibility of the lecturers and designers team that were opened to readjust the schedule according to students' team work. Furthermore, they were very attentive to integrate in their presentations all new questions and means that the students asked for. Second, students were very active on the sessions to share their proposals, making derived questions emerge naturally, some of them planned in the a priori design, some others that extended the initial proposal. Regarding the *media-milieu dialectics*, in the case of this SRP, we took several decisions along the implementation of transforming the media offered to students to help them in the modelling process and also to observe the impact new media had on students' milieu. Nevertheless, giving students the chance of using their own ICT tools, as was decided for the second experimentation, enriched the *media-milieu dialectics*, since it helped to arise other different answers that had not happened during the first experimentation. We may insist again on the role played by very important contributions, such as collaboration with other subjects, focusing some workshop sessions on discussing external answers that students brought, the creation of applets to foster students' experimental work, among other interesting aspects.

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