The ecological relativity of modelling practices: adaptations of an study and research path to different university settings

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This paper focuses on the problem of the ecology of mathematical modelling practices at university level through the systematic variation of teaching institutions. Our aim is to deal with the variety of constraints appearing when modelling is implemented in university classrooms, and to study the way new teaching proposals can overcome them. Within the framework of the anthropological theory of the didactic, a teaching and learning proposal in terms of study and research paths in tertiary education shows new possibilities to surmount some of these constraints. The paper presents the design and successive adaptations of an SRP about an urban bike-sharing system according to the specificities of different university institutions and the reactions obtained by the students and lecturers.

Keywords: Modelling; anthropological theory of the didactic; research and study path; ecology; institutional relativity.

INTRODUCTION

The starting point of this research is delving into the problem of studying the variety of constraints appearing when mathematical modelling proposals are implemented in university classrooms, impeding their regular development, and to study the way new teaching proposals can overcome them. Several research projects have highlighted the existence of strong constraints impinging on the *large-scale dissemination of mathematics as a modelling activity* in current educational systems at all school levels (Doerr & Lesh, 2011; Kaiser & Maaβ, 2007). We use the term *ecology* to refer to the institutional conditions allowing and the constraints hindering the way a given activity is produced, transposed, taught and learned in a given educational setting.

In previous research developed in the framework of the anthropological theory of the didactic (ATD), we propose the use of a general frame to detect and place the institutional constraints hindering the possible large-scale dissemination of modelling activities based on a hierarchy of levels of didactic co-determinacy (Chevallard, 2002). In Barquero, Bosch and Gascón (2013), we use this general frame to detect constraints appearing at different levels, from the specific ones related to how mathematical contents are proposed to be taught at school, to the more general ones regarding the general organisation of school activities and the role assigned to schools in our societies. This ecological analysis shows how institutional constraints are anchored in deep-rooted practices and are difficult – for teachers and also for researchers – to notice since they appeared as "the natural way of doing". For instance, Barquero et al. (2013) characterise and empirically contrast the predominance of "applicationism" as the dominant way of interpreting, describing and conceptualizing mathematical modelling in natural sciences university degrees. Under its influence, modelling is understood as a mere application of previously constructed knowledge, as if the construction of knowledge were independent of its use. At a more general level, in many schools the prevailing pedagogy is still strongly influenced by the paradigm of "visiting works" (Chevallard, 2015), according to which school knowledge organisations are presented as interesting monuments to visit, instead of as useful tools to provide answers to problematic questions.

In this paper, we focus on going one-step to study the *ecological relativity* of modelling practices in university institutions. As it is described in Castella (2004) and Sierra (2006), each institution endures an institutional relation with knowledge, in particular, with mathematical knowledge. Consequently, each institution establishes a set of specific conditions and constraints that can favour or, on the contrary, prevent certain teaching and learning processes and knowledge constructions to be appropriately developed. It is in this aspect where we want to look more carefully. Therefore, we focus on analysing the emergence, persistence and scope of the conditions and constraints for development of modelling through a variation of university institution. In our research, we work on the use of the *study and research paths* (SRP) as epistemological and didactic model (Chevallard, 2015; Winslow et al., 2013; Barquero et al., 2018) where mathematics are conceived as a modelling tool for the study of problematic questions. We here present an SRP based on an urban bike-sharing system inaugurated in Barcelona in 2007 that has been experimented in three different university settings. The starting point of this SRP is the difficulty to get a homogeneous distribution of bicycles in a city with many sloping streets. We present the successive transformations of the SRP to three different university settings, according to the specificities of each institution, and to the reactions from students and lecturers. Some of the commonalities found show the stable constraints hindering the development of the SRP, whereas the differences detected bring new insights about the conditions to surmount them.

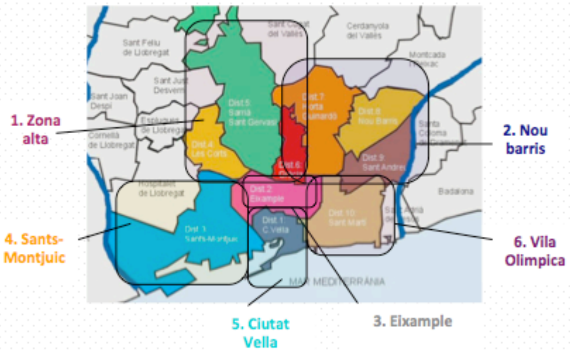
DESIGN OF AN SRP ABOUT A SHARING-BIKE SYSTEM

In the following we describe the initial design of the study and research path (SRP) about the sharing-bike system whose starting point is the generating questions (Q_0) about *how to improve the distribution of bikes in the 'Bicing' system to provide a better service to users*. When working with the *a priori* design of the SRP, there are foreseen several derived questions from Q_0 that needs from a progressive modelling process. In general terms, the modelling project was organised around the following questions that structured the two phases the *Bicing* project:

$Q_{(A)}$: How can we describe the daily flow of bikes between stations? What is the natural behaviour of the system when it is left alone (without redeployment)?

$Q_{(B)}$: How can we predict the bikes' redeployment needs? Which changes can be proposed to improve the current policy of bikes redeployment in the city?

Linked to these questions, we consider real data from *Bicing* about the distribution of bikes among the different bikes' stations. We, the researchers and the experts who collaborated with us, agreed to organise these data in certain city areas according to the similarities different stations shared on the pattern of daily bikes trips and routes followed. Finally, we decided to present the data organised in six areas (as shown in Table 1), which corresponds to the origin-destination matrix (OD matrix) containing the potential number of daily bikes' uses. Each number $\{od_{ij}\}$ means the average of the amount of bike traveling in a day from area j and arriving to area i.



	ARRIVALS	z1	z2	z3	z4	z5	z6
	DEPARTURES	Z_ALTA	N_BARRIS	EIXAMPLE	S_MONTJUIC	C_VELLA	V_OLIMPICA
z1	Z_ALTA	2009	297	2084	1088	589	646
z2	N_BARRIS	207	2356	424	29	149	701
z3	EIXAMPLE	1153	401	4332	1900	2263	1179
z4	S_MONTJUIC	886	32	1649	2594	1071	153
z5	C_VELLA	196	113	2072	895	3572	1264
z6	V_OLIMPICA	101	462	1058	504	1004	2151

Table 1: Origin-Destination matrix with daily bikes' trips

To face the first question $Q_{0(\mathrm{A})}$, and going beyond the descriptive analysis of the data contained in the OD matrix, models based on recurrent sequences of order $d > 1$ can be considered, which are equivalent to matrix recurrent sequences $X(n) = f(X(n\text{-}1))$ where $X(n) = (x_1(n), x_2(n), \ldots, x_6(n))$ is the vector with the bike distribution in each of the six areas at time n. Next we summarize the *a priori* design in terms of hypothesis (H), questions (Q) and answers (A) delimited by the researchers about the models that might be used in an implementation of the SRP.

One of the easier assumptions we can work with is considering that:

$H_{(\mathrm{A})1}$: There is no redeployment of bikes in the system and the bike flows between stations is the same every day.

$Q_{(\mathrm{A})1.1}$: Then, if we deploy different amounts of bikes in each station, what will be the distribution of bikes after 1, 2, 3,…, n days?

The model that can be considered under these assumptions is:

$$X(n) = M.X(n\text{-}1) \rightarrow X(n) = M^n.\, X(0) \text{ for } n > 0 \quad (1)$$

where M is the *transition matrix* (or *transition probability matrix*) obtained from the OD matrix, where $\{m_{ij}\}$ is the percentage of transition between two areas. That is, the potential number of daily travels with origin in j and arriving to i $\{(od_{\mathrm{ij}})\}$ divided by total amount of departures from j ($d(j)$). When working with this first model, several questions can appear:

$Q_{(\mathrm{A})1.2}$: Working with the transition matrix and with different $X(0)$ at the beginning of the day, which traits from the trajectory of $X(n)$ can be underlined?

$Q_{(A)1.3}$: Does it exist any fixed point X^f to which the sequence $X(n)$ converges to? Do all $X(n)$ converge towards a fixed point X^f? Is it possible to calculate X^f in advance?

$Q_{(A)1.4}$: Which relation there exist between X^f and the n-power of the transition matrix?

And, it can easily appear questions about the limitations of the hypothesis assumed and models built, such as:

$Q_{(A)1.5}$: How can include other factors that are important for *Bicing*, such as: the total amount of trips made by a bike, the potential demand of bikes, the available bikes?

Introducing questions about how to improve our hypothesis and the models to be more realistic with the system we want to analyse can open many possibilities. One possible new reformulation of the hypothesis we can work with is:

$H_{(A)2}$: We assume that (1) each bike trip takes about t minutes, (2) the entire fleet of bikes does not move every t min, (3) the total number of bikes that moves in period t depends on: (a) the potential demand for bike trips, and (b) the amount of bikes available.

At this point, there appear more complex models where it is important to frame the time t, for instance, t = 30 minutes (which it is the average of a bike trip in *Bicing*). Then, we can define $B_i(t)$ as the number of bikes in an area at time t and $B(t) = (B_1(t), B_2(t), B_3(t), B_4(t), B_5(t), B_6(t))$ as the vector with the bikes distribution in each area. Then, if we define the departures as $D(t) = (D_1(t), D_2(t), D_3(t), D_4(t), D_5(t), D_6(t))$ and the arrival as $A(t) = (A_1(t), A_2(t), A_3(t), A_4(t), A_5(t), A_6(t))$, $B(t)$ can be modelled by:

$$B(t+1) = B(t) - D(t) + A(t+1) \qquad (2)$$

where $D(t) = \min\,[\text{demand\_trips}(30\text{ min}), B(t)]$ and $A(t+1) = M \cdot D(t)$, with M the transition matrix in time periods t. When this second model is considered, several questions can guide the study process:

$Q_{(A)2.1}$: Using this model (2), and considering different initial distribution of bikes at the beginning of the day $B(0)$, which will be the bike distribution $B(t)$ at the end of the day? And, if the system is left alone, after 2, 3, 4, …, 30 days?

$Q_{(A)2.2}$: Which traits can we underline about the trajectory of $B(t)$ through the simulation of model (2)? Are there also some fixed points to which the sequence $B(t)$ converge?

$Q_{(A)2.3}$: Is there any relation between the fixed points X^f we reach with the ones detected with model (1)?

$Q_{(A)2.4}$: Which relationship is there between the first and second models, defined in (1) and by (2)? Which of the two models do integrate more realistic conditions about *Bicing*?

In the next section we retake this a priori design of the SRP in terms of Q_0 and the likely hypothesis and derived questions $Q_{(A).n}$ to analyse the particular implementation of the SRP about Bicing project in the different university institutions. Besides underlying the adaptations that were necessary to the SRP in each university institution, we focus on the most important conditions (common or not) that favour the development of the SRP, and consequently of the modelling practice. In most of the occasions, these conditions and constraints were phrases by the students and lecturers involved in the implementations or by the survey and interview done at the end of each implementation.

ECOLOGICAL ANALYSIS OF THE SRP IN DIFFERENT UNIVERSITY INSTITUTIONS

First SRP adaptation: The 'Bicing project' at the University of Copenhagen

The first implementation of the SRP about the bikes' distribution in the *Bicing* system took place in the University of Copenhagen (UC). Twenty-three students participated in this implementation. They were taking the course called *MathMod* (Mathematical Modelling), which was an optional course in the third year of the Mathematics degree. The course run over seven weeks, plus two extra weeks to prepare their final team project. The course had three weekly sessions of two hours each. In general terms, the first session was a lecture, the second was a practical or exercise-based session to practice the content introduce in the previous lecture and, the third one, to work in teams in the computer room to simulate by Mapple some models introduced along the course or to work on the team final project. The teaching course proposal was based on the realization of four short projects (mini-projects), linked to some practical activities. These mini-projects mostly consisted of being introduced to some pre-existing models in the lectures sessions to then asked students to put them into practice in the practical sessions. Some example of the project composing the course are: "Mini-project 1: Using the Malthusian and logistic models to predict population evolution" or "Mini-project 4: The Lotka-Volterra models".

In the academic year 2009/10, the author of the paper participated in this course as researcher and the lecturer offered the opportunity of implementing the SRP about *Bicing*. It was integrated as the fifth (and last) project of the course. The SRP implementation ran over two weeks, with six sessions of two hours. At the end of each week, students working in teams had to deliver a report with their temporary results of the *Bicing* project. It was necessary to *break with the above-mentioned organisation* of the course sessions and to set up time for the presentations by the lecturer-researcher and for students' presentation. There, students could compare their proposals and to collectively agree how to follow. During the first week, once the generating question Q_0 was presented by the lecturer-researcher, students agreed to firstly focused on $Q_{0(A)}$ from where students developed most of the path described in the previous section about model (1). In the second week, we (students and instructors) worked on how to reformulate the $H_{(A)2}$ and $Q_{(A)2}$, as most of the groups noticed that in model 1 there were considered some unrealistic assumptions. Due to time restrictions, we could not go further the second model. Finally, each team had to deliver a report one week later the ending of the project with some suggestions for Bicing about how to improve their bike replacing system, $Q_{0(B)}$. Figure 1 summarizes the path followed in this first implementation.

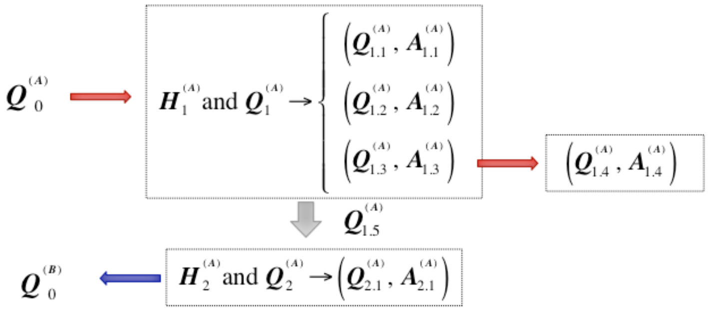


Figure 1: Summary of the path follow in the first implementation of the SRP at UC

We counted on different conditions that favour that the SRP progress fruitfully. First, as it was the fifth project of the course, and the course was explicitly focus on modelling, students and lecturers shared a *common discourse* to refer to modelling. This was an important condition for modelling to be noticed (Barquero et al. 2013). Secondly, the second mini-project was about Leslie matrices and transition matrices. It thus facilitated that students autonomously posed many new questions, such as $Q_{(A)1.3}$ and $Q_{(A)1.4}$ and, thanks to the previous work developed with Maple, students easily worked on calculating and simulating sequences and studying their convergence. On the contrary, there were also some constraints that were manifested by students mostly at the beginning of the SRP implementation. When we started with the Bicing project, students were astonished by the new responsibilities that they were asked, such as: formulating hypothesis, looking for and building models, testing models' appropriateness, formulating new questions, writing a report without any predetermined structure, etc. Although their initial confusion, consequence of a big rupture with the didactic contract established in the course, they started assuming these new responsibilities. In the previous activities of the course, students were only asked to "apply" the models they had been introduced to. So that, breaking some rules of the didactic contract and make students responsible of several new tasks in the modelling process were the main constraints we had to surmount. In fact, the course organisation shown many traits (and constraints) derived from "applicationism" (Barquero et al. 2013). For instance, it was assumed (throughout the course organisation) that the mathematical models had to be introduced in advanced and then applied to different situation, models that are rarely questioned and hardly reformulated. When the Bicing project started, many students' resistances appeared that reflected the implicit assumptions about what modelling was suppose to be and what we (as students and as lecturer) were asked to do. At the end of the course, when students were asked through a survey and with the interviews with some of them, they stated how interesting it was this last project for several reasons. Some of main reasons mentioned by the students were: the openness of the questions, the possibility to delimit the questions to face, the necessity of clearly understanding the modelling process (the hypothesis assumed, the models' construction and their validation), the possibility to compare teams' proposals and results with the rest of the groups who could have been working differently, possibilities to discuss the limitation of the models proposed and make them evolve.

Ecological relativity of the second SRP adaptation implemented at UAB

The second implementation of the SRP was the following academic year at the Universitat Autònoma de Barcelona (UAB). There was a course called "*Mathematical modelling workshop*" which started in 2009/10 with second-year students of Mathematics degree. It was the first edition of the course, which was compulsory, with a total of 45 students participating. The didactic organisation of the course was different from the previously described at the UC. The main aim of the course was to develop a project in working teams (composed of 4-5 students) that students selected from a list provided by the lecturers of the course. Running in parallel, there were planned some short activities about modelling. The first year this course was implemented, one of the modelling activities planned was the 'Bicing project'. It ran over 5 weeks, with two 2-hour sessions per week. We invested more than the double of time than in its first implementation. Similarly, students were asked that at the end of each week they had to deliver a report with a synthesis of their advances in term of: (a) questions they had focused on, (b) hypothesis assumed and mathematical models considered, (c) temporary answers and (d) new questions to follow with). At the end of the Bicing project, each working team had to deliver a final report as summary of the whole modelling work developed. In general terms, the modelling process students and instructors followed in this occasion was not so different concerning $Q_{0(A)}$, although now none of the students' working team tackled the second phase of the project with $Q_{0(B)}$, or posed any questions about the properties of the *n*-power of transition matrices, such as: $Q_{(A)1.4}$ or $Q_{(A)2.3}$.

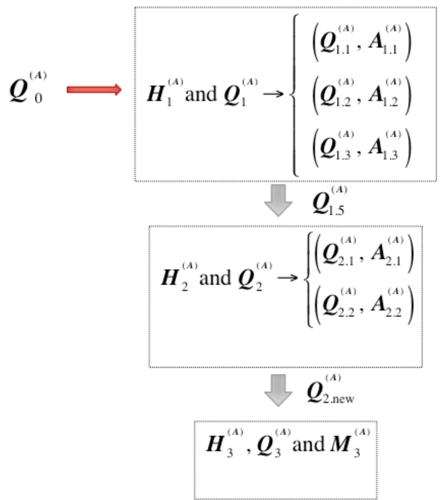


Figure 2: Summary of the path follow in the second adaptation of the SRP in UAB

One important novelty (and extension of the SRP) was that some students asked about the possibility of working with partial matrices, for instance, by considering different OD matrix to describe differently the bikes' flow in the morning and in the afternoon. Students had checked in the web how many bikes were available at different time frames and they had concluded that there were different patterns of bikes disposition depending in the daily time frame. The instructors asked to the experts we worked with about the possibility of having these new data. The external experts provided us two new matrices: one for the morning pattern, from 05:00h to 14:30h, and the other for the afternoon, from 14:30h to 00:00h. With this new data, the modelling process concerning $Q_{0(\mathrm{A})}$ was extended towards the construction of a third model, built upon the two previous ones (1) and (2), and taking into account these two different OD matrices. Figure 2 summarizes the path followed in this occasion and the extension it supposed for the first phase of the Bicing project.

If we focus on analysing the conditions and constraints we detected in the second implementation of the SRP, we have to mention that in this occasion it was the lecturer of the course who expressed more clearly some important constraints. He expressed, in an interview at the end of the implementation, that we had invested too much time with the project. He manifested that students needed to work more independently and there was no need of planning common discussions among all the working groups. His main request was to let students work independently and ask them to present their finding at the end of the course. Reactions that were on an opposite sense than the ones expressed by the Danish lecturer, who expressed that the activity was too open and too less guided for students. We can say that these reactions corresponded to their spontaneous teaching models that both lecturers implicitly defended. In this second implementation, it shared traits of a *modernist* teaching model (Gascón, 2001), by considering knowledge construction as an individual process, also private. That is why the lecturer preferred not planning any teaching device where to share and collectively talk about the modelling work developed, and where to question, debate and agree about the questions, tools and strategies to follow along the modelling process. As the course organisation at the UAB showed, each team was supposed to work most of the time independently in their project, and it was not until the end of the course when they explained their results. We could observe several inconveniences, linked to important constraints, which were more evident in the following courses when the lecturers planned short modelling activities as complement to the working group project of the course. First, students showed a lack of terminology and of a common discourse (shared with lecturers) to talk and write about the modelling activity developed. Second, the main outcome from the students modelling work was their final presentation of the project at the end of the course. It was delivered as a report that mostly contained the final models and models simulation, as if all the intermediate modelling work may remain in the private space of each group. Consequently, most of final reports showed a poor progression of the models considered and of the tools to contrast and validate them.

CONCLUSIONS AND DISCUSSION

It has to be highlighted that the two adaptations of the SRP presented in this paper were done under advantageous conditions. First, it was experienced with students of the Mathematics degree who were taking a course on mathematical modelling and with lecturers who are experts on modelling. Second, in both cases, the schedule and programme of the course were flexible and we had longer sessions (2-hour sessions two or three times per week) than the prevailing university settings use to offer.

Nevertheless, one could think that we may detect similar conditions and constraints in these two university setting, but it is important to see how different institutions established different relations with the knowledge at the stake, in this case, with the teaching of mathematics modelling. Then, for example, some conditions that appear in the first implementation can become strong constraint for the second one. For instance, it was the case of the necessity of sharing a common discourse to talk about and analyse modelling practices, which was an important condition underlined in the first implementation, becoming a constraints in the second one.

But, if we move away from these "optimal" university conditions, do we find similar constraints? Which of them are sensitive to be surmounted? How to overcome some of the most important constraints? To face these questions, and follow enquiring into the institutional relativity of the conditions favouring and the constraints hindering modelling practices, we proceeded with the third adaptation of the SRP. It was redesigned and later implemented with first-year university students of business and administration degree (4-year programme) in IQS School of Management of Universitat Ramon Llull in Barcelona (Spain) during the entire academic year 2013/14. In this occasion, the Bicing project was extended (called now "Cycling project") to become the central project developed along the three terms of the mathematics first-year course. The SRP was broken into three branches. The one described in this paper (in section 2) was implemented during the third term, only focusing on the first model (1). During the entire course, not only the initial structure of the SRP was extended, but also we pay special attention to which teaching devices and strategies could help to overcome some of the most common constraints for modelling and to create appropriate conditions for modelling and for the SRP. We are in the process of analysing them in depth with the aim of extending our knowledge about the ecology of the SRP and its institutional relativity.

ACKNOWLEDGMENTS

The research leading to these results has received funding from the Spanish R & D project: EDU2015-69865-C3-1-R and EDU2015-64646-P.

REFERENCES

Barquero, B., Bosch, M., & Gascón, J. (2013). The ecological dimension in the teaching of modelling at university level. *Recherches en Didactique des Mathématiques, 33*(3), 307-338.

Barquero, B., Monreal, N., Ruiz-Munzón, N., & Serrano, L. (2018). Linking transmission with inquiry at university level through study and research paths: The case of forecasting Facebook user growth. *International Journal of Research in Undergraduate Mathematics Education, 4*(1), 8-22.

Castela, C. (2004). Institutions influencing mathematics students' private work: a factor of academic achievement. *Educational Studies in Mathematics, 57*(1), 33-63.

Doerr, H. & Lesh, R. (2011). Models and Modelling Perspectives on Teaching and Learning Mathematics in the Twenty-First Century. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in Teaching and Learning of Mathematical Modelling* (pp. 247-268). Dordrecht, The Netherlands: Springer.

Chevallard, Y. (2002). Organiser l'étude 3. Ecologie & régulation [Organizing the study 3. Ecology & regulation]. In J.-L. Dorier, M. Artaud, R. Berthelot, & R. Floris (Eds.), *Actes de la XIe École d'Été de Didactique des Mathématiques* (pp. 41-56). Grenoble, France: La Pensée Sauvage.

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In M. Bosch (Ed.), *Proceedings of the 4th Conference of the European Society for Research in Mathematics Education* (pp. 21-30). Barcelona, Spain: Fundemi-IQS.

Chevallard, Y. (2015). Teaching Mathematics in Tomorrow's Society: A Case for an Oncoming Counter Paradigm. In S.J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173-187). Dordrecht: Springer.

Kaiser, G. & Maaβ, K. (2007). Modeling in lower secondary mathematics classroom – problems and opportunities. In W. Blum, P.L. Galbraith P. L., H. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 99-107). New York, NY: Springer.

Sierra, T. (2006). *Lo matemático en el diseño y análisis de organizaciones didácticas. Los sistemas de numeración y la medida de magnitudes.* PhD dissertation. Departamento de Didáctica de las Matemáticas, Facultad de Educación, Universidad Complutense de Madrid, Spain.

Winsløw, C., Matheron, Y., & Mercier, A. (2013). Study and research courses as an epistemological model for didactics. *Educational Studies in Mathematics, 83*(2), 267-284.