Gas kinetics in galactic disk. Why we do not need a dark matter to explain rotation curves of spirals?

Anton A. Lipovka

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Lipovka A.A.
Department of Research for Physics, Sonora University, 83000, Hermosillo, Sonora, México

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Abstract

In present paper, we analyze how the physical properties of gas affect the Rotation Curve (RC) of a spiral galaxy. It is shown, that the observed non-Keplerian RCs measured for outer part of disks, and the observed radial gas distribution are closely related by the diffusion equation, which clearly indicates that no additional mass (Dark Matter) is need. It is stressed, that while the inner part of the RC is subject of the Kepler law, for the correct description of the outer part of the RC, the collisional property of the gas should also be taken into account.

To confirm this fact we suggest both quantitative estimations and exact calculation to show how the outer part of the RC is related with the gas density of the galactic disk. We argue that the hydrodynamic approach is not applicable for the correct modeling of large-scale gas kinematics of the galactic disk and more general diffusion equations should be used. From our result it follows, that if the gas density is high enough (more than $10^{-5}~cm^{-3}$, the RC for the outer part of the disk is formed by "wind tails" of gas.

Proposed calculations are based on solving both the Kepler's and the Fick equations and was carried out for two edge-on galaxies NGC7331 and NGC3198, for which the precise measurements of the gas column densities and RCs are available. An excellent coincidence between the measured column density of gas and that calculated from observed RCs is obtained.

On the basis of the obtained result, we calculate the total masses of the NGC7331 and NGC3198. They consist $32.5 \times 10^{10}~M_sun$ and $7.3 \times 10^{10}~M_sun$ respectively. Consequences for cosmology are discussed.

Keywords: Dark Matter; Rotation Curves; Gravitational Potential; Mass of spiral galaxy; gas kinetics; Mestel's disk.

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1 Introduction

Difficulties in explaining the kinematics of celestial objects within the framework of Kepler's law were first noted by James Jeans and Jacobus Kapteyn in 1922 and then confirmed by Jaan Oort in 1932 and by Fritz Zwicky in 1933. To solve the problem, it was assumed that there is some unobservable, invisible mass affecting the kinematics of the observed objects. It should be noted that these observations were carried out in optics and therefore dark matter was the only way out of this situation. Later, when radio astronomy methods were developed, it became possible to observe the galactic disks in the radio wavelength range as well. It turned out that the large-scale gas motions may also not obey the Kepler's law, and it was quite logical to assume that we are dealing with all the same dark matter in this case. For more than 90 years, the DM nature has been widely discussed by both astronomers and physicists, but no significant progress in this direction was achieved.

As is known, the need for the DM concept was mainly due to two problems: 1) Observed non-Keplerian Rotation Curves (RC)s of spiral galaxies and 2) The presence of additional invisible mass in clusters of galaxies, leading to the observed gravitational bounding of clusters and also to anomalously large gravitational lensing produced by the clusters. However, recently a significant progress has been achieved in finding of the missing mass in clusters, reported by Kovács et al. (2018). Authors argue that the missing baryons reside in large-scale filaments in the form of warm-hot intergalactic medium. It should also be added here that Biernaux, Magain and Hauret (2017), when processing lensed images, showed that neglecting the diffuse lensed signal leads to a significant overestimation of the half-light radius, and therefore to an overestimation of the lensing mass value. For these reasons, it can be recognized that the second item (excessive masses of the clusters) loses its urgency, whereas the RCs of spiral galaxies remains the most intriguing manifestation of the DM.

But the situation with DM in the disks of S-type galaxies is even more difficult, since recently more accurate observational data was published, that sheds light on the dark matter properties. Namely, it was shown that there is a significant correlation between the features of the galactic RC and corresponding spiral structure of the baryonic component: "The dark and baryonic mass are strongly coupled" (Mc Gaugh, Lelli & Schombert 2017; see also Sancisi 2004; McGaugh 2004; Möller & Noordermeer 2006). It is difficult (if at all possible) to realize this coupling within the framework of the conventional DM paradigm in which the DM is coupled with baryonic component by gravitation only, and the distribution of the DM is described by spherically symmetric functions obtained from numerical simulations discussed for example by Navarro, Frenk & White (1996); Merritt, et al. (2006); Katz, et al. (2017); Di Cintio, et al. (2014a) and Di Cintio et al. (2014b).

Moreover, on the one hand, as it was shown by Kroupa, Pawlowski and Milgrom (2012), cosmological models based on warm or cold DM are not able to explain observed regularities in the properties of dwarf galaxies. On the other hand, last year the rotation curves for highly redshifted galaxies were reported and it was clearly shown that a large fraction of massive highly redshifted galaxies are actually strongly baryon-dominated (see Genzel et al. 2017, Lang et al. 2017, and references therein). These data contradict the generally accepted scenario of galaxies formation on the DM halos.

It should be mentioned here also that reported discrepancies between values of the Hubble constant observed at early and late cosmological time (Verde, Treu, Riess 2019) clearly indicate a crisis of the ΛCDM model (Riess 2020). This fact may require a revision of the ΛCDM model.

Search for DM in laboratories is also unsuccessful despite the unprecedented efforts of many international collaborations. The FERMI experiment designed to search for annihilation of DM and anti-DM clearly shows negative result announced by Albert, et al. (2017). As part of the XENON collaboration, the radioactive decay of xenon-124 due to double-electron capture, which has a half-life of 1.8×10^{22} years, was detected (this indicates the highest sensitivity of the method) but no signs of dark matter were found (Aprile E. et al. 2019). SENSEI collaboration reports of world-leading constraints on dark matter – electron scattering for masses between 500 keV and 5 MeV (see Abramoff et al. 2019).

Summarizing, the unsatisfactory situation with the current explanation of the rotation curves of spiral galaxies becomes obvious. All mentioned above clearly indicate a serious problem with the naive simulation of the spiral galaxy dynamics and the mass distribution, based only on the assumption of the dominant role of gravitational interaction. Moreover, it suggests the need for revision of actually used models and argues that more accurate modeling of the galactic baryon component and adequate consideration of all reasonable physical effects are strongly required. At present, such simulations of the density distribution are performed on the basis of the assumption of the overwhelming gravitational force domination (see papers mentioned before). In this case, based on hydrodynamic simulations, it is concluded that the effect of gas kinetics on the rotation curves formation is negligible. Thus, the properties of the gas were not taken into account properly when the quasi-stationary structure of the gas disk was modeled. By taking into account that the rotation curves for the most important - the outer part of the disk are observed mainly in the molecular lines and 21 cm line of neutral hydrogen, it is became clear that the influence of collisions of the hydrogen atoms and ions on the formation of the stationary gas fluxes, should at least be correctly assessed.

As far as we know, such an attempt (to include the gas properties into consideration) was made by Mestel (1963). He proposed a toy model that includes not only the gravitational interaction, but also some general physical properties of the gas that forms the disk. Despite the roughness of the model of a homogeneous isothermal self-gravitating disk, Mestel managed to obtain a solution characterized by flat rotation curves. The toy model of Mestel was considered by Jalali & Abolghasemi 2002, and recently by Schultz (2012), who showed that to form flat rotation curves in Mestel's disk, much smaller masses are needed than previously thought.

It is necessary to emphasize here the fact that Mestel disks with flat rotation curves are observed in the protoplanetary disks where rotation is not Keplerian outside of the inner few AU in spite of the absence of the DM inside it (see Yen et al. (2015a), Yen et al (2015b), Yen et al (2017)).

Unfortunately, the Mestel's toy model suffers from some significant drawbacks (see Demleitner and Fuchs 2001) and probably for the reason it is not widespread. In this case, a new, more detailed consideration that takes into account all significant physical properties of the gas should be developed.

At the first glance, the hydrodynamic approach mentioned above, could become the standard calculation method in this case. But it hardly can be applied because of two reasons.

1) We are not interested in local small scale movements, on the contrary, we need to calculate the global RC of external part of the quasi-stationary galactic disk.

2) Hydrodynamic approach can not be applied because of the extremely low density of gas resulting in the significant free path lengths, that leads to a violation of the conditions of the hydrodynamic applicability $L_k >> l_{fp}$, which must be superimposed to integrate the kinetics equations when we derive the equations of hydrodynamics. Here L_k is the characteristic size of the hydrodynamic calculations (corresponds to the size of step of the computation grid), and l_{fp} is the mean free path of the particle. For example the typical density for outer parts of disk is $10^{-3} - 10^{-4} cm^{-3}$ leads to $l_{fp} = 3 - 30pc$.

For this reason, the hydrodynamic approach becomes inappropriate, since the characteristic lengths L_k on which the values (density, temperature, pressure etc.) are changed considerably, become comparable or even less than l_{fp}.

The only reliable and not cumbersome method can be obtained directly from the kinetic equations the same way as it was made for hydrodynamic equations. In our paper we are going this way.

Namely, the kinetic equations should be first averaged over the volume in order to avoid the limitations imposed on the applicability of hydrodynamic equations, and only after such a procedure they can be integrated over the momentum. Doing so we obtain diffusion equation with modified diffusion coefficient and evaluate the gas kinetic contribution to the RC formation. We prove that to model a disk of a spiral galaxy, the gas kinetics should be taken into account by correct way, because if the influence of the gas kinetics on the RC formation is neglected, it will leads to a completely incorrect (pure Keplerian) model of the galactic disk and, as a consequence, to a wrong estimation of the gravitating masses and their distribution. We show that the solution of the Fick equations strongly implies that the rotation curve of the gas does depend on the gas density. Thus, if such a dependence is observed, this will be direct evidence of the exceptional role of gas kinetics in the formation of the rotation curves of galactic disks. With the example of two edge-on galaxies NGC7331 and NGC3198, we argue that the observed gas distributions do correspond to the rotation curves and they are interrelated by the Fick equations as it should be. Therefore, we show that it is the gas kinetics, that dominates in the formation of the rotation curves of spiral galaxies at large distances. We choose these two edge - on galaxies to minimize the influence of the 3D structure of galaxy on the column density, since we are interested in the disk component only.

Thus, we can conclude that the deviation of the observed rotation curves of spiral galaxies from the Kepler's law can be easily explained if the kinetics of gas is correctly taken into account. In fact the RCs at large distances from the center are just the wind tails of the baryon gas that follows the preceding baryon matter in the case if the gas obeys the conventional laws of gas kinetics. For this reason we do not need the DM concept to explain rotation curves of spiral galaxies.

It should be noted here that a wind in galaxies is not a new concept. For example a starburst-driven galactic wind in starburst galaxies is a well-known phenomenon that is widely discussed in the literature, see for example Jones et al. (2019), Sharp & Bland-Hawthorn (2010), Rekola et al. (2005) and references therein. However, in the case of large-scale movements and extremely low gas densities in outer part of disks that we are interested in, the standard hydrodynamic approach cannot be applied because of the limitations mentioned above. Thus, the wind which forms rotation curves, must be described in a different way.

The article is organized as follows:

In the second section a preliminary estimates are made for the main gas parameters in the disk of a spiral galaxy. From the evaluations obtained here one can see that the gas kinetics plays a dominant role in the formation of the wind tails in the media out of equilibrium. But if so, the dependence of the outer part of RC on the gas density should be clearly manifested. The most interesting and convenient case for integration, corresponds to the constant RC for outer part of the disk. For this reason in the third part we obtain differential equations that describe the gas dynamics. By using the obtained diffusion equations we calculate analytically the gas density as a function of distance from the center of a galaxy, for the outer disk of the spiral galaxy in the case of constant RC.

While solving the diffusion equations gives us the volume abundance of the gas, from observations we can only get the column density along the disk. For this reason in the part 4 we calculate the column density to compare it with the observed one. As an example we consider two edge-on S-type galaxies with measured RCs and optical depth in 21 cm., as function of distance R from the center of the galaxy. We show that the results obtained with the Fick equations, are in excellent agreement with the observational data. In conclusion the main results of the paper are summarized and some important consequences are discussed.

2 Preliminary estimates.

As it was mentioned above, there are two very different components of galaxy population: the stars and the gas, which are used to measure the RC of galaxy in optics and in radio respectively. The first component is driven only by gravitation potential, whereas to describe the second one we should take into account collisions and the gas kinetics should be involved in the consideration in the right way.

To confirm this fact, let's evaluate some parameters of the gas. A rough estimate of the mean free path time for a hydrogen atom $t_{fp} = (N\sigma V_t)^{-1}$ (here N is the density of the gas in cm^{-3} , σ is cross-section for elastic collision and V_t is the mean thermal velocity of the atom) gives $t_{fp} \approx 1.3 \cdot 10^{10}/N$ (sec) $= 4.1 \cdot 10^2/N$ (yrs). For the typical HI density outside of the R_{25} : $N \approx 10^{-3} - 10^{-4}$ we obtain $t_{fp} \approx 4.1 \cdot (10^5 - 10^6)$ yrs.

However, as we know, the intergalactic gas, as well as the hot gas component of the disk of a spiral galaxy is ionized. For this reason, we should estimate the mean free path of the proton. The free path length (see Lang 1974) is:

$$l^*_{fp} = \frac{m^2 V^4_{rms}}{z_1^2 z_2^2 N_e e^4 \ln\Lambda} \approx \frac{3.2 \cdot 10^6 T^2}{z_1^2 z_2^2 N_e \ln\Lambda}, \tag{1}$$

where $z_1 e$, $z_2 e$ are charges of two interacting particles (for the proton and electron we have $z_1 = z_2 = 1$) and N_e is the electron density

$$\Lambda = 1.3 \cdot 10^4 \sqrt{\frac{T^3}{N_e}} \approx 2 \cdot 10^{11}, \tag{2}$$

so $\ln\Lambda = 26$, and the proton's mean free path is $l^*_{fp} = 10^{15} - 10^{16} cm$ for the temperature $T = 3000K$. In this case the mean free path time can be evaluated as $t^*_{fp} = l^*_{fp}/V_t = 10^9 - 10^{10}\,\mathrm{sec} = 30 - 300$ yrs. As one can see these time (t_{fp} or t^*_{fp}) are much smaller than the characteristic time of life of the galaxy, so the collisions must be taken into account.

Thus, one can see that for description of the gas located in the outer $R > R_{25} \gtrsim R_0$ part of galactic disk, the complete gas dynamics equations should be used to explain the observed RCs. Here R_0 denotes a distance to the transition zone at which the contribution of the gas kinetics in formation of wind tails (and hence RCs) begins to dominate, if compared with that caused by Kepler's law and R_{25} is the radius at which the surface brightness of the spiral galaxy falls to 25 $mag\ arcsec^{-2}$ in B-band.

Now let's make another estimation to answer the question : Will the gas be able to follow the underlying falling baryon matter to form the wind tails?. From observations we know that baryon matter of a S-type galaxy moves along a spiral (note that a galaxy is not a stationary object. it has it's beginning, it has the end, and it evolves over time, consuming intergalactic gas).

In this case we can imagine the underlying baryonic matter as a piston (baryon matter in inner $R < R_0$ part of galaxy's disk, the Kepler motion of which is completely due to the inner baryonic mass). It moves in the spiral tunnel with ideal walls (we can apply the homogeneous boundary conditions in this particular case), and is followed by the HI gas (here we will not consider the processes of star formation, that dilute the gas component, but we only note here that accounting for such processes will increase the effects under discussion). The mean acceleration of the "piston" for typical galaxy can be roughly evaluated as $< w >= \Delta V/\Delta t = (200\ Km/s)\ /\ (10^9\ yrs) = 10^{-9}\ (cm/s^2)$. By taking into account the evaluation of t_{fp} (or t^*_{fp}) made before, we can estimate the variation of the piston's velocity ΔV during the mean free path time of the hydrogen atom. Even in the worst case of t_{fp} for neutral component, we have $\Delta V =< w > t_{fp} = 10^{-9} \cdot 2 \cdot 10^{10}/N = 20/N$ (cm/s). For typical density $N = 10^{-3}$ (cm^{-3}) we obtain $\Delta V = 2 \cdot 10^4 (cm/s) << V_t \approx 10^6 (cm/s)$. So one can conclude that even the neutral gas will follow the "piston" if the gas density is high enough: $N(cm^{-3}) \geqslant 2 \cdot 10^{-5} \cdot (10^6/V_t)$. We stress here that it is important result which can qualitatively explain the great variety of the RCs shapes because of their dependence on the gas density. From this estimate it can be seen that when the density is small ($N(cm^{-3}) < 2 \cdot 10^{-5} \cdot (10^6/V_t)$), the wind tails will not be formed and the corresponding RC will decrease to coinside with Keplerian one.

These were rather crude assessments, suggested here to show simplistically the physics of the processes under diskussion. To conclude this part we would like to stress that even in consequence with these simple estimations, one can see that the gas, driven by collisions, will follow easily the underlying baryon matter. Actually the gas under consideration forms the wind tail which is rigidly follows the underlying baryon matter which, in turn, is driven mainly by the gravity at distances $R < R_0$. This way the absence of RC of S-type galaxies in early universe, reported by Genzel et al. (2017), can be explained easily. Namely, rough estimate of distance over which the wind tail (or, the same RC) will spread is $t \cdot V_t \approx 10^{10} yrs \cdot 3 \cdot 10^7 \cdot 10^6 \approx 3 \cdot 10^{23} (cm) = 100$ kpc. This trivial evaluation clearly shows why the RC measured with HI line are seen in our epoch, but can not be observed in early universe, characterized by the time $t < 10^{10}$ yrs., as it was recently observed and reported by Genzel et al. (2017).

3 Gas density as a function of distance for constant RC

At present, it is believed that collisions in the gas of a galactic disk can be neglected even in the case of small galaxies (see, for example, Dalcanton & Stilp 2010) and for large spirals it simply does not matter. Such conclusions are based on hydrodynamic simulation and the assumption that the continuity equation, the Euler equation, and hence the Bernoulli equation, are always satisfied. However, this is not so in the case of the highly rarefied gas, and therefore it is hardly possible to trust such calculations. Unfortunately, this error is very common in hydrodynamic calculations applied to galactic disks, the number of published works is huge and therefore we will mention here only a couple of articles as an illustration of the problem under discussion: Joung et al. (2009) ; Rosdahl et al. (2017). For example, the results of simulations presented by Joung et al. (2009) clearly show that the characteristic scale of the inhomogeneities, on which the gas parameters changes significantly, is $L_k = 1 - 10$ pc, while the mean free path of a particle at a density of 10^{-3} (cm^{-3}) is $l_{fp} = 3 - 30$ pc. As one can see, in this case the main condition for the applicability of the hydrodynamic (HD) approach (the Knudsen number $Kn << 1$) is not fulfilled and HD can not be applyed. Let us consider the calculation method in more detail.

It is well known that the hydrodynamic equations are obtained by integrating kinetic equations over momentum. Kinetic equation for one type of particles is:

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x^\alpha}(v^\alpha f) = Stf. \tag{3}$$

Here f is the distribution function and Stf stands for the collision integral.

Integrating over momentum p under assumption that variations of all parameters (density, velocity, temperature, etc.) are small at the mean free path length l_{fp} , i.e. the characteristic length $L_k >> l_{fp}$ (or, which is the same - the Knudsen number $Kn = l_{fp}/L_k << 1$) , we can evaluate:

$$\int \left(\frac{\partial}{\partial x^\alpha}(v^\alpha f) \right) d^3p \approx \frac{\partial}{\partial x^\alpha} \int (v^\alpha f) d^3p,$$

and obtain the equation of continuity:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x^\alpha}(V^\alpha n) = 0. \tag{4}$$

Other hydrodynamic equations can be obtained by the same way. We will not dwell on this now, referring the reader to the standard textbooks on physical kinetics, but note here that all these equations also will suffer of the same restrictions mentioned above:

$$L_k >> l_{fp}, \tag{5}$$

which imposes some restrictions on the hydrodynamic equations applicability (see Landau & Lifshitz v.X). Unfortunatelly many authors who investigate numerically the kinematics of the galactic disks, do not bother to verify that the conditions for the applicability of the equations of hydrodynamics (5) are satisfied and violate these restrictions. Thus, the results obtained by them are hardly credible.

To eliminate this restriction (5), we start with the same kinetic equation (3), as it takes place in the case of hydrodynamics. To obtain eq.(4) we considered condition (5) to be fulfilled. But if this is not the case, i.e. $(v^\alpha f)$ is changed significantly within the scale l_{fp} (this takes place in the case of an extremely rarefied gas, when the distribution function is not well defined and its derivative strictly speaking does not exist), then the only we can do - is average over a volume Ω (recall that we are not interested in small-scale gas motions and therefore we can average) in order to redefine the function $(v^\alpha f)$.

Now we rewrite (3) as

$$\int \int_\Omega \left(\frac{\partial f}{\partial t} + \frac{\partial}{\partial x^\alpha}(v^\alpha f) \right) d^3x d^3p = 0, \tag{6}$$

and integrate it. First term gives the variation of the total number of particles N_{tot} in the volume of integration Ω.

By using the Gauss theorem, the second term can be transformed to $\Delta S^{\alpha}\partial N/\partial x^{\alpha}$ (here ΔS^{α} is the surface area of the integration volume, orthogonal to the particle flow, and N is the averaged density of the particles in the volume Ω, measured in cm^{-3}), so finally we have (see Appendix for details):

$$\frac{dN_{tot}}{dt} = -D\Delta S^{\alpha}\frac{\partial N}{\partial x^{\alpha}}. \tag{7}$$

In this equation we introduce the modified diffusion coefficient $D = V\Delta l$, where V is the bulk velocity of gas in Ω, and Δl is the characteristic size of the integration volume ($\Delta l >> l_{fp}$). This is well-known diffusion equation.

Summarizing, we know that the hydrodynamic equations can not be applied to simulate the dynamics of the rarefied gas. Moreover, we do not need this approach to calculate RC in the case of an established quasi-stationary solution. By integrating (3) over a volume and making the problem insensitive to the small-scale inhomogeneities, we obtain the diffusion equation (7) suitable to describe the large-scale movements of the rarefied gas in outer part of the disk. In polar coordinates, expression (7) can be written as:

$$V_{d\shortparallel} = -\frac{D}{N}\frac{\partial N}{\partial R}, \qquad V_{d\perp} = -\frac{D}{N}\frac{1}{R}\frac{\partial N}{\partial \varphi}. \tag{8}$$

Now, we denote by R_0 the distance at which the Kepler motion ends and the "unphysical" behavior of the baryonic matter (explained by introducing sophysticatedly distributed dark matter) begins. Then the Keppler's speed at this distance is:

$$V_{K0} = \sqrt{\frac{MG}{R_0}}. \tag{9}$$

Consider the movement of a certain part of the gas. Let $N(R,\varphi)$ be a smooth, parametrizied function of the coordinate R and φ. In this case, we can write $R = R(t)$ and $\varphi = \varphi(t)$, where t is a parameter, $dR = dR/dt \cdot dt$ and $d\varphi = d\varphi/dt \cdot dt$. Taking into account that $Rd\varphi/dt = V_{\perp}^{tot}$, we obtain:

$$\frac{1}{R}\frac{\partial N}{\partial \varphi} = -\frac{V_{d\shortparallel}}{V_{\perp}^{tot}}\frac{\partial N}{\partial R}. \tag{10}$$

For this reason from eqs. (8) it follows:

$$V_{d\perp}V_{\perp}^{tot} = V_{d\shortparallel}^{2}. \tag{11}$$

or

$$\frac{D}{n}\frac{1}{R_0}\frac{\partial n}{\partial r} = -\sqrt{V_{d\perp}V_{\perp}^{tot}}), \tag{12}$$

where we introduce $n = N/N_0$, $r = R/R_0$, and $N_0 = N(R_0)$.

Now calculate corresponding gas density distribution in order to compare it with the observed distribution. By other words, we are interested in the question: "which HI column density function corresponds to the case of the constant rotation curve of baryon matter in absence of DM for an S-type galaxy (in the case when the rotation curve is just the wind tail, the movement of which is determined only by baryonic matter, without involving dark matter in consideration)?" If the calculated density distribution coincides with the observed one, it will be a serious argument against the presence of dark matter in the disks of the spiral galaxies. Consider this problem in more detail. In order to facilitate calculations, we consider a galaxy with a flat rotation curvewe, i.e. $V_{\perp}^{tot} = (V_{d\perp} + V_{K0}) = const.$and $\sqrt{V_{d\perp} V_{K0}} = const.$ By taking into account the fact that $D = const$ for very rarefied gas and integrating (12) we obtain:

$$n = n_0 \exp\left\{ -\frac{R_0\sqrt{V_{d\perp} V_{K0}}}{D} r \right\}, \tag{13}$$

where

$$n_0 = \exp\left\{ \frac{R_0\sqrt{V_{d\perp} V_{K0}}}{D} \right\}. \tag{14}$$

As one can see this density distribution depends exponentially on the distance r. Unfortunately we are not able to realize a direct measurement of the gas density of the galactic disk. The only we have are the observed column densities measured in 21 cm., so now we are going to calculate the column density formed by distribution (13).

4 Column density

By definition the column density is

$$N_L = 2\int_0^L N dl, \tag{15}$$

where N is density function. By taking into account that $l^2 = R^2 - \rho^2$, the eq. (15) can be rewritten as:

$$N_L = 2N_0 R_0 \int_{r=\rho/R_0}^{r_{\max}} \frac{nrdr}{\sqrt{r^2 - \frac{\rho^2}{R_0^2}}}. \tag{16}$$

This integral can be estimated if we take into account that the density n decreases exponentially with distance r.

Consider the distance Δl at which the density drops by about 10 times. A trivial estimate gives $\Delta R/R_0 \approx 2/\varkappa$, where

$$\varkappa = \frac{R_0\sqrt{V_{d\perp} V_{K0}}}{D}. \tag{17}$$

Straightforward calculation gives the following estimate for the integral (m.16):

$$N_L(\frac{\rho}{R_0}) \approx \frac{N_0 R_0}{\varkappa} e^{-\varkappa(\frac{\rho}{R_0}-1)} \sqrt{\varkappa \frac{\rho}{R_0} + 2}. \tag{18}$$

The calculated column density (18) can be used to fit the observed one suggested by Begeman (1987, 1989) and Bigiel et al. (2010) for NGC7331 (fig.1) and NGC3198 (fig.2).

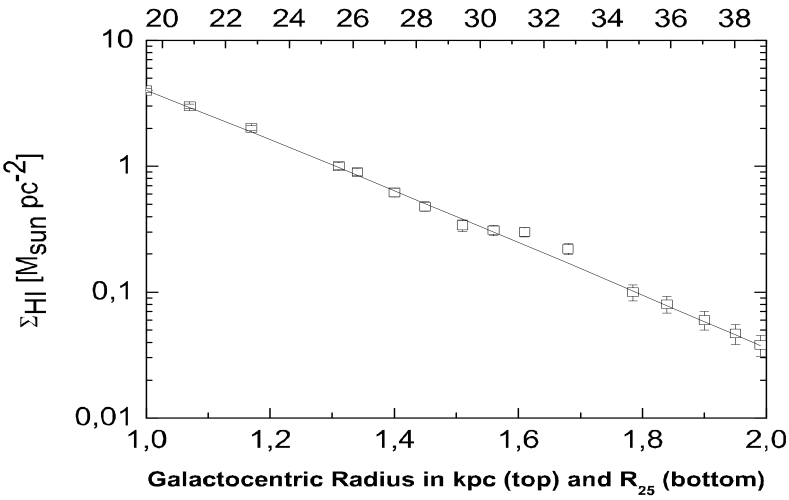

Figure 1: Measured (squares) suggested by Begeman (1987); Bigiel et al. (2010) and calculated with (24) (solid line) HI column density for NGC7331.

These two galaxies were chosen because they are seen edge on by observer. Due to this circumstance, in this case there is no need to take into account the angle of inclination of galaxy, that simplifies the modeling.

As can be seen from the fig.1 and fig.2, the calculated (in assumption of flat RC) column densities perfectly fit the observed ones for very different galaxies, characterized by different mass and slopes of the column density function. So we can conclude that RC at large distances $R > R_0$ are formed by wind tails of gas which obeys the diffusion equation (7). the obtained relation connecting the velocity and density through equation (7) clearly indicates that there is no need to introduce dark matter into the model and we do not need dark matter to explain the rotation curves of spiral galaxies.

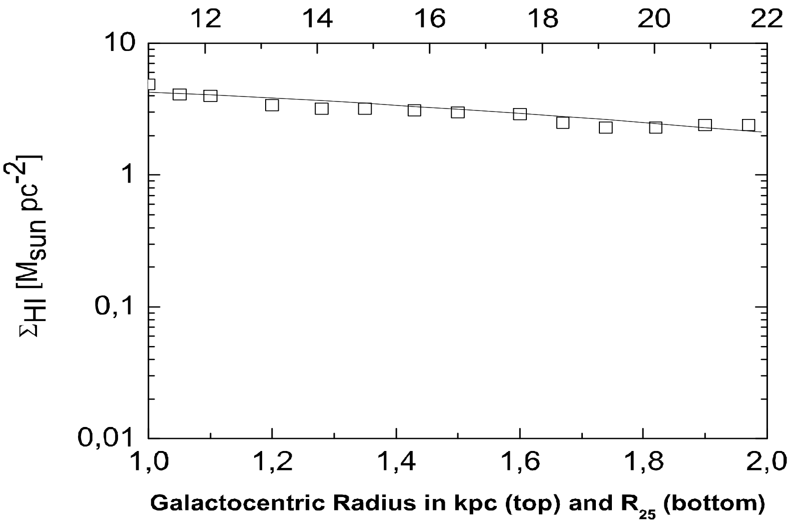

Figure 2: Measured (squares) suggested by Begeman (1987); Bigiel et al. (2010) and calculated with (24) (solid line) HI column density for NGC3198.

Now (by taking into account that RC consists of two parts: 1) $R < R_0$ where the gravitation dominates, and 2) $R > R_0$, where contribution of gas kinetics becomes dominant) we can estimate the masses of two galaxies mentioned above by using their measured RC suggested by Begeman (1987), Begeman (1989) and de Blok et al. (2008), and previously obtained model for the baryon mass distribution (see Lipovka 2018). The coefficients α_k^* and β_k^* we immediately find from approximation of the pure baryonic RCs ($R < R_0$) for these two galaxies by using expansion suggested by Lipovka (2018):

$$V_\perp^2 = \frac{\eta}{R} \sum_k \frac{\alpha_k^*}{\beta_k^*} \left[1 - \frac{\frac{3}{2}\beta_k^* R^2 + 1}{(\beta_k^* R^2 + 1)^{3/2}} \right], \tag{19}$$

where the constant $\eta = 2\pi G 10^{10} M_\odot$.

Figures 3 and 4 demonstrate results of such approximation for NGC7331 and NGC3198 respectively.

The thick straight horizontal line (the constant part of the RC) at the figures 3 and 4 corresponds to the wind tails (rotation curves) formed by gas which obeys the diffusion equations and has column densities shown at figures 1 and

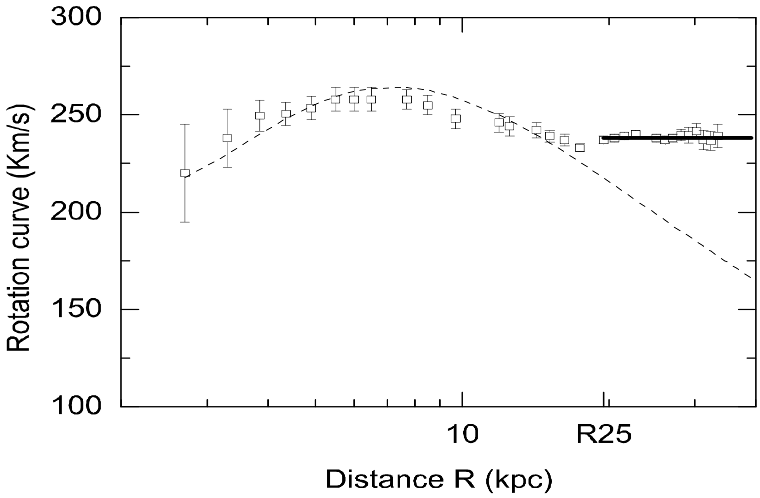

Figure 3: Measured (squares) as suggested by Begeman (1987, 1989), de Blok et al. (2008), and calculated with the model of Lipovka (2018) (dashed line) rotation curve for NGC7331. Wind tail (external part of RC) that corresponds to the HI distribution (see fig.1) is shown by the horizontal bold solid line.The length of the line exactly matches the size of the fig.1.

2. As it can be seen, the wind tails (constant RCs) extend exactly to the distance where the column density function has the exponential form (18).

Obtained coefficients for NGC7331 are $\alpha_1^* = 0.333$, $\beta_1^* = 0.077$, $\alpha_2^* = 7.7$, $\beta_2^* = 29.9$, and for NGC3198 we find $\alpha_1^* = 0.2$, $\beta_1^* = 0.26$, $\alpha_2^* = 0.55$, $\beta_2^* = 6.0$. Now the masses of these galaxies can be obtained immediately with relation for the total baryon mass suggested by Lipovka (2018). In this case for NGC7331 we find $M_{7331} = 32.5 \cdot 10^{10} M_\odot$ and for NGC3198 the total mass is $M_{3198} = 7.3 \cdot 10^{10} M_\odot$.

5 Conclusions and diskussion

In present paper we show that the commonly accepted explanation of the rotation curves of spiral galaxies, based on the naive simulation of the spiral galaxy dynamics and mass distribution in approach of the dominant role of the gravitational interaction, is not complete and can not be considered as satisfactory.

We argue that the influence of the gas kinetics on the formation of rotation curves is important, i.e. the physical properties of the gas must be taken into account to determine the quasi - stationary structure of the gaseous disk named

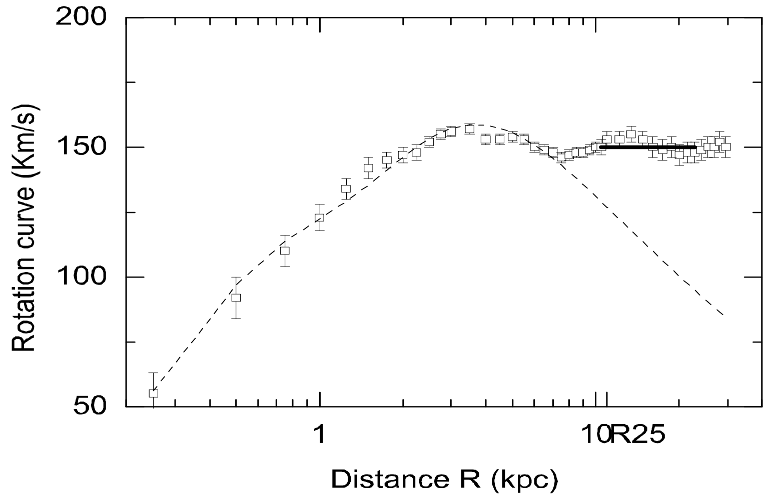

Figure 4: Measured (squares) as suggested by Begeman (1987, 1989), de Blok et al. (2008), and calculated with the model of Lipovka (2018) (dashed line) rotation curve for NGC3198. Wind tail (external part of RC) that corresponds to the HI distribution (see fig.2) is shown by the horizontal bold solid line. The length of the line exactly matches the size of the fig.2.

as rotation curves (recall that the rotation curves for the most intrigue - the outer part of the disk, are observed in the molecular lines and 21 cm. line of neutral hydrogen). Therefore, the influence of collisions of the hydrogen atoms and ions on the formation of the stationary gas fluxes must at least be correctly estimated. In our paper, we suggest such estimations. We show that the solution of the Fick's equations implies that the rotation curve of the gas and the gas density are related by the Fick's equations. Such dependence, if observed, will proves the importance of the gas kinetics in formation of outer part of RC. We consider as an example two edge-on galaxies NGC7331 and NGC3198. It turned out that the observed gas distributions do correspond to the rotation curves and they are related exactly by the Fick's equations. Therefore, we conclude that it is the kinetics of the gas that dominates in the formation of the rotation curves of spiral galaxies at large distances and no DM needs to explain their extended flat RCs.

A couple of words should be said on the rare stars formed in the outer part of the disk. It is known, the RC for outer part of disk is measured not only in 21 cm., or in molecular lines, but sometimes also in optics by using a spectra of rare and young stars that were formed in this region. In this case approximately the same (as the gas has) tangential velocities of remote rare stars that move out of R_0 can be explained as momenta obtained from the gas of which the stars were formed. Elementary estimates show that these young stars in most cases will be bounded, but will move in elliptical orbits.

The main results of the paper can be summarized as follows:

1) It is argued that the hydrodynamic approach (like any other approach based on the hydrodynamic description) is not applicable in the case of a rarefied gas of the outer part of the galaxy disk. For this reason, to describe correctly the observed RCs profile of spiral galaxies, not only gravitation interaction, but also the physical properties of the gas should be taken into account by using correct model.

2) We show that RCs consist of two parts. One (inner part) is formed by collisionless ideal "gas", consisting of stars, and in this case the gravitation interaction dominates, whereas another part (localized in the outer region of disk) is formed mainly by the real gas. In this case, the motion of the gas obeys not only the gravity, but also the gas kinetics, which contribute to the formation of the gas stationary fluxes (and, consequently, to formation of outer part of the observed RC). To model the gas movement, we derive the diffusion equation (first Fick law) with modified diffusion coefficient.

3) On the basis of the Fick's equations, the direct and exact relationship between flat RC and the density function n(R) for the gas is obtained. From the measured rotation curves, we calculate the HI column density as function of distance R for two edge-on spiral galaxies: NGC7331 and NGC3198. The calculated column densities are in excellent agreement with the observed ones.

4) By taking into account the facts proved above, (that the RC consists of two different parts that are governed by Newtonian gravity and the laws of physical kinetics of gases, respectively.) the total masses of two edge-on spiral galaxies are calculated. Our evaluation for the NGC7331 is $M_{7331} = 32.5{\cdot}10^{10} M_{\odot}$ and for NGC3198 the total mass consists $M_{3198} = 7.3{\cdot}10^{10} M_{\odot}$.

In summary, it can be argued that there is no need for the introduction of dark matter to explain the rotational curves of the S-type galaxies. The need for DM arose from the use of an inadequate hydrodynamic model, which, due to initial constraints, cannot be applied to calculate the dynamics of a rarefied gas in the outer regions of galactic disks.

At first glance, the absence of dark matter in nature can dramatically affect cosmological models because the DM is believed to play the key role in formation of the observable structure in the universe. As is known, in the absence of dark matter in the framework of the (pseudo) Riemannian Universe, the cosmological time is not enough for the observed structures to be formed. In this case (in the absence of DM) the only reasonable extension of the existing paradigm, which satisfies the principle of the Occam's razor, is the extension of the (pseudo-) Riemannian geometry to the Finslerian one, that will give the necessary time for the observed structure be formed. Actually the (pseudo-) Riemannian geometry is a very special case of the Finslerian one, and there are no compelling reasons for such a particular quadratic restriction.

Moreover, there are serious arguments in favor of the fact that we live on the Finsler manifold, and not on the Riemannian one.

Firstly, only within the framework of the Finslerian geometry, the cosmological constant appears in a natural way from geometry itself, it has natural explanation and it becomes possible to unify quantum theory and gravity (see Lipovka 2014, Lipovka 2017).

Secondly, on the Finslerian manifold the Planck constant calculated from the first principles (with measured cosmological parameters) coincides with it's experimental value up to the second significant digit, that is, to the measurement errors of cosmological parameters (Lipovka 2017), whereas if it is calculated for the (pseudo-) Riemannian world, we find that the Planck constant differs by factor 3/2 from it's exact value (Cardenas, Lipovka 2019). These are more than serious arguments in favor of the Finsler geometry. If we also add to this the observationally proven lack of dark matter in the early Universe and in clusters of galaxies (see introduction), its absence in the disks of galaxies (this work), then the need to move to the Finslerian world becomes obvious. In this case, of course, the angles of gravitational lensing should also be recalculated using the Finsler metric.

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The author devotes this work to the blessed memory of his teacher of differential and integral calculus S.R. Tikhomirov.

7 Appendix A

We start form the eq. (6) of the paper:

$$\int\int_{\Omega}\left(\frac{\partial f}{\partial t}+\frac{\partial}{\partial x^{\alpha}}(v^{\alpha}f)\right)d^3xd^3p=0, \tag{6}$$

and integrate it. This equation is similar to that we use to obtain the continuity equation (4), but here the averaging over characteristic volume Ω was performed in order to avoid the restrictions $Kn << 1$ that the HD approach suffer of. For this reason eq. (7), obtained from (6) does describe movements of gas, but does not suffer of the restrictions applied to the HD approach, discussed in the manuscript.

Now let us consider transition from (6) to (7) in details.

First term gives the variation of the total number of particles N_{tot} in the volume of integration Ω:

$$\int\int_{\Omega}\frac{\partial f}{\partial t}d^3xd^3p=\frac{\partial N_{tot}}{\partial t}, \tag{A1}$$

Consider second term of eq. (6):

$$\int\int_{\Omega}\left(\frac{\partial}{\partial x^{\alpha}}(v^{\alpha}f)\right)d^{3}xd^{3}p, \tag{A2}$$

By applying the Gauss theorem we obtain:

$$\int\int_{\Omega}\left(\frac{\partial}{\partial x^{\alpha}}(v^{\alpha}f)\right)d^{3}xd^{3}p=\int\int_{\Sigma}(v^{\alpha}f)d^{3}pd\sigma^{\alpha}=\int_{\Sigma}(V^{\alpha}N)d\sigma^{\alpha}, \tag{A3}$$

Where Ω is the volume, Σ is the corresponding surface, $d\sigma^{\alpha}$ is a surface element, V^{α} is the bulk velocity in the volume, and N is the density averaged over Ω.

By taking into account that the velocity V^{α} everywhere on surface $d\sigma^{\alpha}$, is approximately constant and by applying the Gauss theorem, we can write:

$$\int_{\Sigma}(V^{\alpha}N)d\sigma^{\alpha}=V^{\alpha}\int_{\Sigma}Nd\sigma^{\alpha}=V^{\alpha}\int_{\Omega}\frac{\partial N}{\partial x^{\alpha}}d\sigma^{\alpha}dx^{\alpha}, \tag{A4}$$

Now, to obtain the linearized equations we suppose that N is slowly changed function of the coordinate and for this reason in the linear approximation we can evaluate the integral as:

$$V^{\alpha}\int_{\Omega}\frac{\partial N}{\partial x^{\alpha}}d\sigma^{\alpha}dx^{\alpha}=V^{\alpha}\Delta l\Delta S^{\alpha}\frac{\partial N}{\partial x^{\alpha}}=D\Delta S^{\alpha}\frac{\partial N}{\partial x^{\alpha}}, \tag{A5}$$

Where Δl and ΔS^{α} are the characteristic length and corresponding orthogonal surface respectively and the coefficient D stands for $V^{\alpha}\Delta l$. Here $\Delta l >> l_{fp}$ and therefore the restriction (5) is lifted.

Thus the final equation can be written as

$$\frac{\partial N_{tot}}{\partial t}=-D\Delta S^{\alpha}\frac{\partial N}{\partial x^{\alpha}}, \tag{7}$$

Here N_{tot} is the total number of particles in the volume Ω, and N is the averaged density of the particles in the volume.

In the case when we have a diffusion velocity V_{d}^{α} through the boundary surface ΔS^{α}, then the variation of the total number we can write as

$$\frac{\partial N_{tot}}{\partial t}=V_{d}^{\alpha}N\Delta S^{\alpha}, \tag{A6}$$

and (7) becomes

$$V_{d}^{\alpha}=-\frac{D}{N}\frac{\partial N}{\partial x^{\alpha}}, \tag{8}$$

Thus we obtan equations (8).

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